

# Did Gentzen Prove the Consistency of Arithmetic?

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In 1936, Gerhard Gentzen famously gave a proof of the consistency of Peano Arithmetic. There is no disputing that Gentzen provided us with a mathematically valid argument. This talk will address the distinct question of whether Gentzen's result is properly viewed as a proof in the epistemic sense: an argument that can be used to obtain or enhance justification in its conclusion. Although Gentzen himself believed that he had provided a "real vindication" of Peano Arithmetic, many subsequent mathematicians and philosophers have disagreed, on the basis that the proof is epistemically circular or otherwise inert. After gently sketching the outlines of Gentzen's proof, I'll discuss a number of "informal equivalence theses" that seek to equate various formal mathematical systems with intuitive conceptions of mathematical subject-matters. In light of this discussion, I will argue that the truth lies somewhere in between the claims of Gentzen and his critics: although the proof is indeed epistemically non-trivial, it falls short of constituting a real vindication of the consistency of Peano Arithmetic.