SET PROBABILITIES

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We wish to define what it means for a property A of sets to have a probability of being true of a set y in the set theoretic universe V. We identify properties with their extensions, so that A ranges over all proper and improper classes in V. In other words, we want to develop a theory of the probability of events of the form $A(y)$, where A is a class and the variable y is a *random variable*, ranging over all sets.

We are interested in uniform probability distributions. That is, all singleton events in the sample space V should be equiprobable: the probability of $y = a$ should be the same for every set a. In addition, we want the resulting probability functions to be regular: only properties that do not hold of any set should get probability 0. Also, we want the resulting functions to be total: we want every property to determine a measurable class. Of course this means that the sought-for probability functions cannot be Kolmogorov probability functions. They will turn out —on the construction that is proposed here— to be non-Archimedean class functions.