Social groups and social network formation [Online Appendix]

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This appendix provides supplementary material for the paper "Social groups and social network formation" by Bassel Tarbush and Alexander Teytelboym.

This appendix presents further simulations for different parametrizations and variants of the model against the mean-field approximation results derived in the main paper. Section A presents simulations of the model described in Section 2 of the main paper for various parameter values. Section B presents simulations of the model when the link formation process is governed by preferential attachment. The preferential attachment variant of our model was discussed after Proposition 2 and in Appendix II of the main paper. Finally, Section C describes an alternative version of our model for which the mean-field approximation results derived in the main paper still apply, and we show how well our analytical results perform against simulations of this alternative model.

In every section of this Online Appendix, we compare our analytic mean-field approximations against the average of 100 runs of the simulation of the relevant variant of our model, and in line with the running example in the main paper.[1] That is, we restrict ourselves to 2 social categories: There are 20 type a students, 180 type b students, 50 type c students, and 250 type d students. The average over 100 runs of the simulation is shown in black, and the corresponding mean-field approximation is shown in color ($q^1 = 0.1$ and $q^2 = 0.8$ in blue; $q^1 = 0.4$ and $q^2 = 0.4$ in purple; $q^1 = 0.8$ and $q^2 = 0.1$ in red).[2] The parameter p is varied to take the values 0.99, 0.9925, and 0.995.

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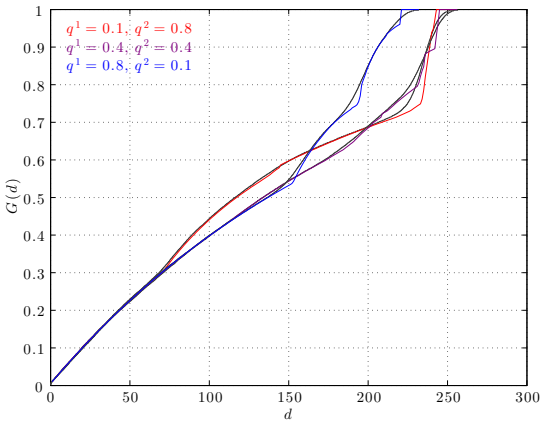
[1] The MATLAB code for running the simulations is available online at http://users.ox.ac.uk/~scat3580/MATLABFriending.zip.

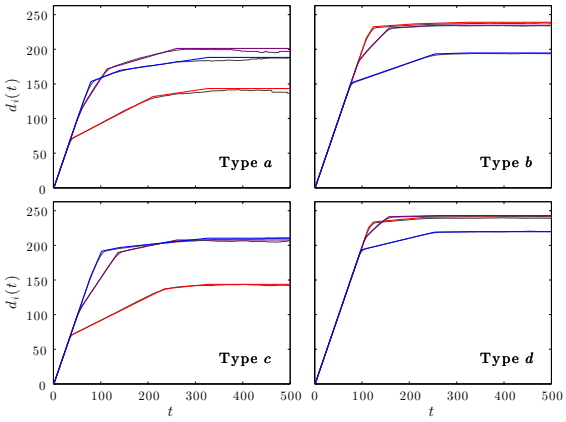
[2] The effective size of social groups is a key determinant of the dynamics of our process, so even though we have fixed the size of the social subgroups, varying $\mathbf{q}$ varies the effective group sizes.

A Further simulations of the baseline model

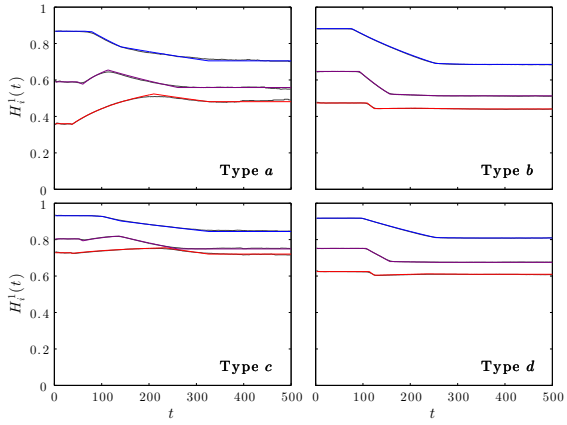
This section presents simulation results for the baseline model described in Section 2 of the main paper. Figure 1 presents the results for $p = 0.99$, Figure 2 presents the results for $p = 0.9925$, and Figure 3 presents the results for $p = 0.995$.



