

## Derivation of aggregate demand curve in Mundell-Fleming IS-LM model

We define the components of aggregate demand as the following:

$$C=C_0+c(1-t)Y$$

$$I=I_0-\delta r$$

$$G=G_0$$

$$NX=X_0+\gamma e-m(1-t)Y$$

$Y$  is output,  $c$  is the marginal propensity to consume out of post-tax income,  $t$  is the proportional income tax rate,  $m$  is the marginal propensity to import out of post-tax income,  $C_0$ ,  $I_0$ ,  $G_0$  and  $X_0$  are autonomous consumption, investment, government spending and exports respectively,  $r$  is the real interest rate. The real exchange rate is  $e=E(P^*/P)$ .  $E$  is the nominal exchange rate; this is given as the price of foreign currency in terms of domestic currency. The real exchange rate is therefore the real price of foreign goods in terms of domestic goods ( $P^*$  is the foreign price level,  $P$  is the domestic price level). An increase in  $e$  is an increase in competitiveness because when foreign goods become relatively more expensive, domestic and foreign demand shifts to the domestic economy. Using the condition for goods market equilibrium  $Y=Y_d$  we can rearrange this model to give the equation for the IS curve:

$$Y=(1-(c-m)(1-t))^{-1}(C_0+I_0+G_0+X_0-\delta r+\gamma e)=\kappa(C_0+I_0+G_0+X_0-\delta r+\gamma e)$$

From this condition, it is clear that the IS curve has the following properties (under the assumptions of this model, which are of course simplifications):

- It is linear and downward sloping. Increases in  $r$  reduce output, with the size of the output reduction depending on the Keynesian multiplier  $\kappa$ , and the sensitivity of investment to the real interest rate, given by the coefficient  $\delta$ .
- Increases in any of the autonomous components cause parallel outward shifts with the size of the output increase being equal to the change in the autonomous component multiplied by the Keynesian multiplier  $\kappa$ .
- Increases in competitiveness (i.e.  $e$ ) cause parallel outward shifts, with the size of the shift depending on both the Keynesian multiplier and the sensitivity of net exports to the exchange rate, represented by the coefficient  $\gamma$ .

We derive the LM curve from the equations for real money supply and real money demand. We let  $N$  represent the nominal money supply, which is assumed to be fixed by the authorities:

$$M_S=N/P$$

$$M_D=\alpha Y-\beta i$$

Where  $i=r+\pi_e$  (i.e. the nominal interest rate on bonds must be equal to the real interest rate plus the expected inflation rate).

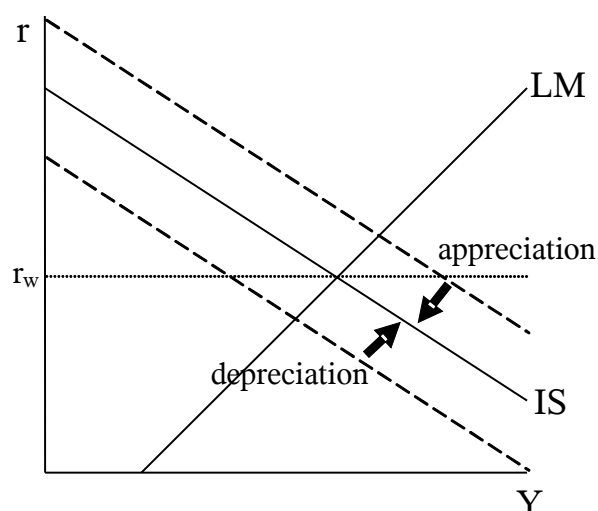
Since we have defined money demand as a linear function of output and the nominal interest rate, the LM curve will be found by setting  $M_S=M_D$ . Therefore:

$$Y=\alpha^{-1}(N/P+\beta(r+\pi_e))$$

It can therefore be seen that the LM curve has the following properties:

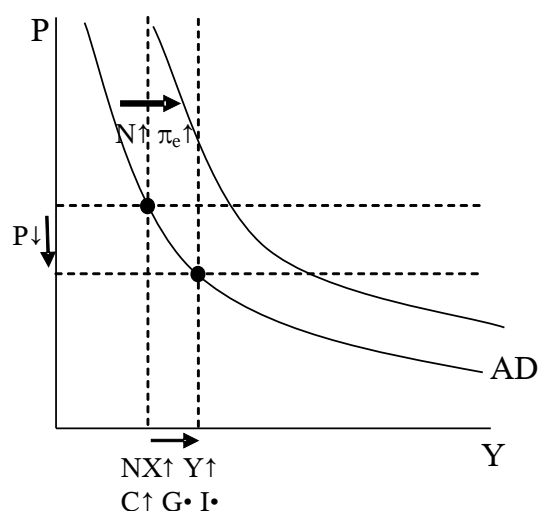
- It is upward sloping.
- Increases in the nominal money supply cause parallel outward shifts.
- Increases in the expected inflation rate cause parallel outward shifts.
- Increases in the price level cause parallel inward shifts.

The way in which we derive the IS-LM equilibrium in a small open economy with perfect capital mobility depends upon whether that economy has a floating or a fixed exchange rate regime. Let us first take the case of a floating exchange rate regime. Here, the government can fully control the position of the LM curve (because it controls the nominal money supply, it can offset any shock to inflation or prices by altering the nominal money supply appropriately). However, because the nominal exchange rate is freely floating, it is determined in markets beyond the control of the government. Whenever the domestic interest rate is below the world interest rate  $r_w$ , huge outflows of capital will be causing the exchange rate to depreciate and domestic competitiveness to improve ( $e$  goes up). This shifts the IS curve outward. Whenever the domestic interest rate is above the world level, massive capital inflows cause an appreciation ( $e$  goes down) which shifts the IS curve inwards. Essentially, therefore, it is the intersection of the LM curve with the world interest rate that determines the short run output level. This means that investment is always equal to  $I_0 - \delta r_w$ . It is the competitiveness level  $e$  that changes when monetary policy changes. For example, when the domestic nominal price level  $P$  decreases, this causes a rightward shift in the LM curve. This in turn leads to a depreciation which shifts the IS curve, “magnifying” the effect of the monetary expansion. So, in a fully open economy with a floating exchange rate under the assumptions made here, net exports and consumption (via the Keynesian multiplier) respond to monetary expansions and contractions whilst investment and government spending remain constant.



If we now think about the derivation of the aggregate demand curve, it is clear that a drop in the price level, with all other variables such as the nominal money supply, fiscal policy, world interest rate etc. staying constant, causes an outward shift of the LM curve and therefore an increase in output. As we saw above, this increase in output is shared between net exports and domestic consumption, with investment and

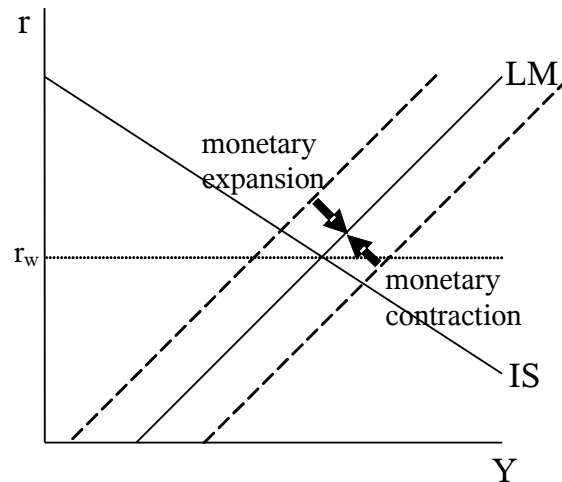
government spending remaining constant. It is also clear that an expansion of the nominal money supply will shift the AD curve to the right (by shifting the LM curve to the right for any given nominal price level), as will an increase in the expected inflation rate.



