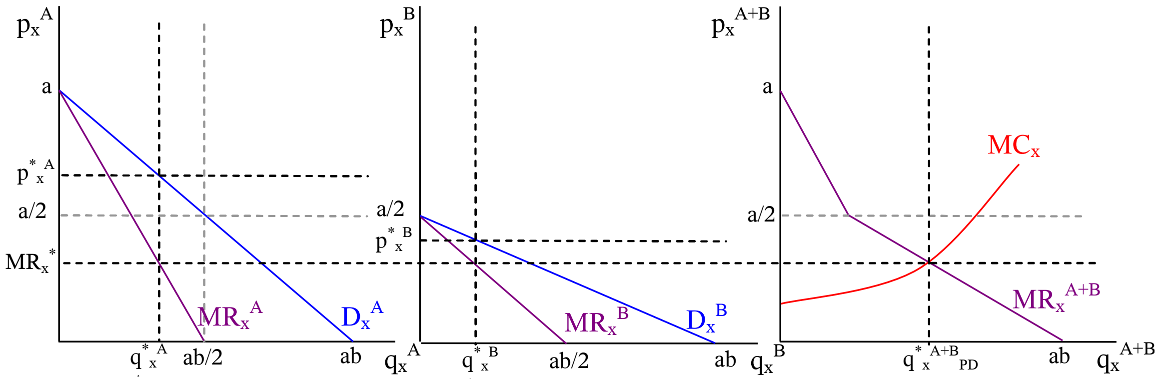
Notes on 3rd Degree Price Discrimination

4. A monopolist which produces in two different markets, where the two sets of consumers cannot engage in arbitrage (i.e. cannot buy or sell the monopolist's good from each other) and can be distinguished by the monopolist, will generally choose a different market price in each market. It is engaging in **third degree price discrimination**. Marginal revenue will have to be equalized in both markets because otherwise the firm could raise its revenue by moving production from one market to the other (whilst keeping total production and therefore total costs fixed). The marginal revenue in each market must be equal to the marginal cost at the total quantity (i.e. the sum of the quantities in the two markets). This means that we can characterize the optimal production decision as the intersection between the marginal cost curve and the horizontally summed marginal revenue curve. The height at which the two curves intersect is the optimal marginal revenue in both markets, MR_x^*. To find the quantities produced in each market we look at the point where marginal revenue is equal to MR_x^* in each market to give q^*_{xA} and q^*_{xB}. To find the price in each market we find the value of the inverse demand curve at these optimal quantities to give p^*_{xA} and p^*_{xB}.



We can show algebraically that the market with the lower price elasticity of demand will be charged the higher price. The monopolist's profit maximization problem is (where π is profits, q_1 and q_2 are the quantities supplied to the two markets, p_1 and p_2 are the prices charged in the two markets and $c(q)$ is the total cost function, with $q=q_1+q_2$):

$$max_{q_1,q_2}\{\pi=p_1(q_1)q_1+p_2(q_2)q_2-c(q_1+q_2)\}$$

To solve this problem, we differentiate with respect to q_1 and q_2 to find our two first order conditions, both of which must be satisfied if the monopolist is maximizing its profit in the two markets simultaneously (we use the product rule in these derivations):

$$\frac{\partial\pi}{\partial q_1}=\frac{d\,p_1}{d\,q_1}q_1+p_1(q_1)-\frac{\partial c}{\partial q}=0$$
$$\frac{\partial\pi}{\partial q_2}=\frac{d\,p_2}{d\,q_2}q_2+p_2(q_2)-\frac{\partial c}{\partial q}=0$$

Now we can use the formula for elasticity to rearrange these two conditions:

$$\epsilon_{q_1}=\frac{d\,q_1}{d\,p_1}\frac{p_1}{q_1}\Rightarrow\frac{d\,p_1}{d\,q_1}=\frac{p_1}{\epsilon_{q_1}q_1}\qquad\qquad \epsilon_{q_2}=\frac{d\,q_2}{d\,p_2}\frac{p_2}{q_2}\Rightarrow\frac{d\,p_2}{d\,q_2}=\frac{p_2}{\epsilon_{q_2}q_2}$$

These derivations use the inverse function rule which states that:

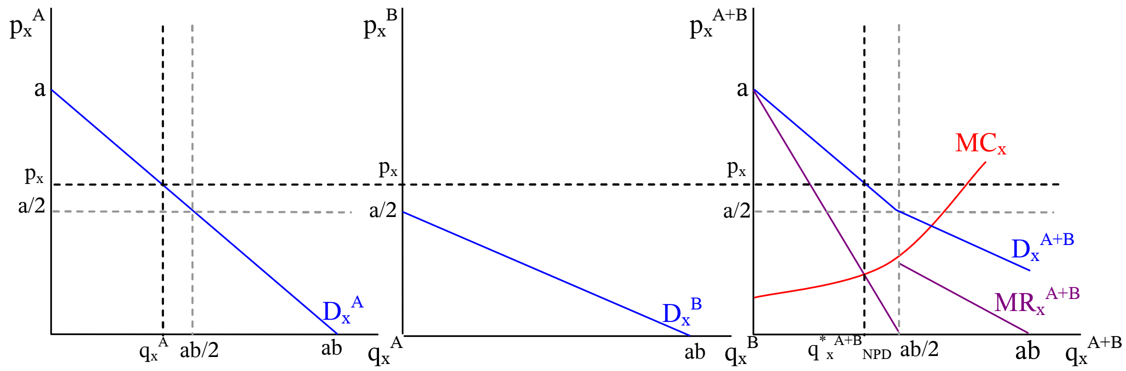
$$y(x)\Rightarrow\frac{dy}{dx}=\frac{1}{\frac{dx}{dy}}$$

Combining the above formulas yields the relationship between marginal cost and elasticity of demand in the two markets (note that the maths is essentially the same as in the normal monopoly mark-up case except that we partially differentiate with respect to the two quantities):

$$p_1^* = \frac{\frac{dc}{dq}}{\left(1+\frac{1}{\epsilon_{q_1}}\right)} \qquad p_2^* = \frac{\frac{dc}{dq}}{\left(1+\frac{1}{\epsilon_{q_2}}\right)}$$

Remembering that, for a good which obeys the Law of Demand, the elasticity as defined above will be negative, we can see from inspection that the market which has more elastic demand (a larger negative value in absolute terms) will have the lower price.

When the consumers in the two markets can engage in arbitrage, the price must be equalized in both markets because otherwise consumers in the market with the lower price would buy goods from the monopolist and sell them on to consumers in the market with the higher price, thus preventing the monopolist from selling goods at the higher price in the other market. We work out the marginal revenue curve differently to when price discrimination is possible; rather than horizontally summing the marginal revenue curves, we horizontally sum the two demand curves to give a total demand curve D_x^{A+B} and then derive the marginal revenue curve from this curve. In the example we see below, this creates a discontinuity in the marginal revenue curve because the elasticity of the total demand curve suddenly changes at quantity ab/2. It is important to remember that this process of deriving the MR curve generally results in a different MR curve to horizontally summing it when arbitrage is not possible (i.e. different prices can be charged).



It is generally the case that the price of the good with the lower price in the price discrimination equilibrium (which is also the good the more elastic demand) increases (or stays the same), and the price of the good with the higher price decreases (or stays the same) when price discrimination is prevented, so the joint non-discrimination price lies in between the two prices under price discrimination. In the case illustrated above, the price in market A decreases and the price in market B rises. The rise in the price in market B leads to nothing being sold in that market. So, consumers in market B are made worse off and consumers in market A better off when price discrimination is prevented. The monopolist will clearly have lower profits (as shown by the fact that when it has the ability to charge different prices, it does so, so taking that ability away must make it worse off). In this case, the effect on the quantity produced from moving to the non-price-discrimination equilibrium is ambiguous. If total quantity sold to both markets were to increase after the removal of price discrimination, we know that total surplus must be unambiguously increased by the prevention of price discrimination due to the one final factor to take into account: The introduction of different prices in the different markets causes allocative inefficiency because if goods could be moved from the market with the lower price to the market with the higher price, total surplus would be raised, because individuals in market A value each marginal good more than market B. So, if allowing price discrimination reduces quantity sold, it unambiguously reduces total surplus because not only are less goods produced, they are also inefficiently allocated, both of which increase the deadweight loss. If, on the other hand, total quantity sold increases with price discrimination, the overall effect on total surplus is ambiguous. Further illustrations of where price discrimination increases, decreases or has no effect on total surplus are provided overleaf.