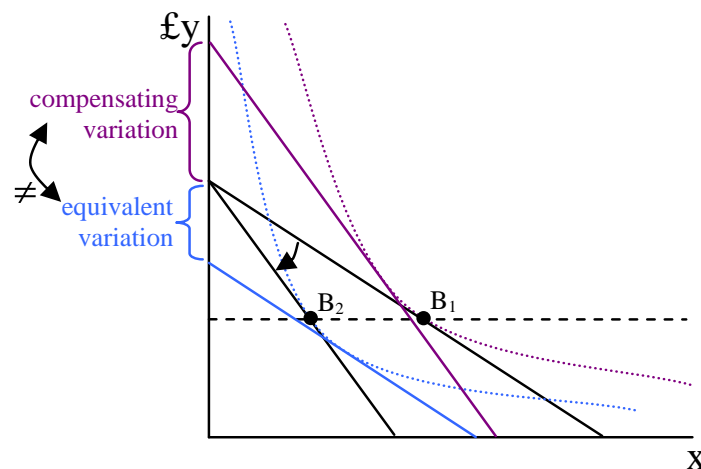
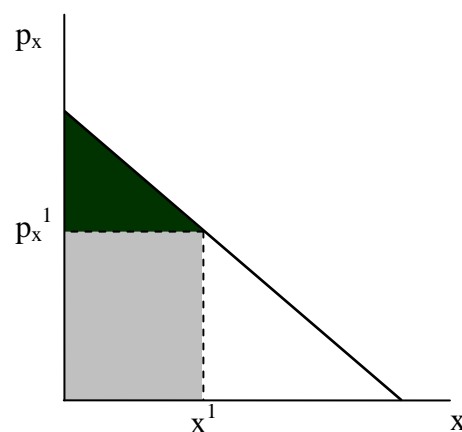


Week 3: Demand Theory and Welfare Analysis

1. Suppose the price of good x increases so that the optimal chosen bundle changes from B_1 to B_2 . If we think of good y as a numeraire good so that $p_y=1$, then the compensating variation is the amount of good y (in money terms) that would need to be given to the consumer at the new price level to put them back on their original indifference curve. The equivalent variation is the amount of good y that would have to be taken at the original price level to put the consumer on the indifference curve that they end up on after the price change. The diagram below illustrates that the compensating and equivalent variation will usually be different for a consumer with consistent and well-behaved preferences (in fact, as we will show later, for normal goods it is always the case that $CV \geq EV$):-



The consumers' surplus is the area under the inverse demand curve but above the price. The idea is that it represents the total gain in welfare received by the consumer from buying the good (and, when the demand curve is made up by lots of consumers, the total welfare gained by all of them). It is the dark triangle in the diagram below. The light rectangle below the triangle is **not** included because consumers must pay p_x for each unit of good x (so, with y as the numeraire good, each unit of good x has an opportunity cost of p_x^1 units of good y , and so the area of the red rectangle ($x_1 p_x^1$) can be thought of the total opportunity cost of buying x^1 units of good x in terms of $\pounds y$, which is also the same thing as the total revenue received by the firm which supplies good x).



The concept of consumers' surplus is based on the assumption that the amount of cash that the consumer pays for the last (marginal) unit of good x at any value of p_x is equal to the marginal utility that the consumer gets from that unit. This is usually not precisely true. The price ratio p_x/p_y is equal to the MRS at the optimal bundle, which in turn is equal to MU_x/MU_y (if the marginal utility of y is high, then the consumer requires **less** of good y to compensate them for losing a marginal amounts of good x). So, setting y as a numeraire good, so $p_y=1$, we will have $p_x=MU_x/MU_y$. Clearly $p_x=MU_x$ can only occur if MU_y is always 1 whatever the value of p_x . This occurs in the special case of quasi-linear preferences, where the utility function takes the form:-

$$U(x,y)=f(x)+y+c \text{ (where } c \text{ is a constant)}$$

$$MU_y=\partial U/\partial y=1$$

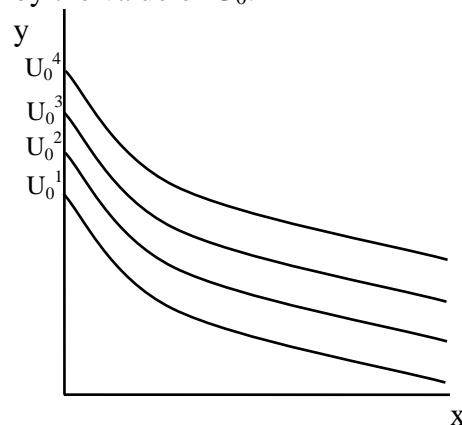
$$MU_x=\partial U/\partial x =\partial f(x)/\partial x$$

Without loss of generality, we can define $f(0)=0$, because any constant can go into the c part. The $f(x)$ part must be a function of x only so that its derivative with respect to y is always 0. Indifference curves take the following form with quasi-linear preferences (assuming $c=0$, which we can do since any positive increasing transformation of a utility function represents the same preferences / indifference curve map):-

$$U_0=f(x)+y$$

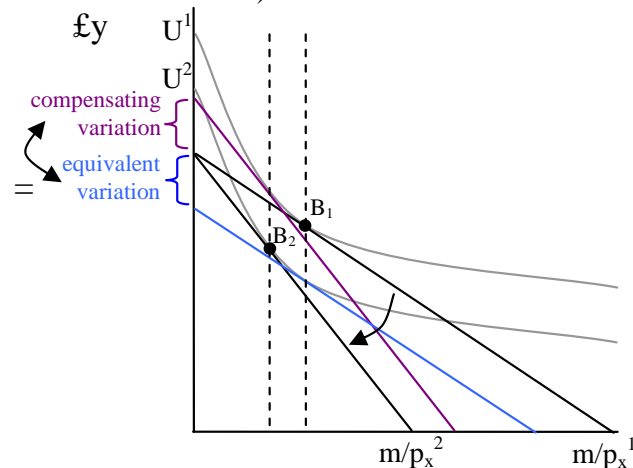
$$y=U_0-f(x)$$

So, the indifference curves are all the same shape, except they are vertically shifted up and down by the value of U_0 :



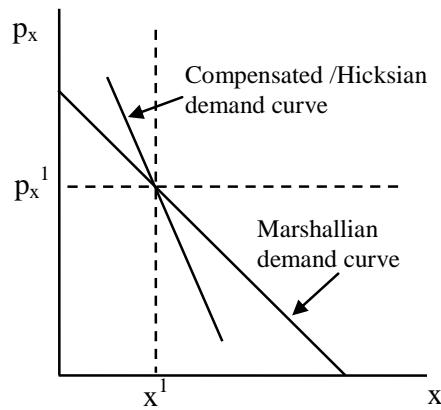
It also turns out that for quasi-linear preferences the CV and EV are always equal to the change in consumers' surplus. The intuition for this is not difficult: because the indifference curves are parallel (which means that the MRS at a particular quantity of x is the same on every indifference curve) then the distance on the y axis between the tangents to any two indifference curves at a particular value of x must be equal to the constant vertical distance between them (which is U^1-U^2 in terms of the value of the utility function). So, because the consumer's indifference curves are parallel, they will always require the same amount of good y, U^1-U^2 , to move back to their original indifference curve (the compensating variation), regardless of the values of p_x before and after the price change which changed their optimal bundle (and therefore utility level) (with non-quasi-linear preferences, this amount will change depending on the precise value of p_x , as we see in the earlier diagram illustrating the general case of non-equality between the CV and EV). This value $U^1 - U^2$ is also the amount of cash income that the consumer would be willing to pay to avoid having the change in the price of good x in the first place (the equivalent variation).

Another way to get to grips with the significance of quasi-linear preferences is to see that the income effect (i.e. parallel shift of budget constraint with no change of slope) only affects the amount of £y consumed (again, because the MRS at each value of x is the same for every indifference curve). So, the effect of a price change on **good x** is captured fully by the substitution effect (here we see the importance of explaining the income and substitution effect and their differing effects on the amount consumed of **both** goods). The significance of this is that it does not matter in the case of quasi-linear preferences whether we assess the size of the substitution or income effect for good x at the original “purchasing power” level or the new level (which is the essence of the difference between CV and EV).