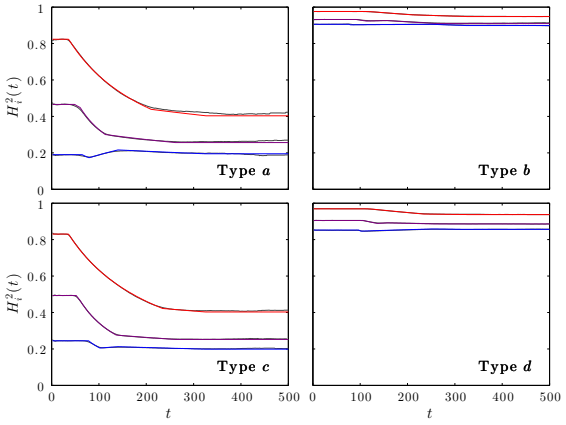
(a) Degree distribution



(b) Degree over time

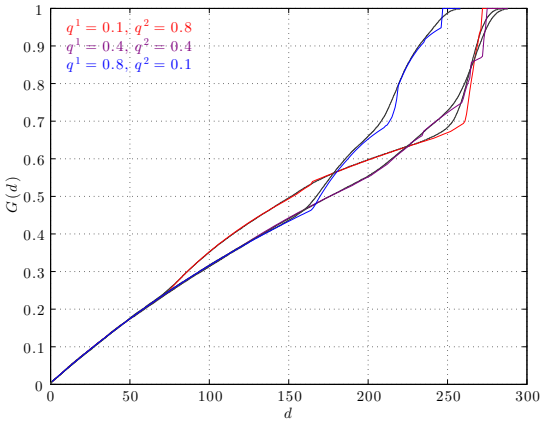


(c) Homophily in social category 1

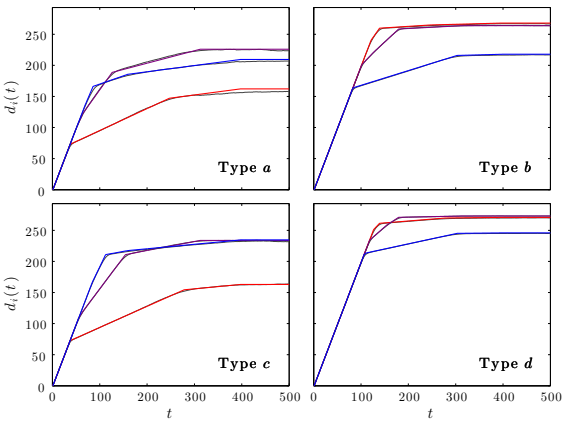


(d) Homophily in social category 2

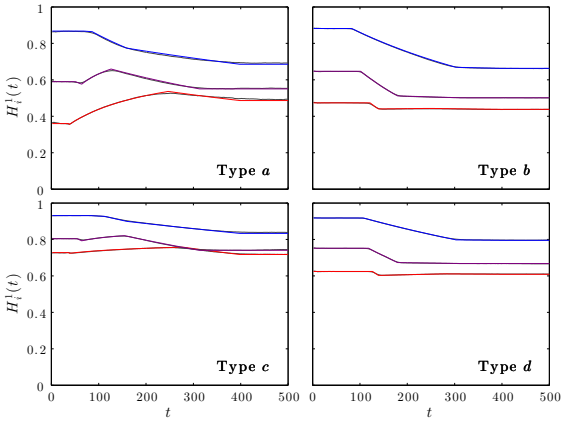
Figure 1: The mean-field approximations (in color) against 100 runs of the simulations (in black) [$p = 0.99$]



