The Microeconomic Basis of Imperfect Altruism

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Abstract

Individuals in the real world exhibit imperfect altruism. There are two distinct kinds of imperfections. The first is that individuals appear to weight the utility of some individuals more than others. We refer to these types of imperfection as "asymmetries of altruism". The second kind of limitation upon altruism is "paternalistic". Individuals place different weightings not only on different people, but on different goods. They do not value other individuals' consumption of goods according to the other individuals' subjective weighting, but according to their own. In other words, even individuals with symmetric altruism disagree as to which bundle of goods production and consumption maximizes the common good. Both these types of imperfection are shown to result in the empirical prediction that individuals will reduce their level of altruistic giving of resources to other individuals relative to the case of perfect altruism where all individuals agree upon the common good and care for every individual symmetrically. In the case of asymmetric altruism this occurs because individuals care for themselves and those close to them more than the rest of humanity. In the case of paternalistic altruism because individuals will be more inclined to aid those individuals whose view of the common good more closely matches their own. This paper explores the relationship between these two types of limitation upon altruism, and argues that there are good methodological and philosophical reasons to adopt paternalism rather than asymmetry as the basis of a microeconomic theory of imperfect altruism.

1 The Self-Interest Assumption

The assumption of rational self interested behaviour has been a highly fruitful one in economic theory. It has a simplicity and elegance that has allowed economics to live up to its founders' project of showing how a socially desirable spontaneous order can arise in market society which, as eloquently expressed by [Hayek, F. A. 1960], encompasses the knowledge and aims of many individuals.

What remains unsatisfactory is the degree to which the standard paradigm of welfare economic analysis continues to assume that whilst social institutions may ideally be designed by social planners with a utilitarian outlook, the ordinary individuals who act within these institutions usually remain highly rational but at the same time highly selfish. The issue of whether this approach is adequate for normative theoretical analysis in economics takes us right to the heart of issues in the philosophy of the social sciences.

Much of the initial attraction and success of the self interest assumption, as well as the apparent explicit basis for the more recent work which has questioned or modified it, is based on a narrowly "positivistic" view of economics. This label is intended not to refer to a specific detailed position on the ontology and epistemology of social sciences, but rather to the view, most famously exposed by Karl Popper, that the value of the social sciences over and above mere superstition is their ability to make testable predictions [Popper, K. 1959].

In this light, the attractiveness of the self interest assumption is clear; it provides a good basis point for a research programme in the social sciences. This concept was introduced by one of Popper's followers Lakatos. He distinguished [Lakatos, I. 1970] between the "core" of a research programme, a set of firm theoretical principles which are not questioned, and the peripheral ideas that are brought into a theory as it is tested against reality and modified. As time goes on and the possibilities of adaptation of the core principles become exhausted, it will become clear which areas of reality are difficult to explain by a particular research programme. Eventually, "progressive" research programmes become "degenerative" ones, with the peripheral assumptions becoming increasingly incoherent. With this approach, Lakatos sought to explain the discontinuities in scientific progress which Popper's theory found more difficult to explain.

The core of the standard research programme in economics could therefore be thought of as individual rationality and self interest, whereas asymmetric information, and different equilibrium modelling techniques would correspond with peripheral components introduced and altered more freely in order to improve the fit between theory and reality. The recent increase in interest in dropping the self interest assumption in empirical applications could be seen as a sign that this research programme has begun to exhaust the possibilities, so that in certain empirical areas it is time to consider a different research programme.

There are problems, however, with making this the philosophical justification for taking economics into the arena of altruistic motivation. Firstly, as a predictive framework, the self interest framework is still highly productive. It would seem that if altruism is to be introduced into the body of the economic theory on this pretext, it will remain fairly ad hoc, in the sense that it will only be used when the self interest assumption evidently fails empirically. Secondly, one senses that the attraction of altruism is not merely that it provides an alternative predictive research programme. It can be argued that economics has neglected a large area of human potential, and left the consideration of human morality and socialization largely in the hands of sociologists, when there is nothing inherent in the economist's arsenal of rigorous tools and techniques which prevent them from being

applied in this area. This kind of thinking, however, requires a more subtle view of the role and potential of the social sciences in the body of human intellectual endeavour.

