

A TYPE-FREE THEORY OF TRUTH, MODALITY, AND SATISFACTION

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In [1] we introduced a semantic dependency relation for sentences of a first-order language \mathcal{L}_{Tr} with truth predicate Tr (\mathcal{L}_{Tr} might e.g. be the language \mathcal{L} of arithmetic plus Tr): for $\varphi \in \mathcal{L}_{Tr}$ and $\Phi \subseteq \mathcal{L}_{Tr}$, φ is defined to depend on Φ :iff for all $\Psi_1, \Psi_2 \subseteq \mathcal{L}_{Tr}$: if $Val_{\Psi_1}(\varphi) \neq Val_{\Psi_2}(\varphi)$ then $\Psi_1 \cap \Phi \neq \Psi_2 \cap \Phi$. ($Val_{\Psi}(\varphi)$ is the truth value of φ as being given by an expansion of a ground model for \mathcal{L} such that Ψ is used as the extension of Tr). Transfinite iteration of the operator $D^{-1} : \Phi \mapsto \{\varphi \in \mathcal{L}_{Tr} \mid \varphi \text{ depends on } \Phi\}$ leads to a least fixed point set Φ_{lf} of sentences the truth values of which may be said to *depend on non-semantic states of affairs only*. Any standard Tarskian definition of truth for \mathcal{L} can be extended to a definition of truth for \mathcal{L}_{Tr} in a way such that for all sentences $\phi \in \Phi_{lf}$ a T-biconditional of the form $Tr(\ulcorner \phi \urcorner) \leftrightarrow \phi$ is derivable from the truth definition (given the metatheory).

In our talk we will show that analogous theories can be introduced for cases where either (i) a necessity predicate \Box of sentences or (ii) a satisfaction or “instantiation” predicate \in of open formulas is added to the arithmetic language. As far as (i) is concerned, this will give us a manner of coping with Montague’s paradox and related paradoxes that seem to affect the treatment of modalities in terms of predicates of sentences. (ii) might be interpreted as giving us a theory of properties or classes. The main aim of this investigation is to give a general account of what instances of Tarski’s T-scheme, of the usual modal axiom schemes, and of the comprehension scheme for instantiation are “natural” if truth, modalities, and satisfaction are expressed by type-free first-order predicates.

REFERENCES

- [1] Leitgeb, H. (2005): “What truth depends on,” *Journal of Philosophical Logic*, forthcoming.

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