CLASS FORMS OF CANTOR'S THEOREM

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Cantor's theorem is generally glossed as the claim that a set has strictly more subsets than elements, and it is an immediate consequence of either of two statements. One is the statement that there is no function from a set A onto the power set of A (Not Onto), and the other is the statement that there is no one-one function from the power set of A into A (Not 1-1). Paul Bernays explained how to regiment a class form of Not Onto in a predicative theory of classes, but one may wonder how to do the same for Not 1-1. We look at a non-trivial strengthening of Not 1-1 against the background of an impredicative theory of classes. We then explore some ramifications of class forms of Cantor's theorem for traditional philosophical puzzles raised by the likes of Russell's paradox of propositions and Kaplan's paradox.