SOME MISCELLANEOUS TOPICS FROM THE PROBLEM OF IMPLICIT COMMITMENT IN ACCEPTING SET THEORY

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Gödel's incompleteness theorem indicates that our mathematical thoughts and commitments cannot be exhaustively captured by any single formal system. When we accept a mathematical system, we thereby implicitly commit ourselves to, say, its consistency at the same time. However, by Gödel's theorem, no formal system can prove its own consistency and therefore our commitment to consistency of a mathematical system can never be realized within the system itself.

Not only consistency may be implicitly accepted in one's acceptance of a system. Depending on one's standard or purpose of her "acceptance", she might presupposes and then commits herself to various other desirable properties of a system S in her initial acceptance of S; for example, in addition to consistency, she may require refection principles for S (for the sake of "soundness"), quantification over definable sets in S, and the truth of S, all of which are, however, not realized (more precisely, neither derivable nor expressible) in S itself.

Consequently, we may well argue that, whenever we accept a formal system S, we implicitly commit ourselves to and accordingly accept stronger or richer systems than S in general. Conversely, whenever we justifiably accept a formal system, we can justify our acceptance of certain stronger or richer systems on the same fundamental ground.

The question is: How much more should be and can be accepted on the same fundamental ground by one's initial acceptance of a system? This problem is well-investigated concerning systems of arithmetic by the works of Turing, Kreisel, Feferman, Schmerl, Friedman, Beklemishev, and Strahm (maybe some others). However, not so much attention has been paid to the same problem concerning systems of set theory. I will present some miscellaneous topics and results on this problem particularly for set theory, hoping to be able to inspire a new research area in mathematical logic motivated by this philosophical problem and interests.