It has been proposed to block Fitch’s paradox by disallowing a predicate or sentential operator of knowledge that can be applied to sentences containing the same predicate or operator of knowledge. Furthermore it has been claimed that this move is not ad hoc as there is independent motivation for this restriction, because this restriction provides a solution also to paradoxes arising from self-reference like the paradox of the Knower. A solution to paradoxes arising from self-reference is only needed if knowledge is treated as a predicate that can be diagonalized. However, if knowledge and possibility are conceived as such predicates with type restrictions, a new paradox arises. Very basic, jointly consistent assumptions on the predicates of knowledge and possibility yield an inconsistency if [a typed version of] the verifiability principle is added.

1 MANIFESTATIONS AND A TREATMENT OF EPISTEMIC PARADOX

Paradoxes arising from diagonalization like the paradox of the Knower (Kaplan and Montague 1960) pose threats to philosophical theories comprising
merely basic assumptions on the notion of knowledge. Most philosophers agree that some sort of treatment against paradoxes of this sort is required.

Philosophers like Williamson (2000) deny Fitch’s paradox a similar status: in their view it does not qualify as a paradox at all, but merely as an ailment of verificationism. Consequently they do not bother to look for some medication against Fitch’s paradox; rather they recommend to give up verificationism altogether.

Most proposals to cure Fitch’s paradox have been rejected by the opponents of verificationism as useless in a hopeless case. But even they have to admit some treatment of epistemic paradox, as paradoxes arising from diagonalization affect also accounts that are not verificationist. What if the preferred remedy against the paradox of the Knower and other paradoxes of self-reference also help to resolve Fitch’s paradox? In that case even a diehard opponent of verificationism would have to agree to a treatment that could save verificationism from Fitch’s paradox. One cannot withhold the remedy from verificationism while giving it to other philosophical accounts. If the solution of the paradox of the Knower and related paradoxes solves also Fitch’s paradox, then verificationism is saved from Fitch’s paradox.

Since Tarski’s work on truth, typing has been thought of as an beneficial medication against the unpleasant side-effects of diagonalization. Typing may not be the most sophisticated curative against the Liar or the Knower paradox, but in many cases it has proved safe and effective. So let’s try, the proponent of verificationism may propose, to administer typing against both: the paradoxes from diagonalization and Fitch’s paradox. The predicate of knowledge could be sensibly applied only to sentences not containing the predicate of knowledge. One does not have to restrict the formation rules of sentences in the language in any way, but axioms that allow one to prove that sentences containing the knowledge predicate are known would have to be rejected.

If typing proves good for both kinds of paradoxes, then verificationism is in a fairly stable state: it is up to the opponent of verificationism to explain why Fitch’s paradox is in any way worse than the epistemic paradoxes from diagonalization, because both, verificationism and other conceptions of knowledge are threatened by inconsistency and both can be cured with the same antidote. So anyone attacking verificationism on the grounds of Fitch’s paradox would have to explain why he rejects typing as a remedy to the paradox of the Knower and related paradox. In this paper it will be
shown that the advocate of verificationism is ill advised to pursue this line of argument: even the typing of knowledge does not save verificationism. Fitch’s paradox will disappear once knowledge is typed; but other symptoms, as unpleasant as Fitch’s paradox, will appear in the case of the verificationism but not necessarily in other cases. Fitch’s paradox cannot be solved by typing knowledge.

Paseau (2007) and Linsky (2007) have proposed to resolve Fitch’s paradox by typing. Paseau explicitly justifies typing by appealing to the paradox of the Knower.

2 MODAL PARADOX

At the core of verificationism, as I understand it here, is a modal principle: if \( p \), then it is possible to know \( p \). There are several variant formulations, but verificationism needs some modal component. Thus in order to give a formal treatment of verificationism, one needs to provide a formal treatment of necessity as well as knowledge: the verificationist’s formal framework will comprise axioms for necessity, axioms for knowledge and ‘mixed’ axioms like the verification principle.