Given that consumers' surplus does not generally accurately measure the total welfare gained from buying that good unless preferences are quasi-linear (a seemingly very special and specific case), why is consumers' surplus so central a concept in welfare economics? There are a number of very important reasons:-

- I. The CS is empirically observable (the demand function can be estimated by looking at combinations of price and demand observed at different times and, put rather crudely, plotting the line of best fit through it – this is an example of econometrics, the application of statistical techniques to fitting economists' models to the real world.) By contrast, the CV and EV cannot be determined unless we precisely know the consumers' preferences. We never observe preferences directly of course, only behaviour.
- II. It turns out that for normal goods the consumers' surplus is always somewhere in between the CV and EV. Imagine we could alter prices whilst at the same time altering income to keep the consumer on the same indifference curve. This would produce a **Hicksian demand curve** or **compensated demand curve** (see Katz and Rosen p. 115-118). Since along a compensated demand curve there is no (Hicksian) income effect, only a (Hicksian) substitution effect, the effect of an increase or decrease in price will (for a normal good) have a **smaller** effect on the amount of x demanded than on the standard demand curve (which is usually called the **Marshallian demand curve** if there is need to distinguish it from the Hicksian), because the normal demand curve measures both the substitution effect **and** the income effect. Since the income effect from a price decrease increases purchasing power (i.e. expands the budget set) and thus will increase the amount demanded of a normal good, and by the same token a price increase will decrease the amount demanded, for a normal good demand is **less elastic** along the compensated demand curve than along the Marshallian demand curve.

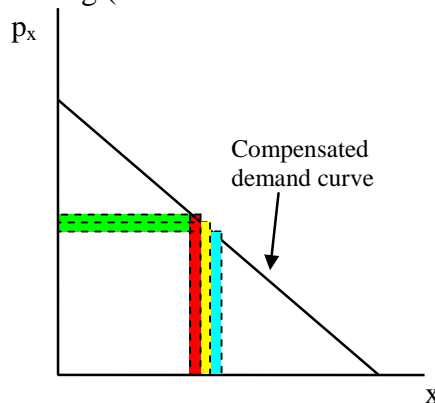


Along the compensated demand curve, as the amount of good x is increased (corresponding to a decrease in the price of x , i.e. a flattening of the budget constraint but remaining tangential to the same indifference curve), good y is taken away so that the following equation approximately holds (with the approximation getting better and better as Δx and Δy get smaller and smaller):

$$\begin{aligned} \Delta x \text{MU}_x &= -\Delta y \text{MU}_y \\ \Delta x (-\text{MU}_x / \text{MU}_y) &= \Delta y \\ \Delta x \text{MRS} &= \Delta y \\ \Delta x p_x &= \Delta y \end{aligned}$$

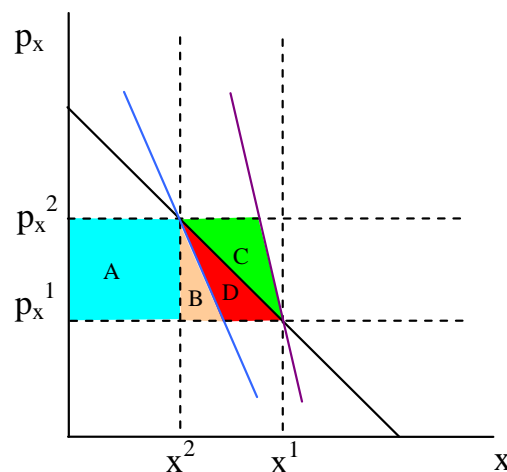
(due to the properties of the optimal bundle at an interior solution, note: we are assuming we are not at a corner solution)

As Δx is made arbitrarily small, $\Delta x p_x$ approximates the change in area under a demand curve, as shown by the three differently shaded rectangles to the right of the revenue rectangle which are approximating the increase in the area under the compensated demand curve as the price is decreased in the diagram below. Note that this is the change in the **total surplus** = *producers' + consumers' surplus* from the production and consumption of **good x** (under the assumption that the consumer **must** be kept on the same indifference curve). It has increased even though the consumer has stayed on the same indifference curve because we must assume that the cash paid by the consumer to the firm is spent by the firm on good y , so the firm gains $\Delta y = \Delta x p_x$ as described by the equation above, by being able to spend the extra revenue it receives. The rectangle above the revenue rectangle, on the other hand, although it represents revenue lost by the firm due to the cheaper price at which it must sell all of the other units of good x , is welfare gained by the consumer because they don't have to pay so much to get the units of good x they were already purchasing (so it does not cause a change in total surplus).



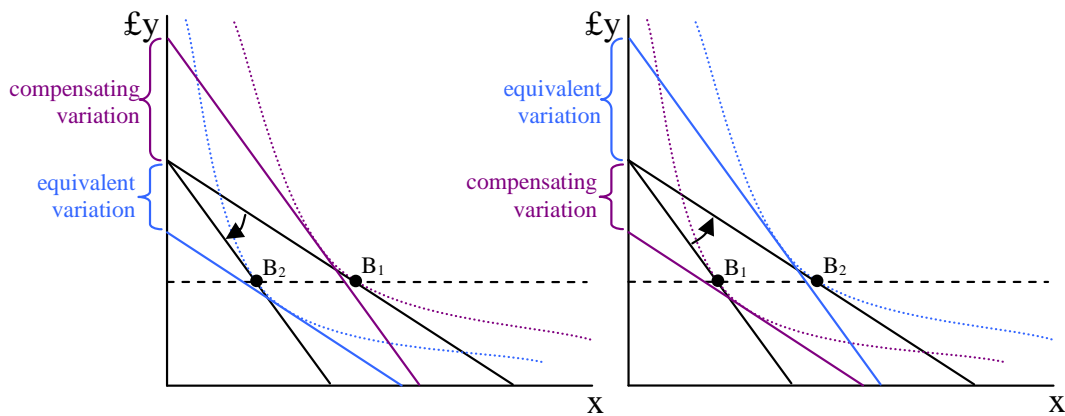
If it seems nonsense that extra welfare could just be “pulled out of thin air”, then this is because the above analysis is missing out an important consideration. In the graph above we are looking at the welfare created by the buying and selling of good x in isolation from good y . Although the total surplus from the buying and selling of good x increases by Δxp_x when the price drops, we must remember that $\Delta y = \Delta xp_x$ units of good y are being taken away from the consumer in order to keep them on their original indifference curve, so the total surplus from selling and consuming good x and good y remains equal (as we break up the price change into arbitrarily small blocks of Δxp_x). Note in this simple analysis that the firm has no significant role: it is simply there to spend the amount of money the consumer is willing to forego in order to buy the amount of x they choose in their optimal bundle on good y at the fixed price of $p_y=1$. It is the *consumer* who does all the resource allocation. In week 5, we will begin to model the activities of the firm more usefully, in terms of how different goods are **produced**.

The above analysis tells us that the total area under the compensated demand curve but above the price is equal to the amount of good y which would be required to compensate the consumer if they could no longer buy any of good x (i.e. if p_x went to infinity). As p_x is reduced or increased by a finite amount, the roughly trapezoidal area between the old and new prices and the compensated demand curve is equal to the amount of good y that the consumer would be willing to pay to cause or prevent the price change. So, for a price increase, when this area is evaluated using the compensated demand curve at the original price, it is the compensating variation ($A+B+C+D$), and when this area is evaluated using the compensated demand curve at the new price, it is the equivalent variation ($A+B$). The change in consumers’ surplus is the change in the area under the Marshallian demand curve, which is $A+B+D$. Hence we can see that for a normal good with a price increase $CV \geq CS \geq EV$. When preferences are quasi-linear, there is no income effect on the amount of x demanded, and so the three demand curves are identical, and so $CV=CS=EV$.