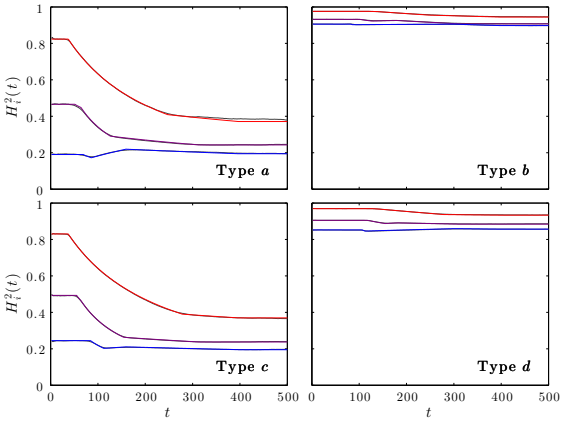
(a) Degree distribution



(b) Degree over time

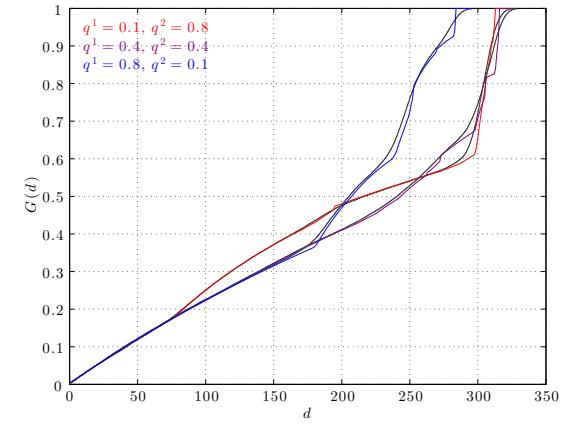


(c) Homophily in social category 1

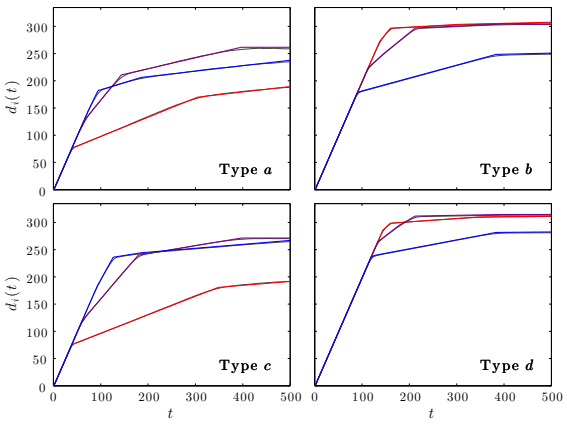


(d) Homophily in social category 2

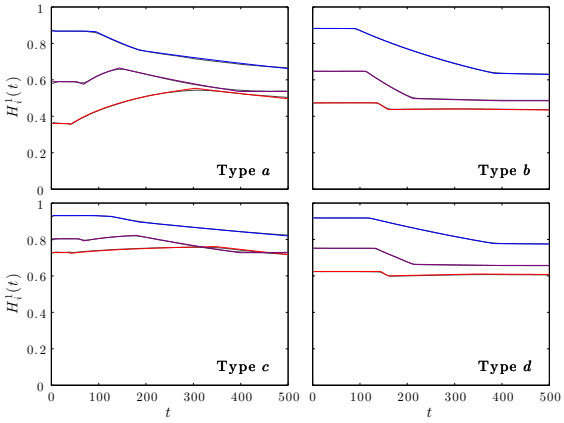
Figure 2: The mean-field approximations (in color) against 100 runs of the simulations (in black) [$p = 0.9925$]



