Topic 1: Saving and Investment

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The key to thinking about how to relate these concepts together in the framework of the Keynesian neo-classical synthesis is to use a number of important distinctions. Firstly, one must distinguish between potential output \bar{Y} and actual output Y. In the usual model, output can in the short run be below or above potential output. If it is below, there is unemployment and deflation. If it is above, there is inflation. Secondly, one must distinguish between gross domestic product, equivalent to national output, and gross national income, which consists of gross domestic product minus the payment of the national external debt (interest payment paid by both government sector and private sector to abroad) plus interest payment from abroad (again to both government and private sectors). When an economy is closed, there is no trade in goods. This means that there can be no borrowing or investment abroad in order to buy foreign goods or sell domestic goods abroad. (Foreign money is useless to domestic consumers and domestic money is useless to foreign consumers.) This means that national income and output are the same.

1 Closed Economy

1.1 "The equality of saving and investment is a national income accounting identity."

Imagine we have and economy that produces and capital good K and a consumption good C. Its production function is defined as follows.

$$F(K,L) = Y \tag{1}$$

where Y is a (K, C) bundle, and L is the labor supply. Imagine that all of C is consumed each period and that all factors of production are used to

produce something each period (full employment of productive resources). If we define saving as the factors of production that the economy does not put towards consumption, then necessarily (since we have assumed full employment of resources) saving is equivalent to investment. All that which we do not put towards the production consumption goes to the production of capital.

In the national income accounting identity, saving refers to cash income saved and investment refers to expenditure on real capital goods (but could arguably include intangibles like human capital). The identity derives from the fact that any expenditure on investment must be financed by saving somewhere in the system, and any income saved must be invested somewhere in the system. The financial system operates as the intermediary by channeling savings (e.g. bank deposits, purchases of equity, purchases on bonds, etc.) into investment (e.g. expenditure on buildings, employee training, stockpiles of goods, knowledge). It is important also to realize that the identity is between both planned and unplanned savings and investment. For example, if a firm purchases a stock of goods it intends to sell, but is unable to do so, then this would be counted an unplanned inventory investment.

Assume a constant proportional income tax at rate t, and that a constant average proportion of pre-tax income c, is consumed, and a constant average proportion of pre-tax income s=1-c-t is saved (so that c+s+t=1), and let G be government expenditure on all goods (both services and investment) and G be private investment expenditure. The national income accounting identity can then be expressed as:

$$C + I + G = cY + I + G = Y$$

$$\Longrightarrow cY + I + G = Y(c + s + t) \tag{2}$$

The government budget surplus B is:

$$B = tY - G$$

Combining these two equations gives us:

$$I = sY + B \tag{3}$$

This identity tells us that private sector investment is equal to private saving plus government saving. A budget surplus implies government saving because the government is raising more in tax revenue that it is spending.

The government must be using this surplus to pay off its debt, so it is buying bonds and other assets from the private sector. The money it is giving to the private sector to purchase back this government debt (or build up government asset holdings) is either being consumed or invested. Since we have assumed that consumption is a constant proportion of income/output, private sector consumption will not increase despite the increase in government saving provided the economy remains at potential output so that Y remains unchanged after the tightening of the budget. An improvement in the government budget surplus therefore implies an increase in private sector investment, if private sector consumption and savings and overall output remain unchanged. (This is obvious when we consider from the perspective of aggregate supply that the only way to make room for more I at full employment when C is fixed so that $\bar{Y} = \bar{C} + \bar{I} + \bar{G}$ is to make G smaller and thus improve the government budget surplus.)

1.2 "The equality of saving and investment is an equilibrium condition."

Now note that if Y is below potential output \bar{Y} then, given a particular government spending level (fixed G) and savings rate, investment I=sY+B must be below potential investment $\bar{I}=s\bar{Y}+\bar{B}$ (higher output will also improve the government budget position by increasing tax revenues). It is therefore not the case by identity that actual savings and investment equal potential savings and investment. It is also, as we have seen, not the case by identity that desired savings equal desired investment. (We do not have to be at the intersection of the demand and supply curves for saving for the identity to apply; we can be anywhere on the diagram because actual saving always equals actual investment, even when the market is in disequilibrium).