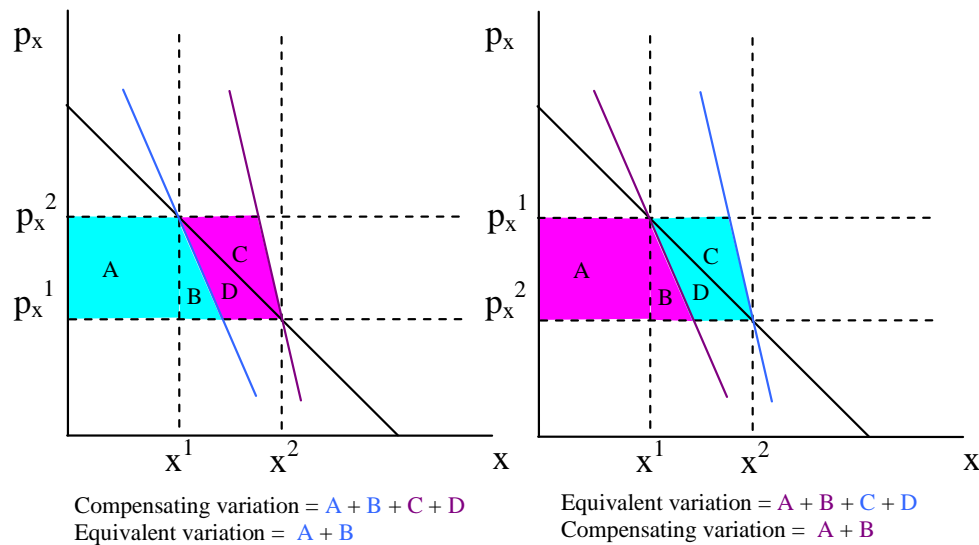


For a normal good, for an *increase* in the price of good x , the compensating variation must be greater than or equal to the equivalent variation because once p_x has increased, with p_y remaining constant at 1, it **must** cost more to compensate the consumer in get them back to their original indifference curve than would be required

to take from them at the original price level to take them from the old to the new indifference curve, because both goods are at least or more expensive than before (another way of saying this is that the marginal utility that the consumer gets from each additional unit of cash income must be lower at the new price level, so more cash income must be given to get the consumer back to their original utility level). For a *decrease* in the price of a normal good, this result is reversed. Now the equivalent variation must be greater than the compensating variation because the marginal utility of income is higher at the new price level, so less cash income needs to be taken away in order to get the consumer back to their original utility level. (Note that we have illustrated the CV and EV with Cobb-Douglas preferences with no cross-price effects since this ensures that both goods are normal and thus allows us to illustrate accurately and graphically the difference in size between the CV and EV.)



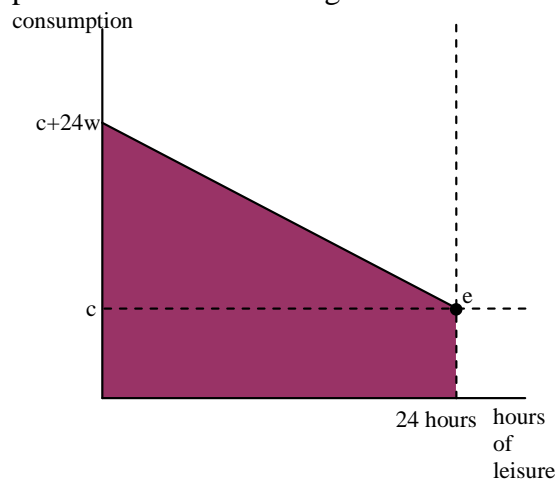
In terms of the interpretation using the compensated demand curves, if we reverse the increase in price to a decrease in price, the compensating variation of the price increase becomes the equivalent variation of the price decrease, and the equivalent variation of the price increase becomes the compensating variation of the price decrease, hence the inequality $CV \geq EV$ becomes $EV \geq CV$:-



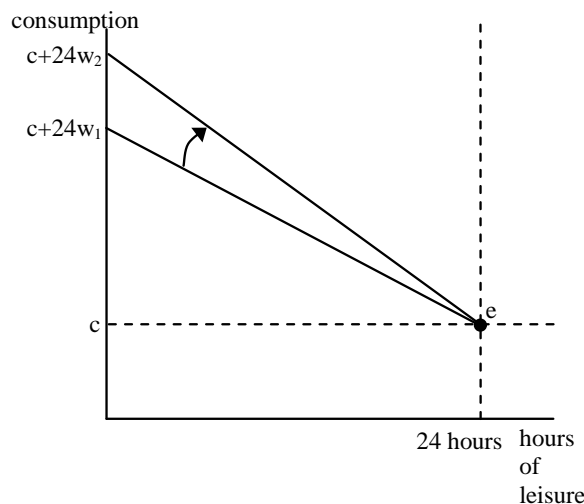
III. We have seen that the consumers' surplus is in principle observable, and always lies somewhere between the EV and the CV for a normal good. The third reason why it may not be such bad measure of the change in welfare after all is that provided the good whose demand we are looking at makes up a small part of consumer expenditure, the income effect of a price change will be small, and so the compensated demand curves will be virtually parallel to the Marshallian demand curves, and so the CV, CS and EV will all be so close together that the change in CS to all intents and purposes represents the change in welfare whichever way we look at it. Another way of saying this is that for a good which makes up a small part of total expenditure, a quasi-linear utility function may be a good model (provided we are only interested in that good and the numeraire good, i.e. only have a 2 good model).

2. (a) In order to apply a two-good model with well-behaved preferences, the good on the x-axis cannot be a "bad" – labour supply. Therefore we think of leisure as the "absence of labour" and plot leisure on the x-axis against consumption on the y-axis.

(b) (i) The slope of the budget constraint depends on the wage as illustrated with the daily budget constraint faced by a worker who has a fixed hourly wage rate w and fixed unearned consumption income c . The budget set is shaded.



When the wage increases, the budget constraint pivots around the endowment point e as illustrated below:-



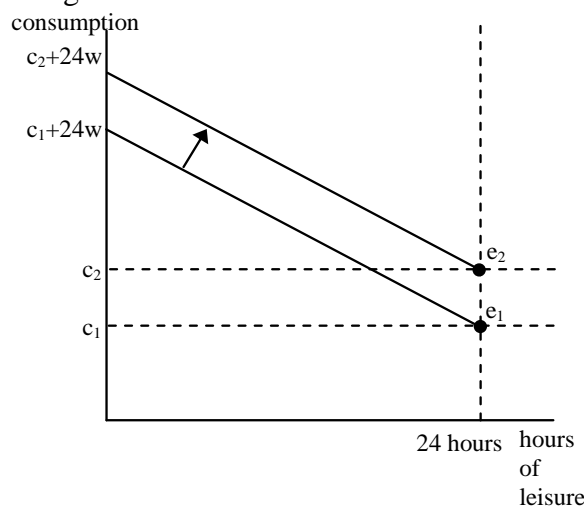
(ii) If the hourly wage changes depending on how many hours the worker works (e.g. overtime pay or different hourly tax rates, as with different tax bands in progressive taxation) then the budget constraint will be kinked.

(iii) If the worker cannot freely vary their hours (e.g. the employer and/or union fixes the length of shift, which the worker must then either take or leave) then the budget constraint may be discontinuous. There may also be fixed costs which do not depend on the number of hours worked (e.g. travel to work). Additionally, once workers work a certain number of hours, they may lose entitlements to lump sum benefits, causing a sudden jump in consumption.