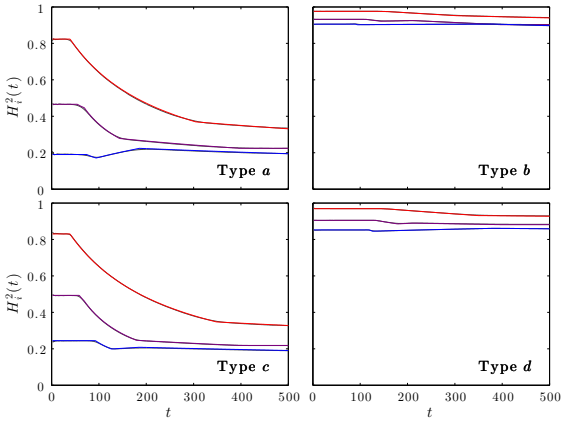
(a) Degree distribution



(b) Degree over time



(c) Homophily in social category 1

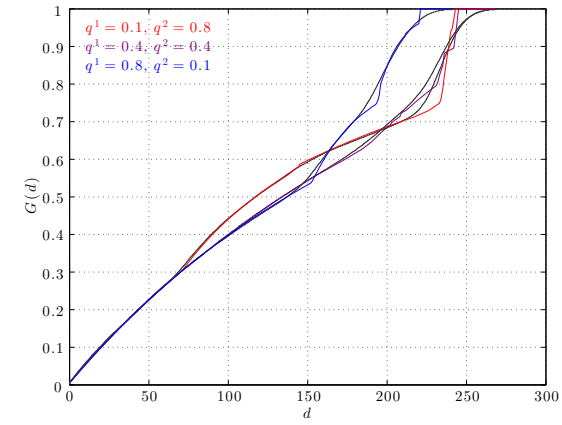


(d) Homophily in social category 2

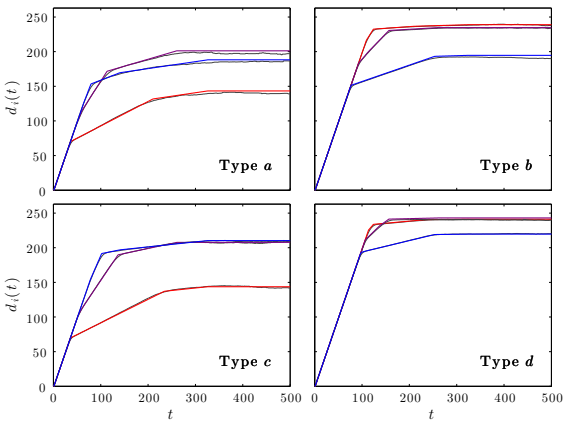
Figure 3: The mean-field approximations (in color) against 100 runs of the simulations (in black) [$p = 0.995$]

B A preferential attachment variant of the model

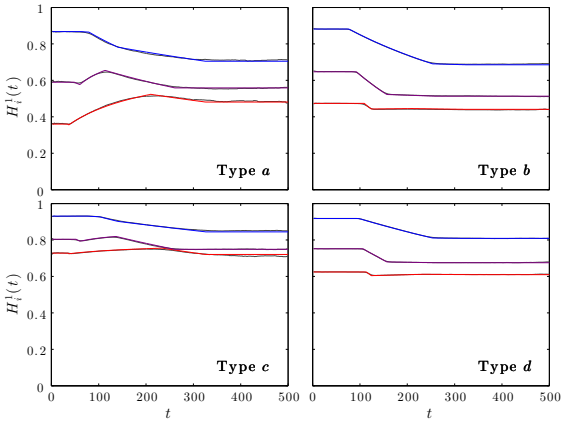
This section presents simulations of the model when the link formation process is governed by preferential attachment. The preferential attachment variant of our model was discussed after Proposition 2 and in Appendix II of the main paper, and it was shown that the mean-field approximations derived for the baseline model still apply when the link formation process is governed by preferential attachment. Figure 4 presents the results for $p = 0.99$, Figure 5 presents the results for $p = 0.9925$, and Figure 6 presents the results for $p = 0.995$.