To say that the equality of saving and investment is an equilibrium condition implies more than that actual (planned and unplanned) savings equals actual (planned and unplanned) investment. It refers to a microeconomic equilibrium in the sense that interest rates have brought savings and investment into line so that desired savings equal desired investment. The equality between desired savings and investment is a property of the equilibrium in terms of the Keynesian multiplier model, as discussed below. However, when output is away from its potential value, the interpretation of the equilibrium of the Keynesian multiplier model must be that some markets in the economy are distorted from their competitive equilibria, leading to inefficiencies in many different markets. The interpretation of \bar{Y} in the Keynesian

neo-classical synthesis model is the level of output which would pertain if the labour market (along with all other markets) is in equilibrium (i.e. no "excess" unemployment). In this special case, then, the quality of savings and investment is part of a general equilibrium in the microeconomic sense. However, when output is away from its potential level, the equality of savings and investment is only a partial equilibrium in the microeconomic sense. If there is "excess" unemployment (the definition of which is fairly complex and a topic in its own right) then savings and investment must be away from their true general equilibrium levels (probably both too low).

We have not so explicitly far presented a simple Keynesian model of how the actual levels of consumption, saving and tax revenues are determined. This is provided by the Keynesian multiplier model. The model equates actual output/income Y with demand for output Y_D . Whereas the average consumption rate c introduced above is to be thought of as the rate which manifests itself after all macro-economic variables are co-determined, consumption demand is modelled as being equal to an exogenous component C_0 plus a proportion of income $m_c(1-t)Y$ where m_c is referred to as the marginal propensity to consume out of post-tax income. We also define $m_s = 1 - m_c$ Since in equilibrium actual consumption must equal desired consumption, we have that:

$$cY = C_0 + m_c(1-t)Y \Longrightarrow c = \frac{C_0}{V} + m_c(1-t)$$
 (4)

The idea is that if someone's income is 0, they still make some expenditure demand, either by spending cash benefit transfers, or running down their savings. On the other hand, as they are given more income, some of this is saved and only some consumed. We can now solve for the Keynesian equilibrium:

$$Y = C_0 + m_c(1 - t)Y + I + G$$

$$\implies Y = \frac{C_0 + I + G}{1 - (1 - m_s)(1 - t)}$$
(5)

The value $\frac{1}{1-(1-m_s)(1-t)}$ is referred to as the Keynesian multiplier. When autonomous investment, consumption of government spending increases by 1 unit, output will increase by more than 1 unit because a proportion of the income generated by the extra expenditure is re-consumed. Hence the overall effect is an infinite geometric series (of finite size).

1.3 "More investment requires more saving."

Equation 3 shows us that the only way for private sector investment to increase is for either government savings or private savings to increase. Note however, that this does not imply that the private sector savings rate, s, must increase. If output increases then this increases private sector saving and government saving. This means that in a recession, policies to stimulate aggregate output may be the correct remedy to increase investment. On the other hand, if the economy is at potential output \bar{Y} already, the only way to increase investment \bar{I} in the long run is to increase the savings rate s or decrease government expenditure G. This is because the economy cannot (indefinitely, at least) operate at greater than potential output.

1.4 "More investment requires a lower rate of interest."

So far, we have not introduced an interest rate as one of the determinants of investment. The simplest way to do this is to make investment I(r) a function of the interest rate r. The change in the interest rate is one way of looking at the mechanism by which an increase in the amount of savings available in the system translates into an increase in investment expenditure. If we think of an investment good as producing a continuous stream of benefit of size one, the present value of the entire benefit stream is equal to $\int_0^\infty e^{-rt} X dt$, where X is the value of the per-period benefit. Solving this integration gives a value for the total stream of $\frac{X}{x}$. This is the price that the investment will sell at in the present period. Suppose for simplicity that all investment plans cost 1 and pay the same benefit X forever, and only differ in the value of X. This means that those investment plans for which X > r will be profitable. In order for more investment plans to become profitable, and hence for investment to increase, we require that r decrease. We can think of r as being determined within the financial system. Governments as well as private firms offer bonds. Private sector investors then decide whether to buy government bonds or invest in private sector assets. If the government offers a higher interest rate on its bonds (equivalent to lowering the price whilst keeping the stream of payments unchanged) then this will result in less private sector investments being profitable, and thus a decrease in investment expenditure. In a Keynesian equilibrium, we assume that desired investment equals desired savings. This imples that the interest rate has adjusted to bring these two into line. However, it should be noted that in a more complex model, investment can be related to factors other than the interest rate (e.g. current and expected future output levels), and so it is not always the case that more investment requires a lower rate of interest.