(c) The following table illustrates the effect of a wage increase and an increase in fixed income on the amounts of consumption and leisure consumed depending on whether consumption and leisure are normal or inferior goods (Note: consumption and leisure cannot both be inferior, under the assumption of non-satiation, since a non-satiated consumer must take advantage of the expansion of the budget set to consume more of at least one of the two goods):-

<i>w</i> increases ↑	Effect on amount of leisure chosen			Effect on amount of consumption chosen		
	Substitution effect	Income effect	Overall effect	Substitution effect	Income effect	Overall effect
<i>Consumption inferior</i>	↓	↑	?	↑	↓	?
<i>Leisure inferior</i>		↓	↓		↑	↑
<i>Both normal</i>		↑	?		↑	↑

An increase in c produces a pure income effect because the slope of the budget constraint does not change:-



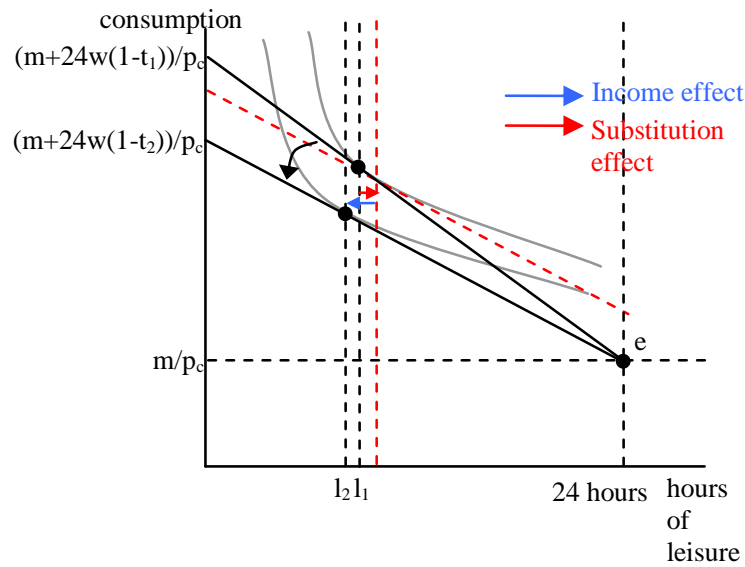
<i>c</i> increases ↑	Effect on amount of leisure chosen	Effect on amount of consumption chosen
<i>Consumption inferior</i>	↑	↓
<i>Leisure inferior</i>	↓	↑
<i>Both normal</i>	↑	↑

There are some important predictions from the theory. Provided people have consistent, well-behaved preferences, we can be sure that:-

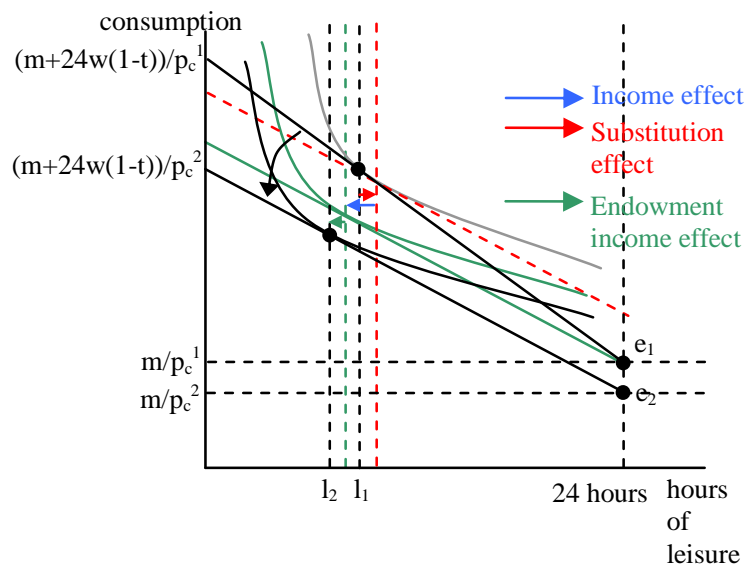
- If leisure is inferior then a wage increase must always reduce the amount of leisure demanded (i.e. increase the amount of labour supplied). So, a theory of labour demand in which leisure is assumed to be inferior would be unable to deal with cases where the opposite occurs in the real world.
- If consumption is inferior then the effect of an increase in c is to reduce the amount consumed at the optimal bundle. This means that none of the increase in guaranteed unearned income is consumed, i.e. that not only is all of the extra income converted into extra leisure, but that so much more leisure is taken in response to greater fixed consumption income that consumption actually *decreases*. This is highly implausible.
- If both leisure and consumption are normal goods, then an increase in the wage rate always increases the amount of consumption chosen. Unlike the most restrictive consequences of the other two assumptions, this does not seem implausible.

(d) We now deepen the analysis slightly by introducing three new variables, the cost of consumption, p_c , fixed cash income m and a proportional tax rate, t . The worker now earns cash income from working which they then use to purchase units of consumption at price p_c . If the value added tax rate is v , then $p_c=(1+v)$. Suppose that the hourly wage rate w is the same whatever the number of hours worked. The worker's cash income will be $w(1-t)(24-l)+m$ where l is the amount of leisure taken. This will purchase a number of units of consumption equal to $(w(1-t)(24-l)+m)/p_c$, which in turn is equal to $(w(1-t)(24-l)+m)/(1+v)$.

(e) (i) An increase in t from t_1 to t_2 will pivot the budget constraint about the endowment point and alter the worker's labour supply decision in the manner illustrated below. Since the income and substitution effect operate in opposing directions, the overall effect will be ambiguous. In this example, the income effect outweighs the substitution effect, and so the worker chooses less leisure after the tax increase (i.e. supplies more hours of labour). This illustrates the important possibility of a backwards-bending labour supply curve, i.e. one where a wage decrease or tax increase increases the amount of labour supplied. However, it is also possible that the substitution effect would outweigh the income effect and so hours worked would be reduced by the tax rise.

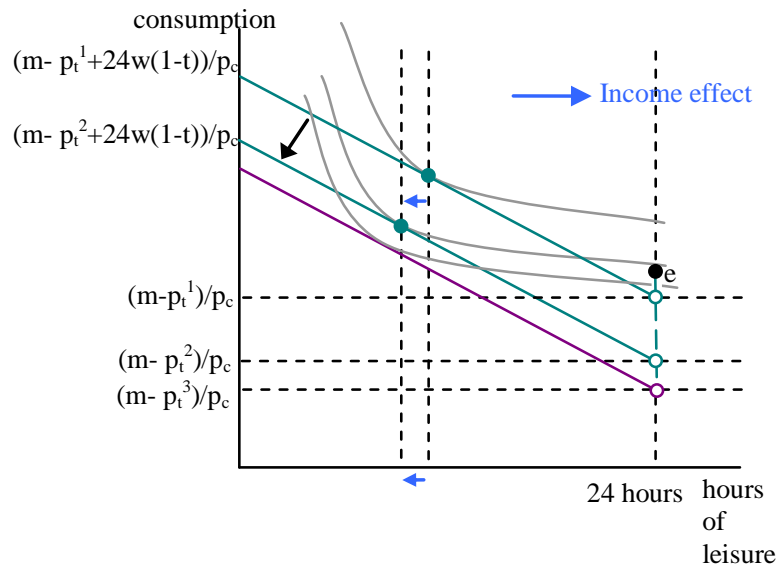


(ii) Now suppose that a rise in indirect taxation causes a rise in p_c :-



There is an additional income effect due to the shift in the endowment point, making it more likely that a rise in indirect taxation will increase the amount of labour supplied.

(iii) Now suppose that the worker commutes to work by train each day. Only one train ticket is bought per day, so the effect of an increase in its (pre-VAT) price p_t is almost identical to a reduction in c in the model in parts (i) and (ii) except that there is a discontinuity in the budget constraint:



If 24 hours of leisure are taken (i.e. 0 hours worked) then the endowment point e is achievable. Once a positive number of hours are worked, consumption must be immediately reduced by p_t/p_c . Suppose the cost of train tickets is initially p_t^1 . If it increases to p_t^2 we can see that the amount of leisure consumed unambiguously decreases (i.e. labour supply **unambiguously increases**). As long as we remain on the continuous part of budget constraint where labour supply is positive, and leisure is a normal good, this must be the case.

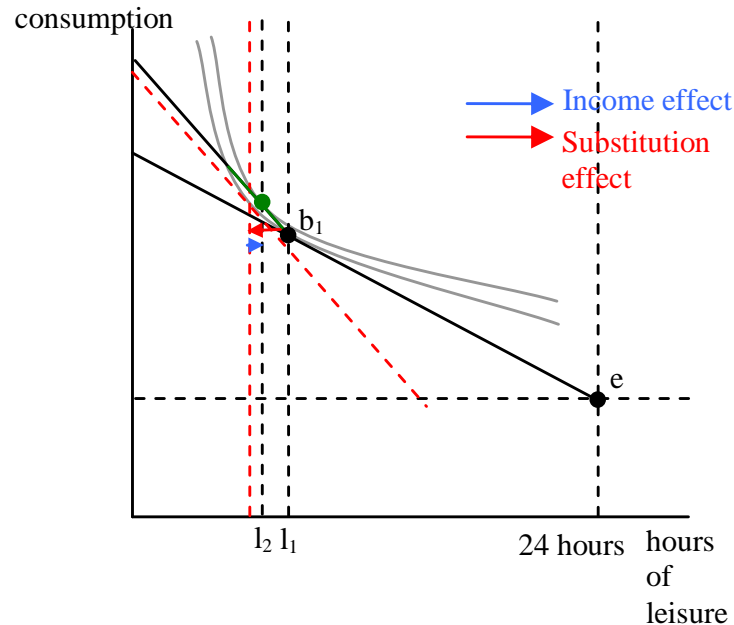
However, suppose the price of tickets were to increase to p_t^3 . At the optimal point on the purple budget constraint where labour supply is positive, the worker is on a lower indifference curve than if they were to choose the endowment point e . So, the optimal choice would be the endowment point (it is on the highest indifference curve which can be reached within the budget set). So, the introduction of discontinuities into the budget set creates discontinuities in behaviour. As p_t is increased, the amount of labour supplied will increase up to a certain point, and then jump to 0 as the worker decides it is better to stay at home.