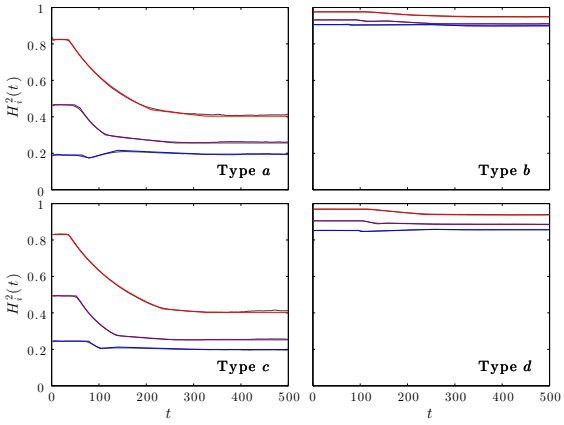
(a) Degree distribution



(b) Degree over time

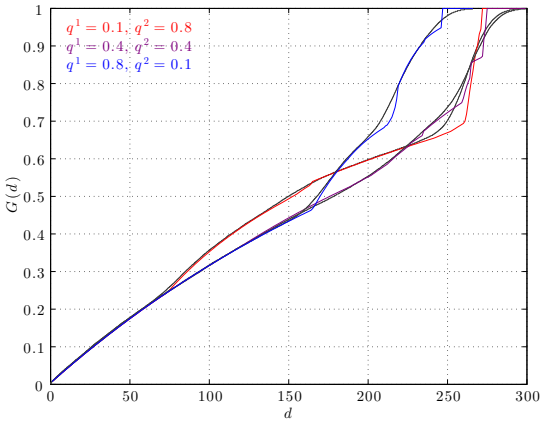


(c) Homophily in social category 1

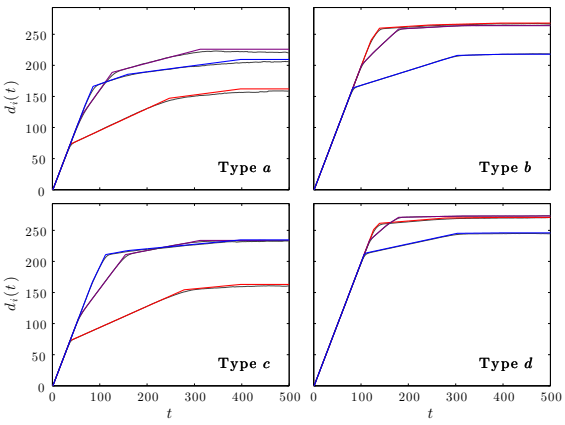


(d) Homophily in social category 2

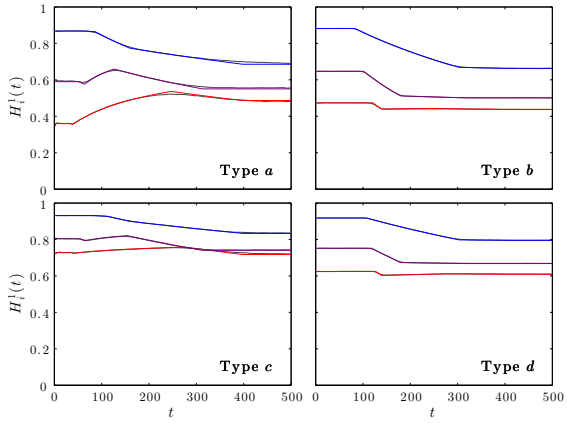
Figure 4: The mean-field approximations (in color) against 100 runs of the simulations (in black) [$p = 0.99$]



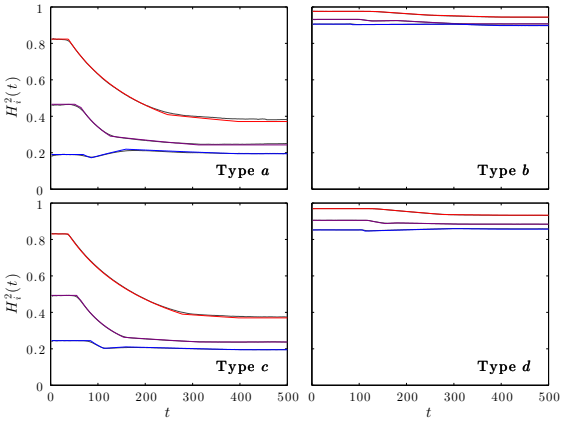
(a) Degree distribution



(b) Degree over time

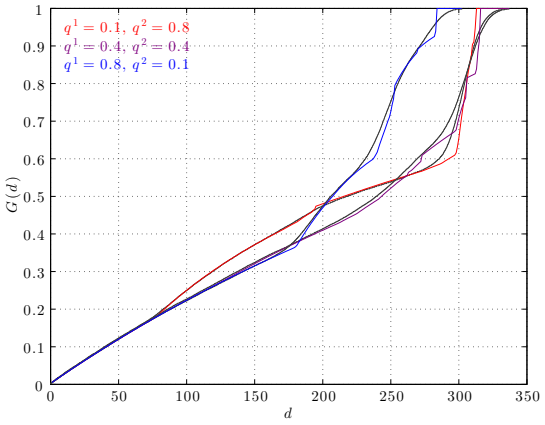


(c) Homophily in social category 1

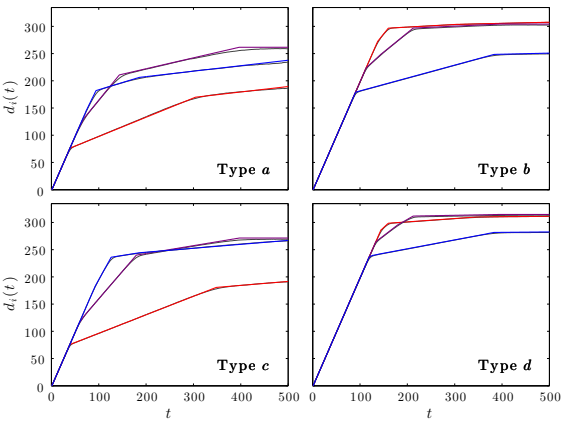


(d) Homophily in social category 2

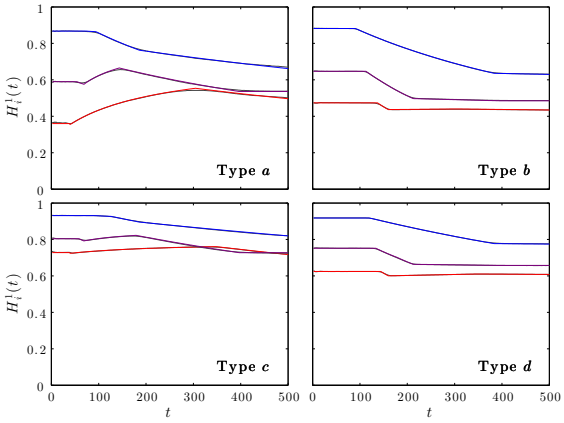
Figure 5: The mean-field approximations (in color) against 100 runs of the simulations (in black) [$p = 0.9925$]