Kuhn took a more radical view of progress in the natural sciences than Popper and Lakatos. He argued [Kuhn, T. 1977] that the decision between what Lakatos would have called "progressive" and "degenerative" research programmes (the process which Kuhn referred to as "paradigm shift") could never be nailed down to evidence is such a simple way that all scientists could agree which programme provided the best potential for future development. Whenever paradigm shifts occur, there are competing considerations or **scientific values** which must be balanced. Ultimately, only the conscience of the individual scientist can decide.

A good example of this is Kuhn's account of the paradigm shift from eighteenth century Phlogiston theory to the modern paradigm in chemistry. When Phlogiston theory was abandoned in favour of the theory based on elements in the early nineteenth century, many older members of the chemistry profession resisted because they remained attracted by the wide degree of physical properties which Phlogiston theory was apparently able to explain. In contrast, the main attraction of the new theory was that despite its limited explanatory scope, it was able to make predictions of mathematically precise proportions of ingredients in chemical reactions. Eventually, the attraction of the value of mathematical precision won the day, but it took more than another 150 years for developments in quantum mechanics to allow the new mathematical chemistry to explain the same breadth of physical features as the old Phlogiston theory.

The relevance of this analogy for the philosophy of economics is that the decision between different research programmes or paradigms in economics must also ultimately appeal to values such as aesthetic appeal as well as sheer "number of facts explained" (which is of course a concept incapable of precise and rigorous operationalization anyway). Another factor which must be considered is the ethical dimension to the social sciences. Although there is an attractiveness to the view, espoused by both Kuhn and Popper, that there is nothing inherent in the social sciences which precludes them from having similar aims and status to those of the natural sciences, it should be admitted that if this view on the ultimately value-based nature of scientific discovery is correct, then the ethical image that the social sciences uphold for humanity must also be part of this value judgement. This links in with the indictment that interpretivist sociologists have made against economics that, to put it crudely, by increasing its intellectual appeal and achievements it helps to create a society of selfish egoists in its own image.

One does not need to accept the radical thesis that there exists no social reality independent of our theoretical constructs (whether through language, customs or social science) to accept that there is a great deal of validity in the idea that there is a "feedback

process" between social reality and the concepts that social scientists use to explain and describe it. Economics has played an ethical role in promoting market societies, because of the view of many economists that individualistic societies produce greater economic efficiency and thus a better way of life than societies where people's economic behaviour is more closely controlled. From this perspective, the "core" assumptions of economics are not merely useful components of a predictive framework. They are, rather, central to the ethical vision of human nature and its potential at the heart of economics.

Consider, for example, the assumption of rationality. In the normative sense that the preferences of the individual should be sovereign, this is not the kind of proposition that can be proved or disproved. The fact that people act as if they know what they want does not imply anything about the moral status of these "desires". The assumption of rational behaviour at the heart of many models in welfare economics is therefore much more than just a predictive modelling technique; it is the embodiment in economic theory of the moral value of a society based on respect for individual autonomy.

Despite its empirical usefulness, the assumption of selfish preferences does not, on the face of it, share the same positive ethical basis. This, arguably, provides a strong reason to bring the modelling of altruism into the heart of welfare economic theory. It also changes the emphasis of the level of analysis from that of empirical explanation of specific phenomena to that of assessing the economic efficiency, and therefore the social desirability, of different levels and forms of altruism in human societies. If we live in societies in which people exhibit partial altruism, or altruism in some contexts but not in others, then this is something that welfare economics must seek to explain, and not be content merely to assume. Indeed, it is probably the undesirability of making ad hoc assumptions about partial altruism that has so far led to the cleaner solution of simple self interest remaining the main workhorse of abstract welfare economic analysis.