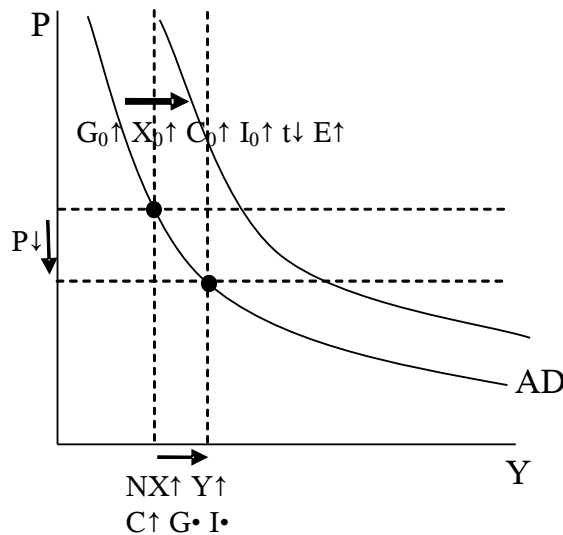
Unlike in a closed economy, however, in a small open economy with a floating exchange rate, fiscal policy, and changes in autonomous consumption, investment or exports do not shift the AD curve (assuming that monetary policy is left unchanged, i.e. there is no accommodating expansion). This is because any increase in  $G_0$ ,  $C_0$ ,  $I_0$  or  $X_0$  will cause an immediate appreciation of the domestic currency (decrease in  $e$ ) in order to shift the IS curve back to meet where the LM curve meets the world interest rate line. This is not to say that these policy tools are not still useful, it is just that they cannot be used for aggregate demand management in the same way as monetary policy can. For example, if a country wanted to reduce its trade deficit, a cut in  $G_0$ ,  $C_0$  or  $I_0$  would immediately lead to a depreciation of the currency leading net exports to “take up the slack” but with total output remaining unchanged. An increase in  $X_0$  (autonomous exports, a measure of the desirability of the country’s exports regardless of the real exchange rate) on the other hand, will cause an immediate appreciation of the currency. Since output must remain unchanged once the IS curve has shifted back to equilibrium position, we know that increased imports (caused by the decrease in  $e$ ) must fully offset the extra injection into domestic demand from the autonomous exports. Although output and the trade balance would remain unchanged in the new short run Mundell-Fleming equilibrium, the country would be better off because it would now be importing more (it’s citizens must be better off because their currency has appreciated). So, the adjustment mechanism of the IS curve would here automatically be translating the increased demand for the country’s exports into additional imports for its consumers.

Let us now take the case of a fixed exchange rate regime. Here the government fixes the value of the nominal exchange rate  $E$ . It can only do this by committing to buy and sell foreign exchange at any quantity supplied or demanded at the fixed rate. This means that the government can no longer control the domestic money supply. The position of the LM curve is no longer under the control of the government. However, the government can now fully control the position of the IS curve (it can offset any shock to private investment or consumption by appropriately altering  $G_0$  or by altering  $E$ , such as by devaluing - increasing  $E$  - in order to boost domestic demand). The LM curve will always shift to meet the intersection of the IS curve and the world interest rate line. This is because whenever the domestic interest rate is below the world one, massive capital outflows cause a huge excess demand for

foreign currency (excess supply of domestic currency), which the Central Bank must sell in exchange for domestic currency, thus causing a contraction of the domestic money supply. On the other hand, whenever the domestic interest rate is above the world interest rate, there is huge excess demand for domestic currency (excess supply of foreign currency) which the Central Bank must provide, thus causing a monetary expansion.