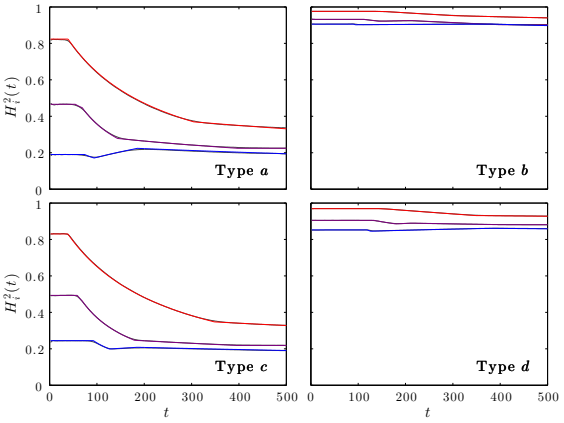
(a) Degree distribution



(b) Degree over time



(c) Homophily in social category 1



(d) Homophily in social category 2

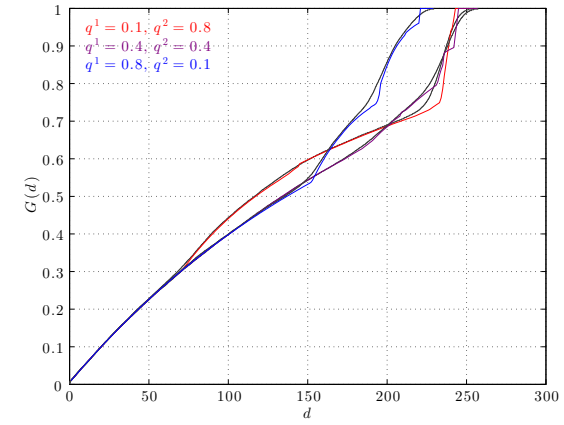
Figure 6: The mean-field approximations (in color) against 100 runs of the simulations (in black) [$p = 0.995$]

C Another variant of the model

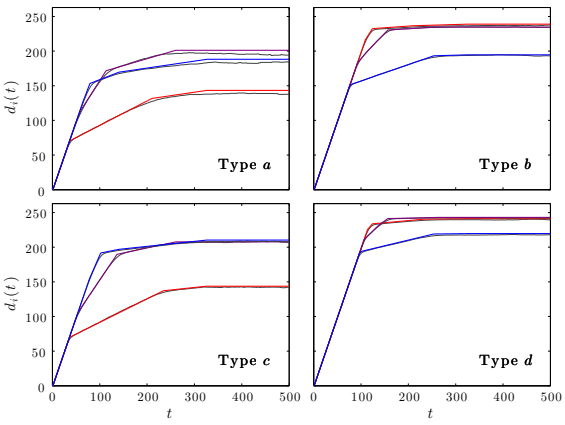
In this section, we describe a variant of our model. We argue that the analytical mean-field results derived in the paper would still describe the average dynamics of this new stochastic process over time, and we show that our analytical approximations perform well against simulations of this variant.

In our description of the network formation process in Section 2 of the main paper, we could have described the interactions as unfolding as follows: In every period, instead of each agent i selecting Γ_i^r with probability q^r independently, suppose that with probability q^r, *all* agents i select the social group Γ_i^r. In the context of our example, in any given period all the students would go to their respective classes, or all of them would go to their respective dorms, and so on. This simple change would allow us to interpret $\mathbf{q}$ as the fraction of time that students allocate to *physically* being in a particular location (Technically, q^r is the probability that all the students interact with their respective r^{th} social group). In this case, one can easily imagine how $\mathbf{q}$ could, to some extent, be determined institutionally by say scheduling or by the location of buildings around the campus.