A model in welfare economics will require a number of properties if it is to satisfy the general prescription laid out above. One of the most important methodological questions which arises concerns the relationship between the social welfare function and the utility functions which the agents in the model seek to maximize. If the level of altruism is to be treated as an endogenous variable which can be altered (e.g. in different societies or via differential socialization processes in the context of an existing society) then the possibility must be left open that each individual could themselves be acting so as to maximize the social welfare function. (This would be a society of perfect utilitarian altruists.) If we are to justify any kind of arrangement as being superior to this perfect utilitarian society, we will clearly need to introduce additional structure to the model (corresponding to Lakatos' peripheral assumptions).

An interesting analogy to the role of the altruism assumption being suggested here

is that of rational expectations in macroeconomics. Just as there seems to be a kind of logical inconsistency between the assumption of rational agents with perfect understanding of an accurate model of the macro-economy and the use by these agents of adaptive expectations, there appears to be a parallel inconsistency between the assumption of moral human beings who design their society along the lines of utilitarianism but then act so as to selfishly maximize their own utility. Just as there are "hidden costs" to processing information which can explain why adaptive expectations are often a more plausible modelling technique in macroeconomics than rational expectations, there are "hidden costs" to individual altruism which may explain why it is socially optimal, despite the possibility of a society of utilitarian altruists within the structure of the model, for people to exhibit imperfections in their altruism.

2 A Model of Imperfect Altruism

The model which we will examine brings out in a clear and rigorous way the fact that asymmetric and paternalistic limitations upon altruism offer alternative "peripheral assumptions" which can both explain why individuals exhibit imperfect altruism, in the sense that they are less generous to one another than would be expected in a society of perfect utilitarian altruists who agree upon the common good. Given the inadequacies of the self-interest assumption outlined above, we will see that paternalistic limitations offer a more coherent and appealing microeconomic basis for imperfect altruism because apparent individual selfishness can be explained as a disagreement over perceptions of the common good, rather than specific discrimination against particular other individuals. Although it would be foolish to deny that real asymmetric limitations upon altruism exist, it is morally legitimate to want to remove these. However, in a liberal society which respects individual autonomy, it is not morally legitimate to want to force everyone to hold the same view of the common good. Using paternalism as the basis of the theory means that any general conclusions we draw for economic theory will apply even in a morally perfected world where everybody cares for everybody else equally and symmetrically. Economic theory built upon this foundation should therefore be free from the accusation that it promotes individual selfishness.

There are N individuals and M goods. Each individual i has the following utility function:

$$U_i = \sum_{j=1}^{N}\left[\theta_{i,j}\left(\sum_{k=1}^{M}\left[\alpha_{k,i}\frac{(C_{k,j})^{\epsilon}}{\epsilon}\right]-\frac{1}{2}\sum_{p=1}^{N}\left[(G_{j,p})^2\right]\right)\right]$$

$C_{k,j}$ is the consumption of good k by individual j. $\theta_{i,j}$ is the weighting placed upon individual j by individual i. So, if $\theta = 0.5$ then individual i cares about individual j 50

per cent as much as themselves. $\alpha_{k,i}$ is the weighting placed upon good k by individual i. We assume that this always takes a positive value. The higher it is, the more individual i cares about that particular good. $G_{j,p}$ is the amount of resources given to individual p by individual j. For simplicity of exposition, the marginal cost of giving is assumed to increase linearly for each individual as more is given to them, so that the cost of giving term is quadratic. (Note that "giving" is being thought of as an input into "production".)