1.5 "An increase in saving decreases national income and thus investment."

The key to understanding this statement is to distinguish between the result of increased saving when the economy has returned to full employment, and the initial short run Keynesian multiplier effect. It is clear from equation 2 and 4 that an increase in the marginal propensity to save out of post-tax income or a decrease in autonomous consumption must decrease the average rate of consumption c, if \bar{Y} is fixed. If government expenditure \bar{G} remains unchanged, this implies that investment I must increase. It is thus untrue in the long run that increased saving decreases national income and thus investment (ceteris paribus, it must increase investment because consumption has decreased). In the short-run, however, equation 5 shows the opposite result, that an increase in the marginal propensity to save and a decrease in autonomous consumption both decrease the level of output. If we make investment a function of income and the interest rate, and investment is positively related to income, we can now get the result that increased saving can, in the short run, decrease output, income and investment. What is going on? The idea is that if a sudden increase in the amount of income saved does not lead to an immediate drop in the interest rate (as represented by the assumption of fixed I in the simplest Keynesian model), then the economy will go into recession in order to bring desired savings in line with desired investment (because if people want to save a greater proportion of their income, and investment cannot increase, they must have less income in order to bring the two back into line). Keynes believed that in a recession, the financial system can fail to translate greater savings into investment for a number of reasons such as if investment becomes insensitive to interest rates.

1.6 "Investment adds to the stock of wealth in an economy and so to national income."

Going back to our model of the supply side of this closed economy, described in equation 1, we can see that saving over time augments the capital input K. This increases the potential output level \bar{Y} (because output is increasing in capital K and labour L). Thus in the long run. increasing the average savings rate s will increase output Y (Note, however, that if s has become very high, although this will increase the growth rate of Y, very little of the output can be consumed. The savings rate can thus be too high

from a consumption-maximizing point of view.) (Also note that although a closed economy can only invest by spending resources on capital formation, an open economy can sell goods abroad, thus building up stock of foreign assets, opening up another avenue for investment to increase national income over time.)

1.7 "Investment depends on the change in national income."

More complex and realistic models of investment expenditure will recognize that investors make decisions based on their expectations of the future conditions of the economy. If national income is expected to increase, this would suggest that investment to expand firm's output capacity will increase in anticipation of greater future selling opportunities. However, if a firm makes a mistake and expands too much relative to demand, it will make a loss relative to other opportunities it could have taken (e.g. buying government bonds). This means that it can be difficult to create an investment-led recovery from a recession, because if private investors are very gloomy about the future prospects of the economy, a lower interest rate may make little difference to their investment decisions. This was one of the reasons why Keynes argued that the government should stimulate the economy itself by increasing government spending or lowering taxes.

2 Open Economy

When an economy is open to external trade, a new component, net exports NX is added to the national income identity:

$$Y = C + I + G + NX$$

Rearranging this identity using our earlier formulae for the budget deficit and consumption C=cY gives us:

$$I = sY + B - NX$$

Assuming that output always remains at potential level

, an increase in the private sector average savings rate or the government budget position can be reflected either in increased investment expenditure,

as we already saw, or in greater net exports NX. If net exports increase this implies that the economy as a whole is earning foreign currency. This must either be being stockpiled (e.g. a lot of Asian countries have huge stockpiles of U.S. dollars) or used to buy other foreign assets such as foreign government bonds, equity, bank accounts, etc. Either way, the economy as a while is increasing its stock of wealth, and will enjoy a higher national income in the future due to the interest payments from its investment.

Within the Keynesian multiplier model, we deal with the opening of the economy by having two new variables, autonomous exports X_0 and the marginal propensity to import out of consumed income m_i . This gives us:

$$Y = C_0 + (1 - t)m_c Y + I + G + X_0 - (1 - t)m_c m_i Y$$
$$Y = \frac{C_0 + I + G + X_0}{1 - (1 - t)(1 - m_s)(1 - m_i)}$$