If we imagine p_t instead represents the loss of lump sum benefits once a worker begins working, this analysis shows that introducing this discontinuity will increase labour supply for workers on the continuous section of the labour supply curve. However, there may also be a group of workers who choose to stop working altogether. This illustrates a common problem in public policy: creating extra work incentives for some workers via taxes/benefits often involves reducing work incentives for others.

- (iv) Now, suppose that the worker is able to earn a higher overtime rate if they work more hours than at b_1 . This creates a kink in the budget constraint around b_1 . By the principle of revealed preference, we know that the dark green section of the new budget constraint lies on higher indifference curves than the one passing through b_1 , so the worker must choose a bundle along this segment of the line. The substitution effect increases the amount of labour supplied (decreases the amount of leisure taken). The income effect increases the amount of leisure taken. However, we know in this case that the substitution effect must outweigh the income effect. If you use the

Slutsky decomposition, then the change in labour supply brought about by the introduction of the overtime rate can be thought of as a pure substitution effect, since the new budget constraint goes through the original bundle B_1 .

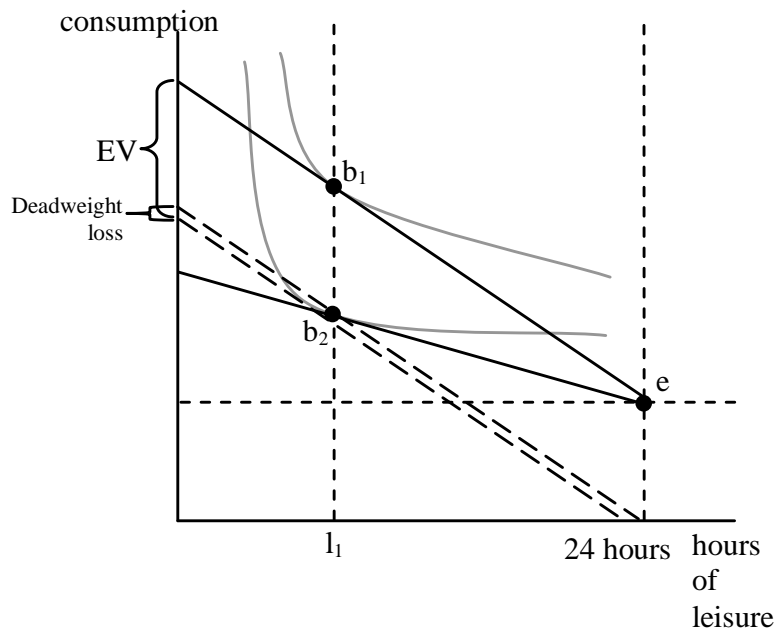
This provides an appealing explanation for why overtime rates are paid to workers: providing a higher rate just on those extra hours produces less of an income effect (which increases leisure taken, i.e. decreases labour supplied) than if a higher rate were provided on all work hours, so it is a more cost effective way for an employer to give the worker an incentive to work extra hours.



(v) The diagram below illustrates the key theoretical result that a given tax revenue can be raised more efficiently by a lump sum tax than an income tax. The equivalent variation of the increase in the income tax is always greater than the tax revenue. Hence there is a deadweight loss relative to a lump sum tax which simply directly takes the revenue away from the worker without altering their marginal tax rate (the slope of the budget constraint).

This model assumes looks at the case of a single consumer. If all consumers were identical, it would indeed be optimal to take away the lump sum tax rather than have an income tax. The reason this does not work in the real world is arguably inequality. Since consumers have different abilities, we cannot levy a lump sum tax on all, since to make it equal would offend most people's sense of justice. Also, we would not be able to raise enough revenue with a lump sum tax that both rich and poor can afford, Hence, in reality there is a trade off; the more progressive the income tax, the higher the tax rate and hence the larger deadweight loss it creates.

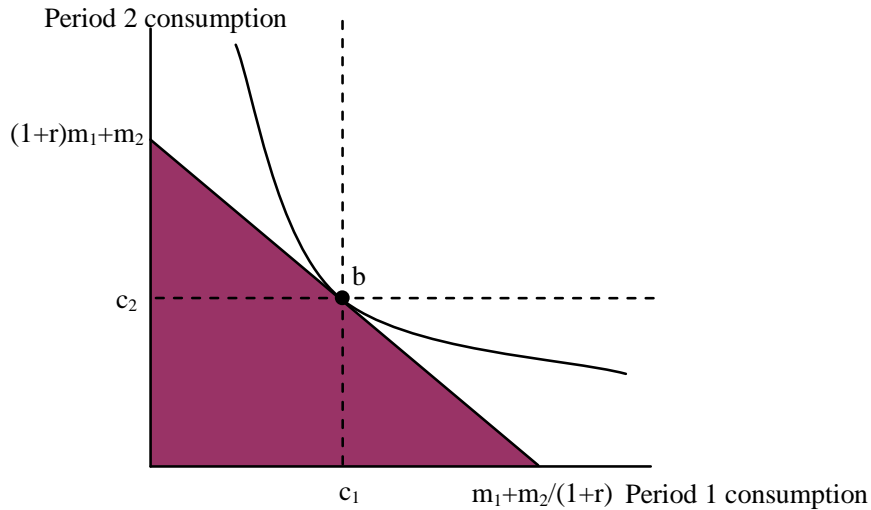
Note also that the argument *does not* depend on the introduction of the income tax altering the labour supply. In the diagram below, the labour supply remains the same before and after the introduction of the tax. However, there is still a deadweight loss from the income tax relative to a lump sum tax. This is because it is the substitution effect that distorts the economy. Just because the income effect counteracts (or even outweighs) it does not mean that decisions have not been distorted at the margin.



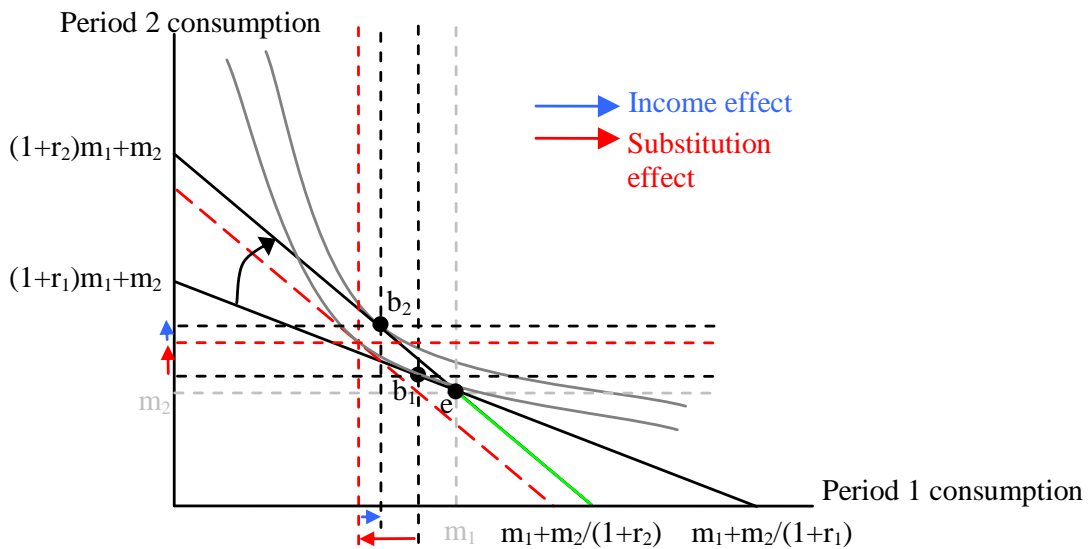
3. (a) The assumption that consumers wish to smooth out consumption between periods is the application of the standard assumptions of strict convexity to the problem of inter-temporal choice. It is also intuitively plausible, since people do save for retirement and borrow when they are poor students, thus smoothing out their consumption between periods. (Note: we are assuming throughout the following analysis that consumption in each period is a normal good).