Each individual is able to spend the total gift they receive on the M consumption goods. The way in which they spend the gift is up to them. The budget constraint for individual i is:

$$\sum_{k=1}^{M} C_{k,i} = \beta\left(\sum_{j=1}^{N} G_{j,i}\right)\left(\frac{1}{N}\right)^{\sigma}$$

The productivity parameter β determines the "gains to trade" from individuals altruistically giving to each other. The higher it is, the more productive is giving. The $N^{-\sigma}$ term reflects the fact that there are fixed factors which limit the productivity of the individuals' gifts to one another. These fixed factors are assumed to be spread evenly among the members of the group. Each individual maximizes their utility subject to their budget constraint by choosing their own $C_{k,i}$s. To solve this problem, we form the Lagrangean:

$$\zeta_i = U_i - \lambda_i\left(\sum_{k=1}^{M} C_{k,i} - \beta\left(\sum_{j=1}^{N} G_{j,i}\right)\left(\frac{1}{N}\right)^{\sigma}\right)$$

There are $k \times i$ first order conditions for consumption:

$$\frac{\partial \zeta_i}{\partial C_{k,i}} = \theta_{i,i}\alpha_{k,i}(C_{k,i})^{\epsilon-1} - \lambda_i = 0$$

We can normalize weightings so that $\theta_{i,i} = 1$. Rearranging therefore gives us that:

$$C_{k,i} = \left(\frac{\alpha_{k,i}}{\lambda_i}\right)^{\frac{1}{1-\epsilon}}$$

Inserting these back into the individual budget constraint gives us:

$$\sum_{k=1}^{M}\left[\left(\frac{\alpha_{k,i}}{\lambda_i}\right)^{\frac{1}{1-\epsilon}}\right] = \beta\left(\sum_{j=1}^{N} G_{j,i}\right)\left(\frac{1}{N}\right)^{\sigma}$$

Therefore:

$$\lambda_i = \left(\frac{\left(\sum_{k=1}^{M} \left[(\alpha_{k,i})^{\frac{1}{1-\epsilon}} \right] \right)}{\left(\beta \left(\sum_{j=1}^{N} G_{j,i} \right) \left(\frac{1}{N} \right)^{\sigma} \right)} \right)^{1-\epsilon}$$

Plugging this back into the formula for each $C_{k,i}$ gives us the following (note we have replaced k and j in the above formula with q and r respectively so as not to duplicate index letters):

$$C_{k,i} = \frac{(\alpha_{k,i})^{\frac{1}{1-\epsilon}}}{\sum_{q=1}^{M} \left[(\alpha_{q,i})^{\frac{1}{1-\epsilon}} \right]} \beta \left(\sum_{r=1}^{N} G_{r,i} \right) \left(\frac{1}{N} \right)^{\sigma}$$

We can see that since $\forall_k \forall_i [\alpha_{k,i} > 0]$, the total amount consumed of each good will always be weakly positive provided the total amount given to each individual is weakly positive. We are now in a position to solve for each individual's gift to each other individual. Each individual is assumed to know the utility function of every other individual, so that they are also able to predict how each other individual will "spend" their gift. We also assume that all individuals declare their gift simultaneously, so that we are looking for a Nash equilibrium with size of gift being the strategic variable. Once all gifts have been declared simultaneously and transferred, each individual decides how to split up their consumption betweent the different goods. We need to partially differentiate each individual i's Lagrangean with respect to their gift to each particular other individual j to give $j \times i$ first order conditions for giving. Each individual, in calculating their optimal gift size, can treat the gifts of all other individuals as fixed, since we are looking for a simultaneous-move Nash equilibrium:

$$\frac{\partial \zeta_i}{\partial G_{i,j}} = \theta_{i,j} \sum_{k=1}^{M} \left[\alpha_{k,i} (C_{k,j})^{\epsilon-1} \frac{\partial C_{k,j}}{\partial G_{i,j}} \right] - \theta_{i,i} G_{i,j} = 0$$