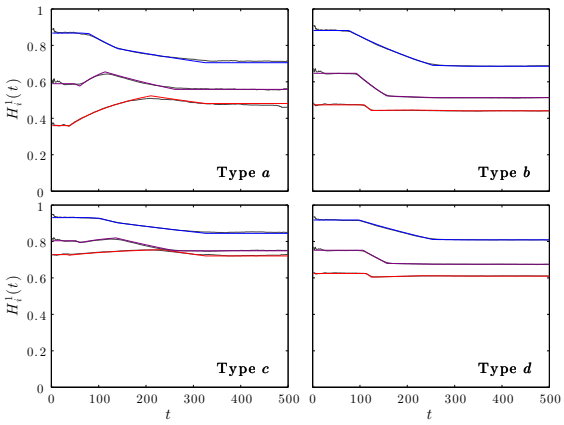
It is straightforward to check that the mean-field analysis of this variant of the model would result in precisely the same analytical results that we derived in the main paper. Figure 7 presents the simulation results for $p = 0.99$, Figure 8 presents the results for $p = 0.9925$, and Figure 9 presents the results for $p = .995$.



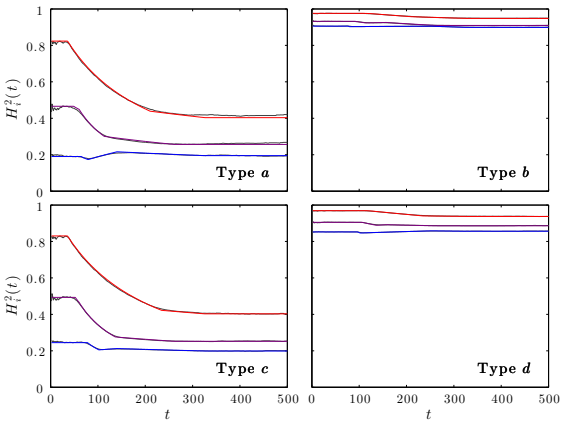
(a) Degree distribution



(b) Degree over time

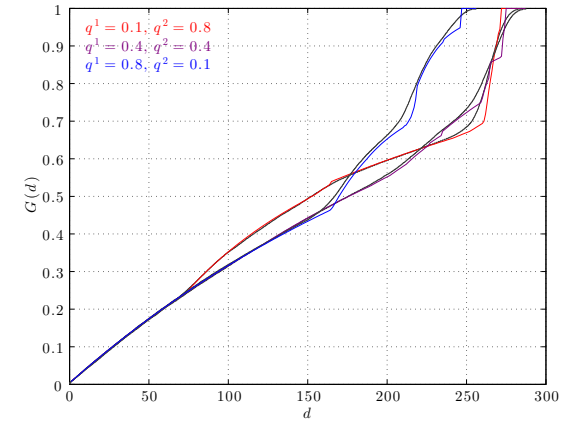


(c) Homophily in social category 1

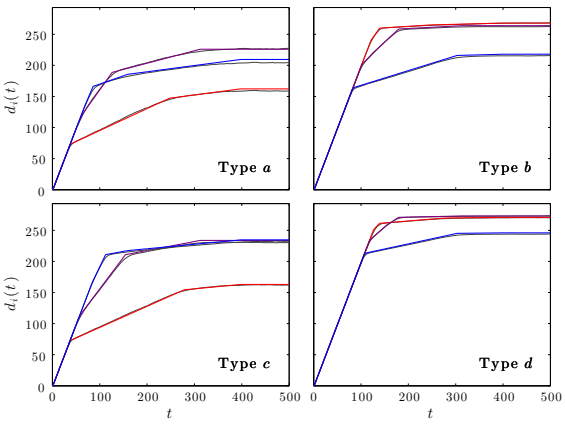


(d) Homophily in social category 2

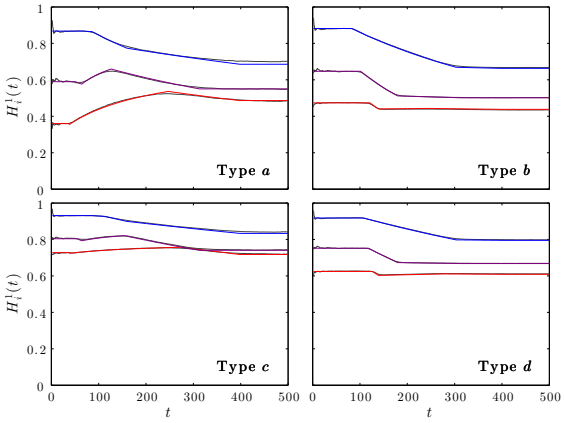
Figure 7: The mean-field approximations (in color) against 100 runs of the simulations (in black) [$p = 0.99$]