(b) If consumption in periods 1 and 2 were perfect complements, there would be complete consumption smoothing (a change in interest rates would then only have an income effect, not a substitution effect, due to the “kinky” solution at the optimal chosen bundle).

(c) Suppose that the consumer has fixed income of m_1 in period 1 and m_2 in period 2. Assume that the consumer may save or borrow as much as they like at the current interest rate, r . So, £1 saved in period 1 will give $£(1+r)$ in period 2. Equivalently, to borrow £1 in period 1 will cost $£(1+r)$ out of income in period 2. So, the present (period 1) value of the consumer’s endowment is $m_1 + m_2/(1+r)$. The inter-temporal budget constraint will look like this. (The vertical axis intercept is the future value of the consumer’s endowment, which is $(1+r)$ times the present value.):-

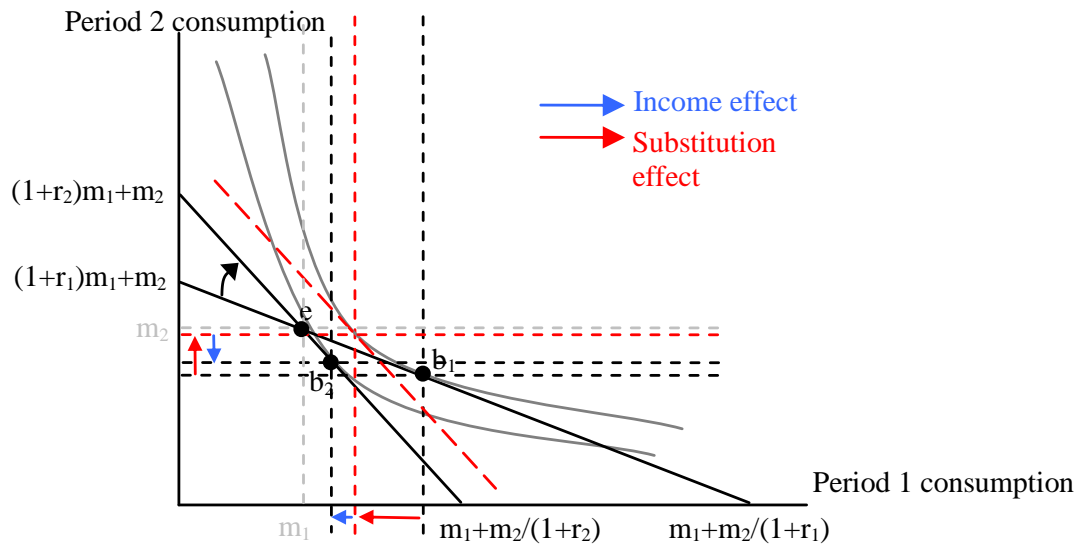


(d) (i) The diagram below illustrates the effect of a rise in the interest rate from r_1 to r_2 when the consumer is already a lender. The effect on first period consumption is ambiguous. The effect on the amount saved/lent is therefore also ambiguous. Second period consumption unambiguously increases.

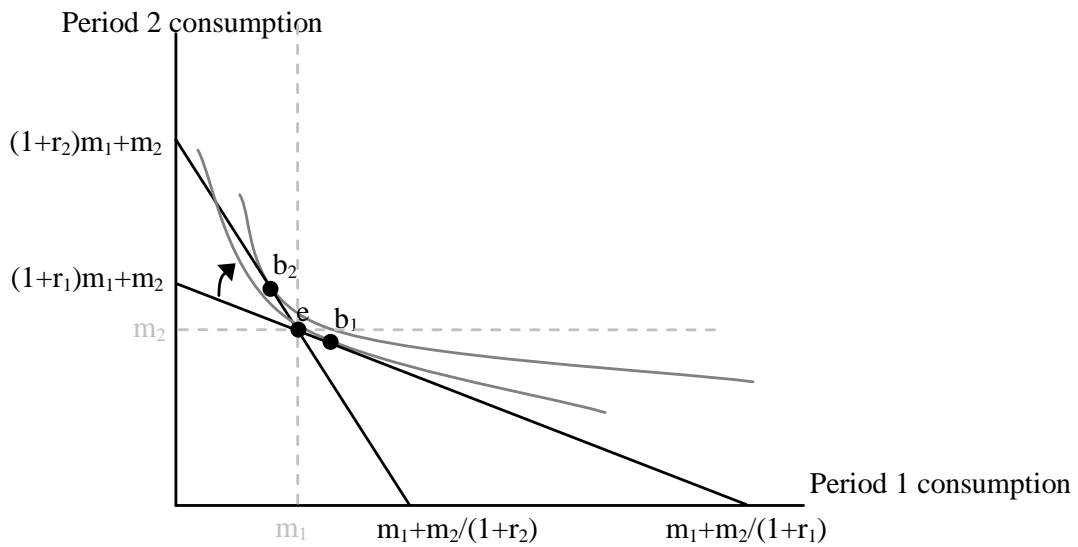


The principle of revealed preference tells us that a lender will never become a borrower, because the green section of the new budget constraint lies within the original budget set, so if any point on it was preferred to b_1 it would have been chosen originally.

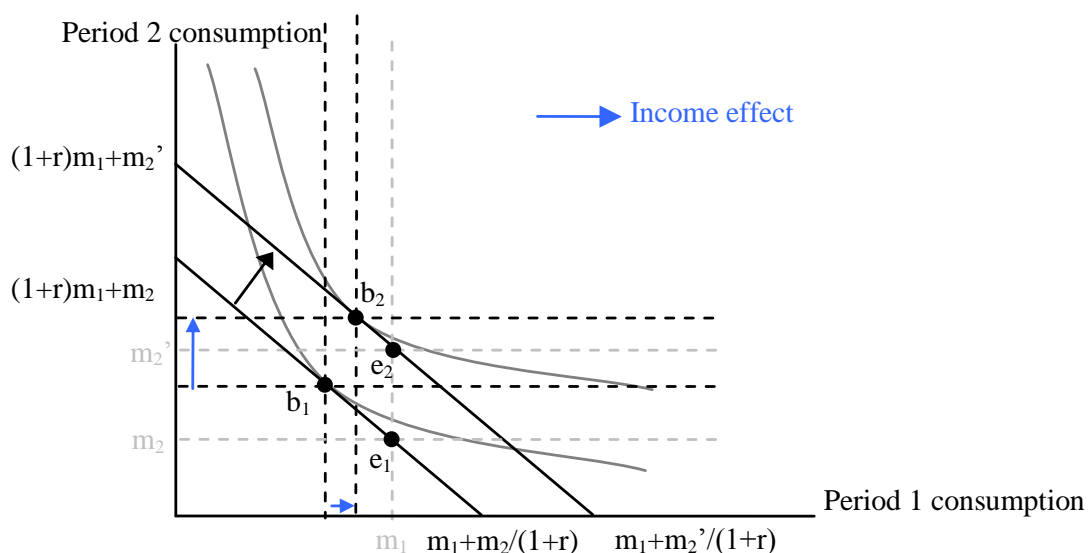
The response of somebody who is already borrowing is shown in the diagram overleaf. Period 1 consumption is unambiguously reduced (and therefore the amount borrowed unambiguously decreases) whereas the effect on period 2 consumption is ambiguous:-



It is possible that an increase in the interest rate could lead a borrower to become a lender, if their indifference curves looked something like this. In this case, they will, by revealed preference, definitely become better off:-

