ADDING STANDARDNESS TO NONSTANDARD MODELS

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Abstract: It has been know for some time that the theory of initial segments of models of first order arithmetic and theories of second order arithmetic are related. For example, a cut I of M is strong if and only if it, together with second order structure inherited from M, satisfies Friedman's theory ACA_0. This suggests that the study of ω-models of second order arithmetic, and in particular the ω-rule, is related to the study of models of the form (M, ω) where M is a nonstandard model of PA. The main objective of this talk will explain these connections in much more detail.

The talk will touch on one or more of three related questions: what can one interpret or define in (M, ω) that cannot be defined in M ; what is the theory of structures such as (M, ω) ; and what reals are coded in the model (M, ω)?

In terms of definability and interpretability, structures (M, ω) interpret ω-models of second order arithmetic, as already mentioned, and also under certain circumstances define truth predicates for submodels of M. It turns out that a weak interpretation also goes the other way: the theory of the structure (M, ω) is locally interpreted in $(\omega, \mathrm{SSy(M)})$.

In terms of the theory, the Henkin-Orey theorem on ω-logic tells us about the theory of all models (M, ω) (i.e. the statements true in all such models) but tells us little about the theory of any specific model. In fact the theory of (M, ω) depends on structural properties of M that are not first order, and so there is a wide range of possibilities for the theory of (M, ω), even for a given completion T of PA. It is perhaps surprising therefore that given a complete theory T extending PA there is a canonical choice for a theory $Th(M, \omega)$ of some $M \models$ T. More surprisingly, this is not hard to prove. We will discuss a few consequences of this result, and further applications of the truth predicates available in (M, ω) will be given.

In a difficult paper in the JSL, Kanovei characterised the Scott sets $\mathrm{Rep}(M, \omega)$ of subsets of ω that are 0-definable in (M, ω), when M is a model of true arithmetic. A similar characterisation of the standard system $\mathrm{SSy}(M, \omega))$ of (M, ω) (i.e. such definable sets, where parameters are allowed from M) is not known. We will conclude with some results and observations on these standard systems, with some open problems for future work.