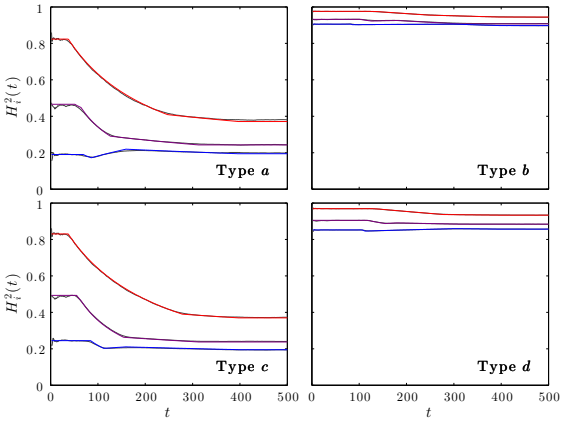
(a) Degree distribution



(b) Degree over time

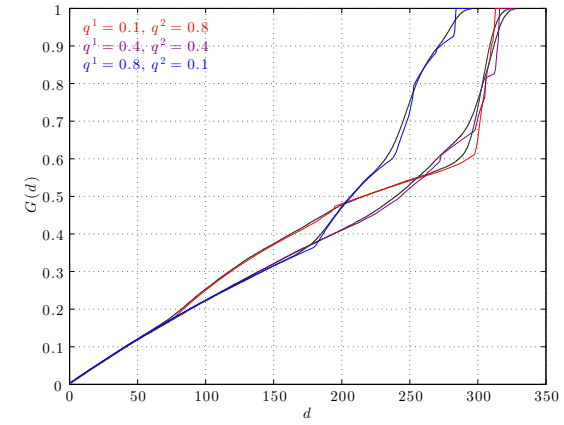


(c) Homophily in social category 1

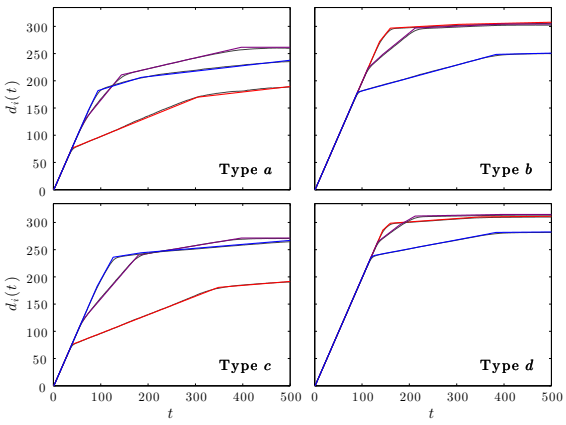


(d) Homophily in social category 2

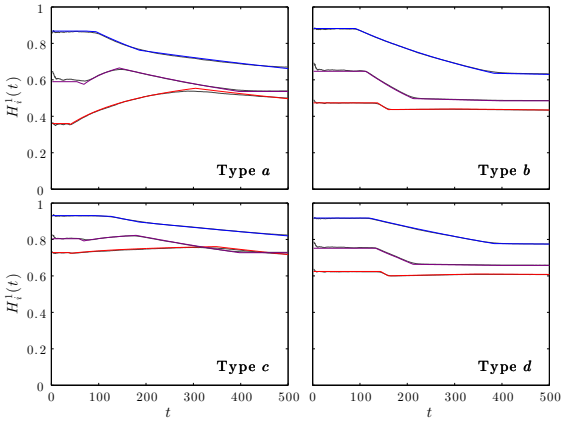
Figure 8: The mean-field approximations (in color) against 100 runs of the simulations (in black) [$p = 0.9925$]



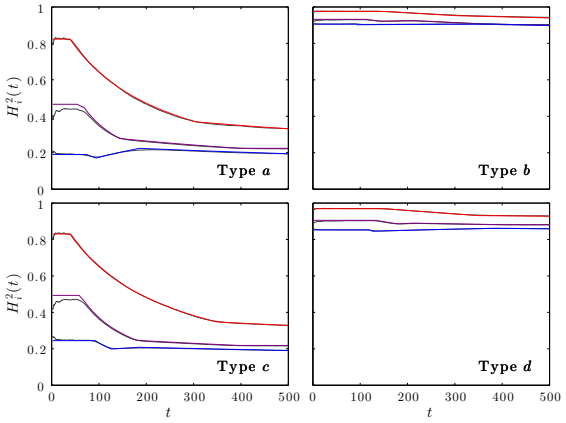
(a) Degree distribution



(b) Degree over time



(c) Homophily in social category 1



(d) Homophily in social category 2

Figure 9: The mean-field approximations (in color) against 100 runs of the simulations (in black) [$p = 0.995$]