# PROOF THEORY OF MARTIN-LÖF TYPE THEORY

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*Abstract* The goal of Hilbert's was to show the consistency of strong axiomatizations of mathematics by finitary methods. By Gödel's incompleteness theorem we know that this is not possible. One solution is taken in ordinal theoretic proof theory, where the consistency of theories is shown by transfinite induction over a suitable ordinal notation system. While this provides some evidence of the consistency for medium sized theories, for strong theories a direct insight into the well-foundedness of the ordinal notation system seems at the moment to be very difficult.

One solution is to replace finitary methods by a theory for which we have some evidence into its consistency. The theory which has been developed most in order to serve for such a purpose is Martin-Löf Type Theory.

A revised Hilbertian program then can be carried out in two steps: First the ordinal analysis of a theory as described before is carried out. Then the well-foundedness of the ordinal notation system used is shown in a suitable extension of Martin-Löf Type Theory.

This requires the development of strong extensions of Martin-Löf type theory such that we have some evidence of their consistency. The application of this approach is the development of proof theoretically strong data structures, which can then be used in general programming.

In this talk we describe some steps in this program: Martin-Löf Type Theory with W-type and one universe which proves consistency of Kripke Platek set theory with one recursively inaccessible; the Mahlo universe, which proves consistency of Kripke Platek set theory with one recursively Mahlo ordinal; and an extension of type theory by the principle of $\Pi_3$-reflection.

We will discuss the controversy about the Mahlo universe, and how one can overcome this problem when switching to a framework of explicit mathematics. This shows that a framework involving partial functions might be more suitable for proving the consistency of proof theoretically strong theories.