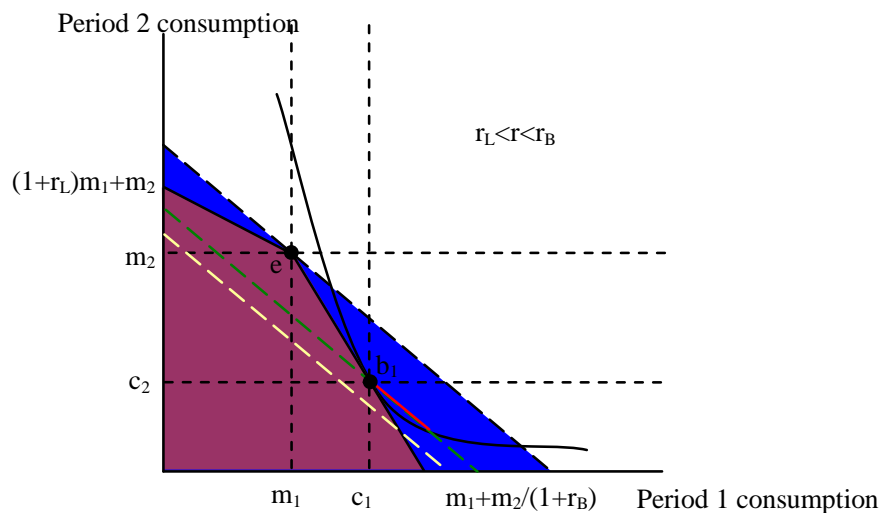


(ii) An increase in period 2 income causes an income effect only. This will increase consumption in both periods whether the individual is a borrower or a lender. So, saving is unambiguously reduced (or borrowing unambiguously increased).



(e) Assume that there is some kind of underlying interest rate r available on all investments made by a bank. The bank, and the government's tax treatment of saving and borrowing, then becomes a kind of intermediary process between the consumer and the underlying investment process. The bank borrows money at interest rate r_L from consumer lenders and lends money at interest rate r_B to consumer borrowers. There are a number of reasons why we would expect r_L to be lower than r_B :

- 1) Banks must pay their operating costs and make profits. They do this by charging more to lend money than they do to borrow it (i.e. the interest rate on loans is greater than the interest rate on savings.)
- 2) Banks lending money to consumers tend to be taking more of a risk than consumers lending to banks (e.g. the consumer is more likely to default or have problems paying on time whilst the bank is very unlikely to go bust). Banks must take into account the fact that a certain amount of consumer loans will go bad by charging a higher interest rate on all of them (similar principle as insurance companies).
- 3) Interest earned on savings is often treated as income and therefore taxed, thus leading the after-tax interest rate for lenders r_L to be lower than the interest rate paid by the bank.



The wedge driven between the two interest rates leads to a kink around the endowment point as shown in the diagram above. The budget set shrinks (from the whole shaded triangle to the light grey area only) relative to the situation where the consumer can save/borrow at the same core interest rate, so provided the consumer's preferences satisfy non-satiation, they must be made worse off. We saw earlier that if the consumer is a borrower, then a rise in the interest rate must reduce the amount borrowed, because the income and substitution effects work in the same direction. The effect on the amount saved by a lender is ambiguous because although the substitution effect leads them to consume more in period 1, the income effect leads them to consume less in period 1 (although period 2 consumption unambiguously decreases). So, the shrinking of the budget set due to this kink will definitely discourage borrowing, and may discourage saving.

We may intuitively feel that the introduction of this kink is a bad thing, but the conceptual tools we have been developing allow us to make a more rigorous argument. Under the kinked budget constraint the consumer optimally chooses point b_1 . Suppose we could somehow cut out the bank as an intermediary and allow the consumer to save/borrow at the underlying interest rate. The consumer would be able to reach a higher indifference curve by choosing a point somewhere along the bright red segment of the line.

Now imagine instead of having private profit making banks the government runs a national bank, and simply takes income directly from the consumer to pay the administrative costs. This would be equivalent to a parallel shift of the original budget constraint inwards. The consumer would be able to borrow at the underlying interest rate and so this **allocative inefficiency** would be avoided. Would this make the consumer better off? Not necessarily, because a government owned bank would have a monopoly and would probably be much less likely to achieve **productive efficiency**. If the shift to the dark grey dotted line were adequate to raise the money required to pay the administrative costs then the consumer would be made better off (this would be the case if the publicly owned bank only used up the same operating costs as a privately owned bank) However, if a shift to the light grey dotted line were required to pay the administrative costs (i.e. the publicly owned bank is less efficient than this), then the consumer would still be better off under a private banking system.

4) Explain how the Weak Axiom of Revealed Preference and the Laspeyres and Paasche price indices could be used to work out if a consumer is better or worse off after a simultaneous change in income and prices. Can we always say unambiguously that a consumer is made better or worse off if (i) the price of only one good changes, (ii) only money income changes and (iii) prices and income change at the same time?

The Laspeyres and Paasche price indices are both measures of the overall price level, calculated as the (nominal – i.e. using actual market prices) cost of a representative bundle of goods during a particular time period (we often think of time being split up into discrete periods in economic theory). There is a difference between the two indices if the bundle of goods chosen changes between two periods, because if we want to compare the price level in the two periods then we must use the consumption bundle during one of the periods to derive the weightings for the prices of the different goods in the representative bundle. If we choose the earlier, or base, period b , we have the Laspeyres price index. If we choose the later period t , after prices and consumer behaviour have changed, we have the Paasche price index.

The Paasche price index is therefore the ratio of expenditure at period t to expenditure at period b using the new consumption bundle at period t to calculate the weightings. For a two good model (goods 1 and 2), the formula would therefore be:

$$P_P = (p_1^t x_1^t + p_2^t x_2^t) / (p_1^b x_1^t + p_2^b x_2^t)$$

The Laspeyres price index is a similar ratio, except that the old consumption bundle at period b is used to form the weightings:

$$L_P = (p_1^t x_1^b + p_2^t x_2^b) / (p_1^b x_1^b + p_2^b x_2^b)$$

It is obvious that if at period t the prices of all goods are equal to or higher than the prices at period b , a consumer (assuming that income remains the same and that they have no endowment of any of the goods which they can sell) will be worse off. Both the Paasche and Laspeyres price indices will be greater than 1, indicating that price levels have unambiguously risen. If all prices during period t are less than or equal to those during period b , then a consumer whose income remains fixed must become better off. Both the Paasche and Laspeyres price indices will be less than 1, indicating that price levels have unambiguously dropped. Since in this case the expenditure index is equal to 1 (since income remains unchanged; see definition in paragraph below) then if all prices remain constant or rise (with income staying unchanged) then $L_P > M$ and $P_P > M$. If all prices remain constant or drop (with income staying unchanged) then $L_P < M$ and $P_P < M$.

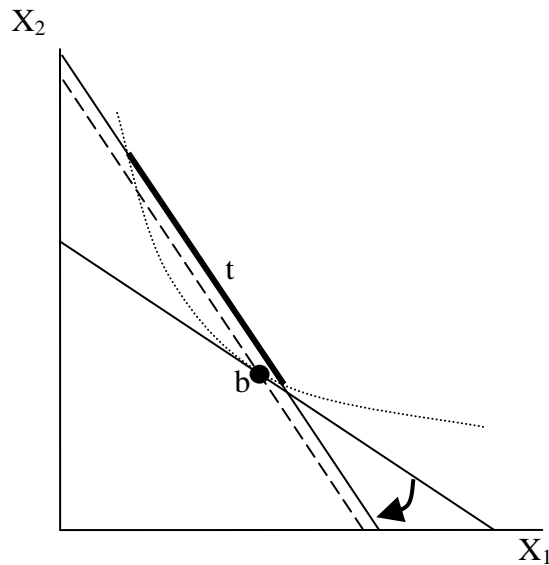
If all prices between period b and t remain fixed, and the only thing that changes is income, then both the Laspeyres and Paasche indices will be equal to 1. The expenditure index is the ratio of income in period t to income in period b which, given non-satiation, is equal to total expenditure. Therefore:

$$M = M_t / M_b = (p_1^t x_1^t + p_2^t x_2^t) / (p_1^b x_1^b + p_2^b x_2^b)$$

So, it is obvious that if $M > L_P$ and $M > P_P$ then (if all prices remain unchanged) the consumer is better off and if $M < L_P$ and $M < P_P$ then the consumer is worse off.