Now, since $\frac{\partial C_{k,j}}{\partial G_{i,j}} = \beta \left(N \right)^{-\sigma} \left(\alpha_{k,j} \right)^{\frac{1}{1-\epsilon}} \left(\sum_{q=1}^{M} \left[(\alpha_{q,j})^{\frac{1}{1-\epsilon}} \right] \right)^{-1}$ and $\theta_{i,i} = 1$, the above first order condition can be rearranged and simplified to give:

$$G_{i,j} = \theta_{i,j} \beta^{\epsilon} N^{-\sigma\epsilon} \sum_{k=1}^{M} \left[\alpha_{k,i} \alpha_{k,j}^{\frac{\epsilon}{1-\epsilon}} \right] \left(\sum_{r=1}^{N} G_{r,j} \right)^{\epsilon-1} \left(\sum_{q=1}^{M} \left[\alpha_{q,j}^{\frac{1}{1-\epsilon}} \right] \right)^{-\epsilon} \tag{1}$$

If we think of taking a particular good k, the vector $\vec{\alpha}_k$ (which consists of N $\alpha_{k,i}$s stacked together) can be thought of as being generated by a multivariate random distribution. The set of such vectors for the M goods therefore forms a sample of M draws from the same underlying distribution. The correlation coefficients between the component

variables of the vector, and any monotonic increasing functions of them, will therefore measure the degree of "agreement" between various individuals over the common good. The correlation coefficient we will be interested in using to rearrange the above equation will be:

$$\rho_{i,j} = \frac{Cov\left[\alpha_{k,i}, (\alpha_{k,j})^{\frac{\epsilon}{1-\epsilon}}\right]}{\sqrt{Var\left[\alpha_{k,i}\right] Var\left[(\alpha_{k,j})^{\frac{\epsilon}{1-\epsilon}}\right]}}$$

Replacing the components of this expression with consistent estimators of them using the sample of M goods, and rearranging, gives us that:

$$\sum_{k=1}^{M}\left(\alpha_{k,i}\alpha_{k,j}^{\frac{\epsilon}{1-\epsilon}}\right) =$$
$$M\left(\widehat{\rho_{i,j}}\sqrt{\widehat{Var_k}\left[\alpha_{k,i}\right]\widehat{Var_k}\left[(\alpha_{k,j})^{\frac{\epsilon}{1-\epsilon}}\right]} + \widehat{E_k}\left[\alpha_{k,i}\right]\widehat{E_k}\left[(\alpha_{k,j})^{\frac{\epsilon}{1-\epsilon}}\right]\right)$$

Substituting this into expression 1 and summing over i and r gives us:

$$G_j = \sum_{i=1}^{N} G_{i,j} =$$
$$\sum_{i=1}^{N}\left[\theta_{i,j} M \left(\begin{array}{c}\widehat{\rho_{i,j}}\sqrt{\widehat{Var_k}\left[\alpha_{k,i}\right]\widehat{Var_k}\left[(\alpha_{k,j})^{\frac{\epsilon}{1-\epsilon}}\right]} \\ +\widehat{E_k}\left[\alpha_{k,i}\right]\widehat{E_k}\left[(\alpha_{k,j})^{\frac{\epsilon}{1-\epsilon}}\right]\end{array}\right)\right]\beta^{\epsilon}N^{-\sigma\epsilon}(G_j)^{\epsilon-1}\left(M\widehat{E_k}\left[\alpha_{k,j}^{\frac{1}{1-\epsilon}}\right]\right)^{-\epsilon}$$

Rearranging, we obtain:

$$G_j^{2-\epsilon} =$$
$$\sum_{i=1}^{N}\left[\theta_{i,j}\left(\widehat{\rho_{i,j}}\frac{\sqrt{\widehat{Var_k}\left[\alpha_{k,i}\right]}}{\widehat{E_k}\left[\alpha_{k,i}\right]}\frac{\sqrt{\widehat{Var_k}\left[\alpha_{k,j}^{\frac{\epsilon}{1-\epsilon}}\right]}}{\widehat{E_k}\left[\alpha_{k,j}^{\frac{\epsilon}{1-\epsilon}}\right]} + 1\right)\widehat{E_k}\left[\alpha_{k,i}\right]\right]\beta^{\epsilon}N^{-\sigma\epsilon}M^{1-\epsilon}\frac{\widehat{E_k}\left[\alpha_{k,j}^{\frac{\epsilon}{1-\epsilon}}\right]}{\widehat{E_k}\left[\alpha_{k,j}^{\frac{1}{1-\epsilon}}\right]^{\epsilon}} \quad (2)$$

