HOW ARITHMETIC IS ABOUT NUMBERS
A WITTGENSTEINIAN PERSPECTIVE

FELIX MÜHLHÖLZER

Many philosophers, but also mathematicians, are worried by the following dilemma which I call the aboutness dilemma. It affects every mathematical theory—and maybe even empirical theories—but I will stick to arithmetic alone. Its first horn states that if one gives the aboutness of arithmetic a precise formulation that makes it transparent, as this is done by means of the model-theoretic notion of interpretation, then we cannot catch it uniquely; we are confronted with a multitude of non-intended interpretations that cannot be avoided. The second horn claims that this non-uniqueness can be avoided in the meta-language in which model theory is expressed, and typically expressed in a non-formalized way; but then the aboutness of arithmetic is not made precise and transparent, it rather remains in the dark. So, for short: If the aboutness is clear and precise, it is not unique, and if it is unique, it is neither clear nor precise. This problem is tailor-made for a Wittgensteinian approach. It can be clarified by acknowledging that the interpretations mentioned in the first horn of the dilemma are simply mathematical functions that do not involve any use of the so-called 'signs' that are interpreted, while the second horn concerns "reference", and according to a Wittgensteinian perspective (see PI §10) the notion of "reference" is essentially tied to the use of signs. There is a categorial difference between "interpretation" and "reference", which in the existing literature has been blurred, however. The Wittgensteinian treatment of the dilemma consists in simply accepting the first horn, as asserting a noncontroversial mathematical result, and in correcting the second horn by presenting a perspective on "reference" that is sufficiently clear (even though not precise). The confusion in the literature can be explained by an insufficient appreciation of this perspective coupled with a distorted view of the relation between "interpretation" and "reference".