HUMAN EFFECTIVE COMPUTABILITY AND ABSOLUTE UNDECIDABILITY

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Abstract: Kreisel [1972] distinguishes the notion of human effective computability from that of algorithmic computability. Roughly, a total function f on the natural numbers is said to be human effectively computable if for any natural number m given in canonical notation, a number n in canonical notation can be found such that it can be proved that $f(m) = n$. Kreisel's notion yields a new variant of Church-Turing's Thesis according to which every human effectively computable function is Turing-computable. Kreisel argues that it is difficult to judge whether this thesis holds because it is not clear exactly which idealisations are involved in the notion of human effective computability.

In this paper, we apply informal rigour (along the lines of Kreisel [1967]) to the notion of human effective computability. To this end, we follow Kreisel's suggestion that human effective computability should be explicated in terms of a notion of provability. More precisely, we suggest that human effective computability can be explicated in terms of the notion of a priori knowability, and that the latter notion can be fruitfully investigated in the formal framework of Epistemic Arithmetic. When this is done, Church-Turing's Thesis for human effective computability is formalised as what is called "Epistemic Church's Thesis" (ECT) in the literature. We then go on to explore an analogue of Gödel's disjunctive thesis for human effective computability: either ECT fails, or there are absolutely undecidable statements of a certain kind. In concluding, we return to Kreisel's question about the idealisations that are appropriately adopted in the investigation of the notion of human effective computability and we investigate whether ECT is true in models that incorporate certain of these idealisations.