If we set the following components as constant coefficients, we can further simplify the expression:

$$\psi = \psi_{i,j} = \frac{\sqrt{Var\left[\alpha_{k,i}\right]}}{E\left[\alpha_{k,i}\right]}\frac{\sqrt{Var\left[\alpha_{k,j}^{\frac{\epsilon}{1-\epsilon}}\right]}}{E\left[\alpha_{k,j}^{\frac{\epsilon}{1-\epsilon}}\right]}$$

$$\gamma = \gamma_j = \frac{E\left[\alpha_{k,j}^{\frac{\epsilon}{1-\epsilon}}\right]}{E\left[\alpha_{k,j}^{\frac{1}{1-\epsilon}}\right]^{\epsilon}}$$

$\psi_{i,j}$ is the product of the ratios of the standard deviation and the expected value of $\alpha_{k,i}$ and $\alpha_{k,j}^{\frac{\epsilon}{1-\epsilon}}$. It thus measures the variability of the preferences of a pair of individuals over the M goods. For simplicity, we will assume it to be identical for any pair of individuals. γ_j is smaller the greater the variance of $\alpha_{k,j}^{\frac{1}{1-\epsilon}}$. This reflects the fact that the greater the extremities of taste of a particular individual, the less likely they are to use resources optimally from the perspective of other individuals, and so the lower the amount of altruistic giving. We also assume this coefficient to be identical for all individuals.

Since $\widehat{\rho_{i,j}}$ and the other estimator components in expression 2 are themselves random variables, in order to further simplify the above expression, we must assume that M is large and take a plim. This raises the problem that the $M^{1-\epsilon}$ component will go to infinity. Intuitively, this is because greater product variety gives more opportunity for specialization and thus greater gains to altruistic giving. To correct for this, we will assume that $\beta = \kappa^{\frac{1}{\epsilon}} M^{1-\frac{1}{\epsilon}}$ where κ is a constant. With this additional assumption, the expression converges as M goes to infinity. The simplified form of our expression for the total amount given to individual j as $M \longrightarrow \infty$ is now:

$$\widetilde{G_j} = plim_{M\to\infty}[G_j] = \left(N^{-\sigma\epsilon}\sum_{i=1}^{N}\left[\theta_{i,j}(\rho_{i,j}\psi+1)\bar{\alpha_i}\right]\kappa\gamma\right)^{\frac{1}{2-\epsilon}}$$

We can now see that if we can only observe the actual amount of gift made, and do not know the underlying altruistic preferences driving this behaviour, then we cannot distinguish empirically between asymmetry and paternalism as the cause of less giving than in the case of perfect utilitarian altruism where $\forall_i\forall_j[\rho_{i,j} = 1 \wedge \theta_{i,j} = 1]$. In the case of perfect utilitarian altruism, we see that the amount given to each individual would be:

$$\widetilde{G_{\text{perf}}} = N^{\frac{1-\sigma\epsilon}{2-\epsilon}}\left((\psi+1)\bar{\alpha}\kappa\gamma\right)^{\frac{1}{2-\epsilon}}$$

If the amount of giving is less than this amount, then there are a spectrum of explanations utilizing asymmetry and paternalism to varying degrees. It is most useful to compare an explanation utilizing entirely asymmetry, with one utilizing entirely paternalism. Assuming that $\forall_i\forall_j[i \neq j \rightarrow \rho_{i,j} = \rho]$ and $\forall_i\forall_j[i \neq j \rightarrow \theta_{i,j} = \theta]$ then as $N \longrightarrow \infty$:

$$\widetilde{G} = lim_{N\to\infty}[\widetilde{G_j}] = N^{\frac{1-\sigma\epsilon}{2-\epsilon}}\left(\theta(\rho\psi+1)\bar{\alpha}\kappa\gamma\right)^{\frac{1}{2-\epsilon}} \tag{3}$$