The above cases are specific examples of the more general rule that the consumer is unambiguously better off if $P_P > M$ and unambiguously worse off when $L_P < M$. However, when there is a simultaneous change of prices and income between period b and t , or if prices change in different directions, it need not necessarily be the case that either of these inequalities be true. In this case, if we knew the positions and shapes of the consumer's indifference curves then we could clearly know whether the consumer is better or worse off. However, given that we do not have this information, the Laspeyres and Paasche price indices by themselves do not give us enough information for us to tell whether the consumer is better or worse off.

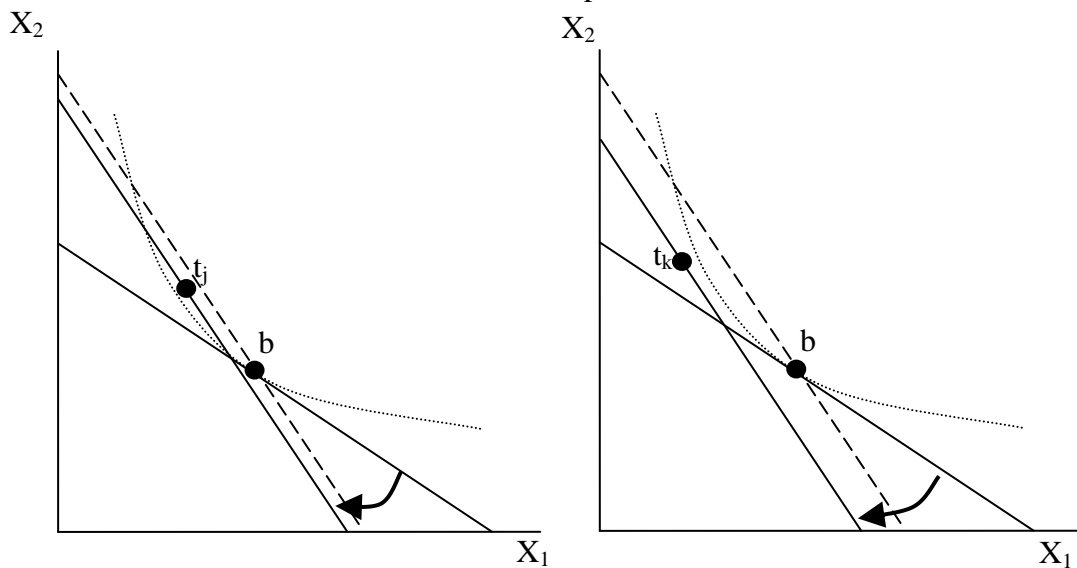
To see why the above inequalities are useful to us, we need to use the Weak Axiom or Revealed Preference. This states essentially that if a bundle was affordable before a change in the budget set, and was not chosen, then it will never be chosen under the new budget constraint, even if it is still affordable. Suppose we know that $L_P < M$. This inequality can be rearranged using the definitions above to give that $(p_1^t x_1^b + p_2^t x_2^b) < (p_1^t x_1^t + p_2^t x_2^t)$. This inequality states that the new chosen bundle after the change in the budget set is worth more at the new price ratio than the old chosen bundle. This means that the old bundle is affordable within the new budget set. However, if it is affordable and not chosen, then the actual chosen bundle must be better than the original bundle. Another way to see this is that geometrically for the new bundle to be chosen over the bundle a, it must lie on a higher indifference curve (i.e. be somewhere along the thick shaded line in the diagram below).



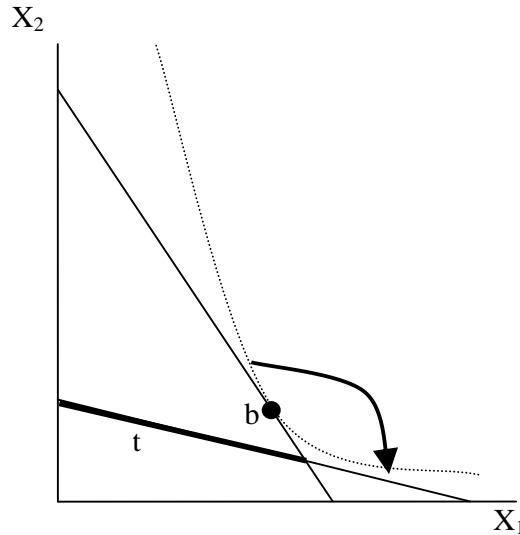
Suppose instead we knew that $L_P > M$ and therefore that:

$$(p_1^t x_1^b + p_2^t x_2^b) > (p_1^t x_1^t + p_2^t x_2^t)$$

Thus we know that the new bundle is worth less than the original bundle at the new price ratio. This tells us nothing because if the new bundle were at point t_j , it would be worth less than the original bundle at the new price ratio, but preferred to it, whereas if it were at t_k , it would be worth less and less preferred to it.



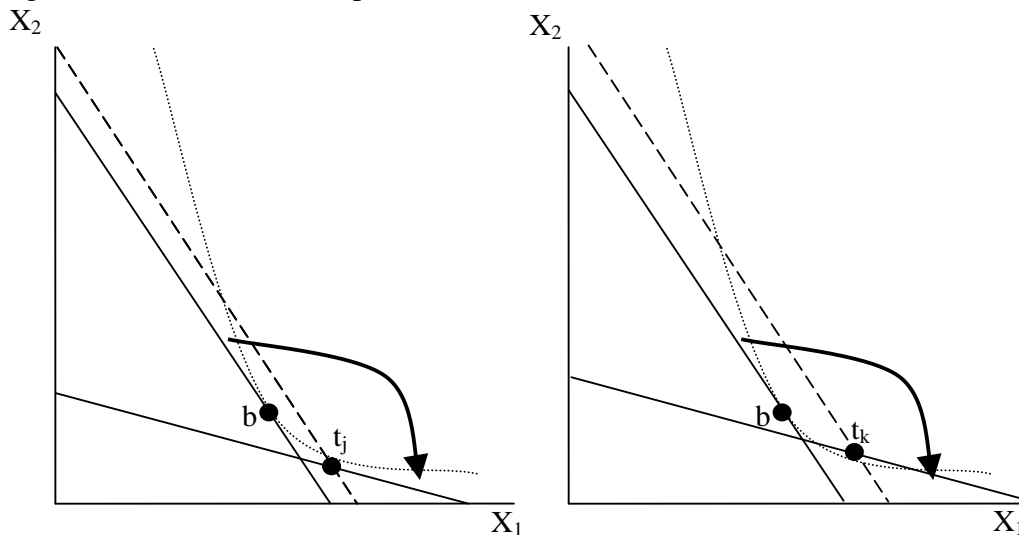
Now suppose we know that $P_p > M$. Rearranging this inequality gives us that $(p_1^b x_1^t + p_2^b x_2^t) < (p_1^b x_1^b + p_2^b x_2^b)$. This tells us that the new bundle is worth less than the old bundle at the original price ratio. For this to be the case, the new bundle would have to be on the thick dark line in the diagram below. However, by revealed preference we know that since this section was affordable under the original budget constraint, it must be less preferred than bundle b.



Finally, suppose that $P_p < M$. Rearranging this inequality gives us that:

$$(p_1^b x_1^t + p_2^b x_2^t) > (p_1^b x_1^b + p_2^b x_2^b)$$

This tells us that the original bundle is worth less than the new one at the original price ratio. However, this does not imply that the new bundle is necessarily better or worse than the original. Points t_j and t_k are both worth more at the original price ratio, but the first represents a case when the consumer is made worse off by a price/income change, whereas the second represents a case where the consumer is made better off.



To conclude, when all prices move in the same direction and income remains constant, or when all prices remain fixed and income changes, one of the two inequalities $P_p > M$ and $L_p < M$ will definitely be fulfilled, and so we can see directly by revealed preference whether the consumer is made better or worse off, without needing to know their actual indifference curve map. However, when price and incomes change together in a more complex way, it is sometimes the case that neither of these inequalities is fulfilled, in which case we would need more information in order to work out whether the consumer is better or worse off.