THREE LEVELS OF INTENSIONALITY

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Results on axiomatic system of first-order arithmetic or extending first-order arithmetic are often used to argue for philosophical or metalogical claims. Here are some examples:

-The unprovability of certain arithmetical sentences is taken as evidence that Peano arithmetic doesn't prove its own consistency.

-The provability of certain arithmetical sentences is taken as evidence that Peano arithmetic proves the sentence that states its own provability.

-The provability of all sentences in an arithmetical sentences expanded by certain truth-theoretic axioms is taken as evidence that certain (informal) assumptions about truth are inconsistent.

-For many paradoxes, the use of the diagonal lemma in the corresponding inconsistency proofs (e.g. in Yablo's paradox) is taken as evidence that these paradoxes involve self-reference.

For the construction of sentences with metatheoretic content certain assumptions have to be made:

(1) A coding of the expressions of the language in the natural numbers has to be fixed.
(2) Provability and other predicates are assumed to be expressed by certain formulae of the object language.
(3) Self-reference can be obtained in the formal system by providing certain diagonal sentences.

I will argue that neither of the three assumptions is unproblematic and usually involve some more or less arbitrary stipulation, as will be shown by appropriate examples.

The results impinge on the interpretation of the Gödel incompleteness theorems, Löb's theorem, results on axiomatic truth theories and the analysis of self-reference.

The talk is work in progress. However, parts of the talk build on the following two papers:

' Self-Reference in Arithmetic I' (with Albert Visser), *Review of Symbolic Logic* 7 (2014), 671-691

'Self-Reference in Arithmetic II' (with Albert Visser), *Review of Symbolic Logic* 7 (2014), 692-712