Necessity is plagued by similar paradoxes from diagonalization as knowledge. In particular, both, knowledge and necessity are affected by Montague’s (1963) paradox. Here I will not discuss these paradoxes arising from diagonal arguments and their potential solutions (see Brogaard and Salerno 2002 and Koons 1992 for the epistemic versions, and McGee 1991 for the modal versions). Given the structural similarity between these paradoxes of knowledge and necessity, the verificationist seems well advised to fight them with the same cure: typing. In analogy to the typing of knowledge one restricts all axioms and rules of inference of necessity in such a way that necessity does not provably apply to sentences containing the necessity predicate.

Many logicians have followed Montague’s (1963) lead: they do not treat necessity (and knowledge) as predicates of objects but rather as sentential operators as in modal logic. A sentential operator has to be combined with a formula not a singular term to yield another formula. If necessity or knowledge are conceived as predicates, diagonalization of the required kind is no longer possible and the respective paradoxes are blocked. Here I do not enter the debate on whether the operator or the predicate conception
are to be preferred (but see Halbach, Leitgeb, and Welch 2003). For if the verificationist attempts to motivate typing not as an ad hoc solution to Fitch’s paradox but rather as a remedy against the paradoxes from diagonalization, then he is not free to reject the predicate conception, even if in the usual presentations of Fitch’s paradox neither necessity nor knowledge are treated as predicates but rather as sentential operators. With a rejection of the predicate conception he would also loose diagonalization and the resulting paradoxes as reasons for typing that also compel the non-verificationist (actually Grim 1993 observes that these claim need at least some qualification; but I take it that the whole point of solving the paradoxes is to allow for a predicate conception of knowledge or necessity). Paseau (2007) has provided a solution of Fitch’s paradox based on operator conceptions of knowledge and necessity. But as long as knowledge and necessity are treated as sentential operators, typing cannot be justified by appeal to the paradoxes based on diagonalization.

Of course the verificationist could treat necessity and knowledge in different ways: he could type knowledge but not necessity and treat the latter as operator. Such an asymmetric treatment seems odd to me. At any rate I shall assume that both, necessity and knowledge are conceived as unary typed predicates.

3 A NEW SYMPTOM

The side effect of solving Fitch’s paradox by typing will be presented in a language comprising the unary predicates $K$ for knowledge and $N$ for necessity. The language has a name $⌜\varphi⌝$ for each of its sentences. The numeral of the Gödelnumber of $\varphi$ can serve as this name in an arithmetical language. Besides the specific axioms for knowledge and necessity only the diagonal lemma will be assumed. Only a single instance of the diagonal lemma will be used in order to derive a contradiction. Here in this short paper I will not repeat the usual formal assumptions. For a more detailed account see, for instance, McGee (1991).

Concerning knowledge I endorse only factivity, which is almost uncontroversial. Since the verificationist has subscribed to typing as his policy on the paradoxes, we postulate factivity only for $K$-free sentences:

\[(F) \quad K^\varphi \supset \varphi \quad \text{for all sentences } \varphi \text{ not containing } K\]
For necessity I assume only necessitation. Since also necessity is typed, necessitation is postulated only for $N$-free sentences:

\[
\frac{\varphi}{\text{N}^\varphi} \quad \text{for all sentences } \varphi \text{ not containing } N
\]

No additional axioms specific to $K$ or $N$ will be needed to derive a contradiction from the verification principle.

Of course one would want to add further axioms for knowledge and necessity. For instance, one may postulate distribution of knowledge over conjunction: $K^\varphi \land \psi \supset K^\varphi \land K^\psi$. This axiom – or rather an operator version of it – is used to derive Fitch’s paradox. Axioms of this kind can be added consistently even with some relaxations of the typing restrictions. Trivial consistency proofs for this axiom with the verification principle (Ver) below, (Nec), and an underlying base theory like Peano arithmetic can be obtained by interpreting the knowledge predicate by the empty set and necessity by the set of all sentences. This will suffice for showing the consistency of the mentioned basic axiom, but supplementary axioms may force a more interesting model. Horsten’s (1998) construction of a model for unknowability and truth (rather than necessity) may serve as an example of a more sophisticated model construction. There are, of course, limitations to further axioms: augmenting the theory with necessitation for knowledge and factivity for necessity leads to an inconsistency, even if type restrictions are observed; see Halbach 2006 for a related inconsistency.

