

NAIVE MODAL SET THEORY

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We explore an alternative approach to set theory that couples a naive conception of set with a notion of mathematical modality. It is well-known that the naive conception of set is inconsistent. In response to the inconsistency of the naive conception, we argue in favour of a modal conception of set. According to this conception, sets are merely possible, or potential, with respect to their members. The basic set existence principle that we articulate is a modal version of the unrestricted comprehension axiom: for any condition P , it is possible that there exists a set of all and only those things that satisfy the condition P . This comprehension principle is formalised in the first-order language of set theory extended with modal operators. Given this modal language of set theory, we explore the following basic proof-theoretic question: What can and cannot be derived from this modal comprehension axiom?