Necessities and Necessary Truths

Volker Halbach

B.Phil. Seminar Logic and Philosophy of Logic

18th February 2019

The problem

What is the logical form of 'that'-sentences or, generally, sentences with *dicta*?

Solutions to this problem impinge on issues in philosophy of language, logic, ontology and further areas.

- It's a priori that 5+7=12.
- It's analytic that all bachelors are unmarried.
- That water is H_2O is necessary.
- Wilfrid knows that water is H_2O.
- It's true that snow is white.
- deontic, temporal etc notions

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I won't sketch a theory of necessity or analyticity; I'll rather sketch a general framework wherein theories of necessity and analyticity can be developed.

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The Predicate Analysis

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Ontological commitment

According to the Predicate Analysis, *dicta* ('that'-sentences) are singular terms denoting objects (propositions or sentences).

Therefore the Predicate Analysis commits one to an ontology of objects that can be necessary, analytic, be believed etc.

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The Operator Analysis

'It is necessary that' is conceived as an operator or adverbial phrase.

$\underbrace{\text{It is necessary that}}_{\text{operator, adverb}}\ \underbrace{\text{water is } H_2O}_{\text{sentence}}$.

According to the Operator Analysis sentences with 'that' can be parsed into an operator (or adverbial phrase) and a sentence.

In English 'necessarily' is an adverb acting in this way.

In modal logic necessity is conceived as the sentential operator □: If ϕ is a sentence $\Box\phi$ is also a sentence.

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Advantages of the Operator Analysis

1. There is no ontological commitment to objects that can be necessary or analytic.
2. Paradoxes of modal predicates are avoided.
3. Paradoxes arising from the interaction of modal predicates are avoided.
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Assume that we have settled for the Predicate Analysis. Assume further that sentences are analytic and that propositions are necessary (here propositions are not sentences but rather what is expressed by sentences).

That water is water is necessary.

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Ontology

That water is water is necessary and analytic.

This sentence is false, because only sentences but not propositions can be analytic.

On the Operator Analysis this problem doesn't arise:

Operator Analysis

Necessarily and analytically, water is water.

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Paradoxes of modal predicates

Montague's Montague (1963) Paradox

If analyticity is conceived as predicates of sentences, then the following assumptions are inconsistent on the basis of a weak syntax theory.

1. axioms: If 'A' is analytic, then A.
2. rule of inference: If 'A' has been proved, one may infer '"A" is analytic.'

The same paradox applies to necessity, a prioricity etc.

If analyticity is conceived as a predicate of propositions, the theory of syntax is replaced by a theory of propositions, which may prove the diagonal lemma.

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Paradoxes from interactions of modal predicates

Interactions of modal (and truth in some cases) predicates can lead to inconsistencies (Halbach Halbach (2006, 2008), Horsten & Leitgeb Horsten and Leitgeb (2001)).

For instance, the theories of analyticity and necessity (as predicates) taken separately could be consistent, while the combination of both theories yields an inconsistency (The example will show why this does not contradict interpolation theorems).
I give a very elementary example:

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Paradoxes from interactions of modal predicates

One might hope to solve Montague's paradox by typing the modal predicates; this move blocks Montague's paradox.

'Necessity–analyticity paradox'

The following axioms and rules are inconsistent with a basic theory of syntax:

- If 'A' is analytic, then A (where A does not contain 'analytic')
- rule of inference: If 'A' has been proved, one may infer '"A" is analytic.' (where A does not contain 'analytic')
- If 'A' is necessary, then A (where A does not contain 'necessary')
- rule of inference: If 'A' has been proved, one may infer '"A" is necessary.' (where A does not contain 'necessary')

Modal logic

In modal logic necessity is treated as a sentential operator. '□A' is well formed *only* if A is a formula; '□*t*' isn't a formula if *t* is a singular term (individual constant, variable etc).

Possible worlds semantics relies on modal logic and thus on the Operator Analysis.

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The quantification problem

Quantified claims such as the following cannot be expressed without nasty ploys on the Operator Analysis:

- All laws of nature are necessary.
- Some necessary propositions are not analytic.
- Volker has a true a posteriori belief.

On the predicate account of modalities, in contrast, these sentences can readily be formalized

- $\forall x(\mathrm{L}aw(x) \rightarrow \mathrm{N}x)$
- $\exists x(\mathrm{N}(x) \wedge \neg \mathrm{A}x)$
- $\exists x(\mathrm{B}vx \wedge \mathrm{T}x \wedge \mathrm{A}post(x))$

Quantified statements of this kind are of particular interest to the philosopher.

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- moving quantifiers into the metalanguage
- ‘substitutional quantification’
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The truth predicate serves the purpose of expressing quantification (generalization) that couldn’t be expressed otherwise.

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Every analytically true proposition is true.

A first (inadequate) attempt

x is necessary

is substituted with

x is necessarily true

For this translation the truth *predicate* is still required. Predicates for necessity, analyticity etc can be eliminated by the respective operator.

Thus necessities are replaced by necessary truths.

Tense as an example

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Problems with iterated modalities I

Iterated modalities are discussed is various contexts, e.g.:

All sentences of the from ‘A or not A’ are necessary.

An analogous claim can be made in terms of propositions.

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Thus a truth predicate is required that can be applied to sentences containing the necessity *predicate*.

The main motivation for the eliminating the necessity predicate was the aim to avoid the difficulties with the semantics of a necessity predicate. Now the substitution of necessities with necessary truths requires a truth predicate for sentences containing the necessity predicate, that is, it requires semantics for the necessity predicate.

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O-translation

Since the first proposal failed, I suggest to modify the translation.

I would like the O-translation to satisfy the following condition:

O-translation

The O-translation of '*x* is necessary' is 'the O-translation of *x* is necessarily true'.

Otherwise the O-translations should not make any changes to the original sentence.

Problem: In order to define the notion of an O-translation, I one seems to require the notion of O-translation. The definition of O-translation seems bound to be circular.

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According to the so called Recursion Theorem one can define computable functions in terms of a code of that function.

One can use the Recursion Theorem in order to define the notion of an O-translation.

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An example

original sentence

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Justifiying the O-translation

I need to show that the O-translation preserves ‘meaning’ in order to show that the O-translation provides a sound reduction of the necessity predicate to an operator and a truth predicate.

This presupposes semantics at least for some sentences (e.g., the non-circular) with the necessity predicate and for language with necessity operator and a truth predicate.

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The benefits of the reduction: ontology

Objects to which necessity, analyticity etc can be attributed are no longer required. The reduction thus preserves this advantage of the operator analysis.

In contrast to the pure operator approach, a truth *predicate* is retained; and therefore objects that can be true true are still needed.

Thus all the ontological problems with the predicate analysis are concentrated on the truth predicate.

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The benefits of the reduction: the paradoxes from the interaction of modal predicates

The paradoxes arising from the interaction of modal predicates (including the truth predicate) are resolved, as only the truth predicate is retained, while all other notions are conceived as predicates.

The benefits of the reduction: modal logic

As necessity and other modal notions are treated as operators, they can be analysed in modal logic. The use of modal logic allows one to apply possible worlds semantics in the analysis of the modal notions. Thereby large parts of philosophical logic, philosophy of language and analytic metaphysics are salvaged.

The benefits of the reduction: division of labour

If the reduction of modal predicates to operators is feasible, then many problems can be concentrated on the theory of truth: The mentioned ontological decisions and the paradoxes would belong completely to the realm of the theory of truth. In the theory of necessity, epistemology one would not have to care about these problems; they could be left entirely to the people working on truth.

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