At any rate, the theory outlined so far is consistent. Only adding the verification principle leads to an inconsistency. Verificationists usually do not restrict the verification principle by some sort of typing, but the following restricted version may be more in line with the above axioms; at any rate, the stronger unrestricted version is not needed getting the verificationist into trouble.

\[
\varphi \supset P^\varphi K^\varphi \quad \text{for all sentences } \varphi \text{ not containing } K
\]

Here I have used $P$ for possibility. $P^\varphi \varphi$ will be understood as shorthand for $\sim N^\varphi \sim \varphi$. Thus (Ver) is the schema $\varphi \supset \sim N^\varphi \sim K^\varphi \varphi$ for $\varphi$ not containing $K$.

The effect of adding the verification principle (Ver) to a theory of typed knowledge and typed necessity is swift and lethal. The following sentence is
an instance of the diagonal lemma:

\[(1) \quad \gamma \equiv \sim P^\gamma K^\gamma\gamma\gamma\]

A contradiction can now be obtained as follows:

\[(2) \quad \gamma \supset P^\gamma K^\gamma\gamma\gamma \quad \text{(Ver)} \]
\[(3) \quad \sim \gamma \quad \text{(1) and (2)} \]
\[(4) \quad K^\gamma\gamma \supset \gamma \quad \text{(F)} \]
\[\sim K^\gamma\gamma \quad \text{two preceding lines} \]
\[N^\gamma \sim K^\gamma\gamma\gamma \quad \text{(Nec)} \]
\[(5) \quad \sim P^\gamma K^\gamma\gamma\gamma \quad \text{definition of } P \]
\[\gamma \quad \text{(1)} \]

The last line contradicts (3).

4 THE POSTMORTEM

The proof in the previous paragraph might leave adherents of verificationism feeling taken in. After all, verificationism might have succumbed to a problem that would carry off other sprightly philosophical theories in a couple of lines of a proof.

It may be suspected, for instance, that the cure, typing, has been administered in the wrong way. The diagonal sentence \(\gamma\) introduced in (2) in the previous section contains the necessity predicate \(N\). In (4) factivity of knowledge \((F)\) has been applied to \(\gamma\), which contains the necessity predicate, at least once \(P\) has been defined away. Perhaps type restrictions should have prevented this application of \((F)\). Perhaps, generally, the necessity predicate must not be applied to sentences containing the knowledge predicate, and, conversely, the knowledge predicate must never be applied to sentences containing the necessity predicate. This radical cure can be shown to be effective against the argument in the previous section. Nevertheless this extreme move has lethal consequences for verificationism, too: according to verificationism, it is possible to know \(p\), if \(p\). In the very statement the modality ‘is possible’ has been applied to a claim containing the knowledge predicate. Thus this strict form of typing makes the formulation of verificationism impossible. Even if one only disallowed the application of the
knowledge predicate to sentences containing \( N \) while allowing applications of the necessity predicate to sentences containing the knowledge predicate, unmotivated as this may be, verificationism would be heavily crippled, as modal knowledge would be ruled out.

The proponent of verificationism might still feel that verificationism was a viable philosophical theory. He may be left with the impression that the paradox in the previous section is not as specific to verificationism as Fitch’s paradox. The new paradox in the previous section belongs to a larger family of paradoxes that arise from the interaction of two or more intensional notions treated as predicates. After all, the typing of necessity and the typing of other notions are known to interact with lethal effects also in other cases (see Halbach 2006; Horsten and Leitgeb 2001). Therefore the paradox may highlight only a general problem with resolving paradoxes by typing. I am happy to concede this: there may be an underlying general problem with resolving paradoxes by typing, if more than one notion is under consideration. But exactly that is the point of this paper: Fitch’s paradox cannot resolved by typing knowledge. The question whether he demise of verificationism is due to the treatment of Fitch’s paradox by typing or due to the frailty of verificationism, is left undecided here.

REFERENCES