How is the aggregate demand schedule derived for a fixed exchange rate? The key to this is that it is the nominal exchange rate  $E$  that is fixed. The real exchange rate  $e$  depends on the domestic and foreign price levels. We can assume  $P^*$  to be fixed. However,  $P$  alters along the aggregate demand schedule. When  $P$  decreases, this improves domestic competitiveness ( $e$  goes up). This adds to domestic net export demand and, via the Keynesian multiplier, to output and to consumption demand:



Although the effect of a drop in the price level is similar in terms of its effects on components of domestic demand, the mechanism by which this occurs is, as we have seen, different in the cases of fixed and floating exchange rate regimes. The factors under the control of the government which shift the AD curve are now also different to the floating exchange rate regime case. It is now fiscal policy tools and shocks to domestic autonomous demand components which cause an outward shift in the AD curve. Also, importantly, the level at which the nominal exchange rate is fixed determines the position of the AD curve. A devaluation ( $E$  goes up), by increasing

domestic export demand at any given domestic price level, causes an outward expansion of the aggregate demand curve.

### **Derivation of aggregate supply curve using Friedman's money illusion**

The easiest way to generate an upward sloping short run aggregate supply curve using a rigorous foundation is to use Friedman's idea of money illusion. Although sticky nominal wages work OK to explain involuntary unemployment (because nominal wages do not fall enough there is excess labour supply), the problem with using sticky nominal wages is that they cannot be used to generate output above the equilibrium level, because if nominal wages are driven below the equilibrium level workers will work less than at the equilibrium level and there will be excess demand for labour. The advantage of money illusion is that it can explain output fluctuations in a simple symmetric way.

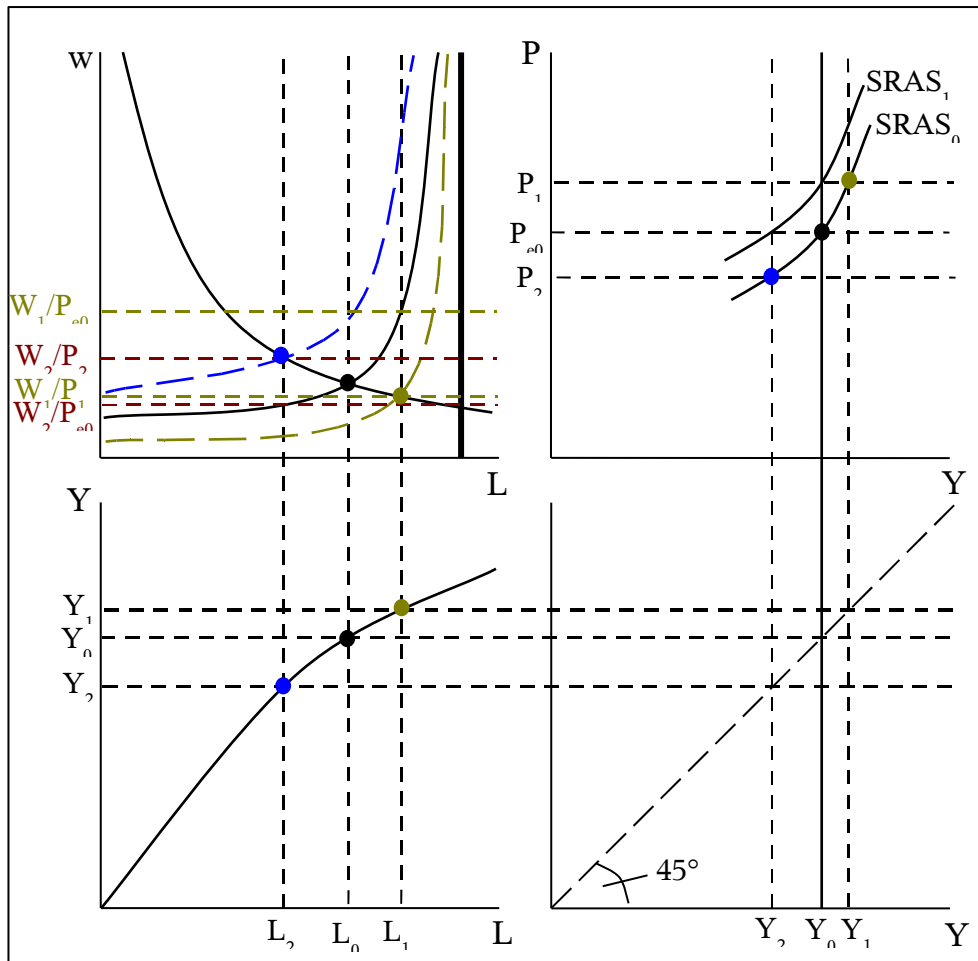
It takes workers time to realize that the domestic price level has changed. This means at any time there can be a discrepancy between the workers' perceived/expected domestic price level  $P_e$  and the actual price level  $P$ . We assume that firms know the actual price level correctly. Labour demand is assumed to be described by a perfectly competitive market. If firms have a Cobb-Douglas production function  $F=K^\alpha L^{1-\alpha}$ , their profit function will be given by:

$$PK^\alpha L^{1-\alpha} - WL$$

(we assume that the capital stock  $K$  is fixed,  $P$  is the nominal price level and  $W$  is the nominal wage).

Maximizing this function gives the first order condition:  $W/P=(1-\alpha)(K/L)^\alpha$  (this condition says that the real wage  $w=W/P$  is equal to the marginal product of labour). We can clearly see from this that the labour demand curve is downward sloping and convex, and goes to infinity as  $L$  goes to 0 and 0 as  $L$  goes to infinity.

If can think of  $L$  as the number of workers who are working, and assume for simplicity that they each work a standard week. The labour supply curve will be derived from the reservation wage required to get each worker to enter the labour market. If there are unemployment benefits we would expect that there would need to be a certain positive level of real wages before any workers will choose to work. Different workers will have different reservation wages because they have different preferences, different situations etc. If we let  $L^\wedge$  be the total number of workers in the labour force, the labour demand curve must climb to infinity before it reaches this line because no level of the real wage can induce more workers to work than actually exist. We are now in a position to derive the aggregate supply curve (short run and long run) from the labour market model. To do this, we assume that the workers' expected/perceived nominal price level is fixed at  $P_{e0}$ . The fact that the nominal price level can deviate away from this means that workers can be "driven away" from their labour supply curve. Because firms know the actual price level, and the market is assumed always to clear, firms must always be on their labour demand curve.



Output can be driven above the potential level  $Y_0$  if labour supply is driven above  $L_0$  (i.e. if unemployment is driven below the natural rate). This can happen if prices rise to  $P_1$  and nominal wages to  $N_1$ . Although the real wage has in fact dropped from  $W_0/P_0$  to  $W_1/P_1$ , workers do not realize that the price level has increased and believe that the real wage level is  $W_1/P_{e0}$ . They therefore supply  $L_1$  labour. They are in fact off of their labour supply curve, but they are on their perceived labour supply curve, the dashed green line lying below the actual labour supply curve. (A similar story applies if the price level drops below the expected price level, except that here the workers' perceived labour supply curve lies above the actual labour supply curve – this is shown in blue.) Once workers realize that the price level has increased, the SRAS curve will shift upwards from  $SRAS_0$  to  $SRAS_1$  to cross the LRAS curve at the new perceived price level. The labour market can only be in long run equilibrium when the workers' perceived price level is the actual price level. Clearly this point will lie on the intersection of the LRAS and SRAS curves. (Note – if workers have rational expectations they always correctly perceive the price level and so we are always on the LRAS curve.)

To conclude, if you need to derive an SRAS curve in the exam, Friedman's money illusion is the easiest way to do it. Together with the open economy IS-LM model presented earlier, you have here a fully worked out model of the economy where aggregate demand fluctuations cause employment fluctuations in the short run, but where the economy is money neutral in the medium to long run.