

INCONSISTENCY STATEMENTS

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Do (in)consistency statements still have meaning when they occur in the context of weak and/or unsound theories? Or rather, what does it mean that a formula is a(n) (in)consistency statement in such cases? I will explain why I think such questions are misdirected.

Sometimes wrong-headed philosophy leads to nice mathematics. One idea to address the meaning question is to say that to be a provability predicate is to fulfil a certain role in reasoning. This role is codified as the satisfaction of certain global modal principles: the Hilbert-Bernays and/or the Löb Conditions. I will briefly criticise this idea (mainly by giving examples). However, as I will illustrate, the idea leads to an interesting technical result.

Let U be a base theory, like Peano Arithmetic. (We will explain the conditions on base theories in the talk.) Let's say that A is an inconsistency statement for U if A is the inconsistency statement of some predicate that satisfies both the Löb and the Hilbert-Bernays conditions for U . We say that a sentence B is interpretable over U if U interprets $U+A$. One variant of the Second Incompleteness Theorem, due to Solomon Feferman, says that all inconsistency statements for U are interpretable over U . We will show the converse: all sentences interpretable over U are inconsistency statements of U .

Thus, it turns out that there is a connection between interpretability and inconsistency (in the sense of the role conception). We will further explore this connection. We can use our result to define interpretability in terms of inconsistency statements. Can we reprove known results about interpretability only using insights about proof predicates? Yes, we can. I will illustrate this idea by discussing the connection with the interpretability logic ILM.