Therefore:

$$\frac{\widetilde{G}_j}{\widetilde{G_{\text{perf}}}} \longrightarrow \left(\frac{\theta(\rho\psi+1)}{\psi+1}\right)^{\frac{1}{2-\epsilon}}$$

Here again we can see that in order to infer a particular θ from empirical data, we would have to assume a particular ρ and ψ. We can now form an equation to derive the apparent value of θ, $\check{\theta}$, which would be generated by a given actual value of ρ, if we were to incorrectly assume that $\rho = 1$, when in reality $\theta = 1$ and $\rho < 1$.

$$\check{\theta}(\psi+1) = (\rho\psi+1) \Longrightarrow \check{\theta} = \frac{\rho\psi+1}{\psi+1} \tag{4}$$

Individuals in a group who disagree over the common good will therefore appear to be more altruistic towards one another, the more correlated are their valuations (i.e. the less they tend to disagree over the common good). If we assume that $\alpha_{k,i}$ is independently drawn from the same distribution for all individuals i and goods k, then the value of ρ will be 0. This means that $\check{\theta}|_{\rho=0} = \frac{1}{1+\psi}$. Since the expected values of $\alpha_{k,i}$ and $\alpha_{k,j}^{\frac{\epsilon}{1-\epsilon}}$ must be positive, ψ must necessarily also be positive. This implies that the average level of apparent pure asymmetric altruism (i.e. the parameter $\check{\theta}|_{\rho=0}$) is always positive but less than 1, and is decreasing as the variability of individual valuations increases.

Taking natural logarithms on both sides of equation 3 and setting $\theta = 1$ gives us:

$$\ln[\widetilde{G}] = \left(\frac{1-\sigma\epsilon}{2-\epsilon}\right)\ln[N] + \left(\frac{1}{2-\epsilon}\right)\ln[\rho\psi+1] + \left(\frac{1}{2-\epsilon}\right)\ln[\bar{\alpha}\kappa\gamma]$$

Here we have an equation which can be estimated using an econometric panel model of the form: $\ln[\widetilde{G}] = \eta\ln[N] + \vec{\delta}'\vec{\Gamma} + c$. $\vec{\Gamma}$ is a vector of dummy variables, such that one and only one element equals one, and all others equal 0. $\vec{\delta}$ is a vector of coefficients on these dummy variables such that, for a particular type of group v, $\vec{\delta}_v = \frac{1}{2-\epsilon}\ln[\rho_v\psi+1]$ where ρ_v is the value of this coefficient specific to that particular group and ψ is assumed to be common to all groups. $\eta = \frac{1-\sigma\epsilon}{2-\epsilon}$ and c is the constant intercept which is assumed to be common to all groups.

By guessing a value of ϵ, the value of σ is pinned down by the estimated coefficient η: $\sigma = \frac{1}{\epsilon}(1-(2-\epsilon)\eta)$. The guessed value of ϵ therefore also determines the vector $\vec{\mu} = (2-\epsilon)\vec{\delta}$, so that $\vec{\mu}_v = \ln[\rho_v\psi+1]$. Finally, it can be shown (see appendix) that assuming that all αs are unformly distributed between 0 and an upper bound Φ, that $\psi = \sqrt{\frac{1}{3}\frac{\epsilon^2}{1-\epsilon^2}}$. If we assume values of ϵ so that $\rho_v\psi$ is small then $\ln[1+\rho_v\psi] \approx \rho_v\psi$. We can therefore use the value of the dummy variable for a particular group, which represents the extra intercept for that particular group, to estimate the difference in ρ for different groups. Whether plausible assumed values of ϵ produce plausible differences in ρ will therefore give us an idea of how well the model fits the data.

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