Mushy Credences I

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Daniel Rothschild
daniel.rothschild@philosophy.ox.ac.uk

1 Standard Bayesian picture

On the Bayesian view we represent an agent's beliefs by a probability function over the set of relevant possibilities W.
Some motivations:

- Belief seems often to be a graded affair. (e.g. I'm more confined that it's a 2012 than I am that it's thursday).
- High confidence not maintained under conjunction. I'm highly confident that each student passed, but I'm but I'm not highly confident that every student passed (e.g. paradox of the preface). On the Bayesian view we can assign .9 to a series of propositions while assigning 0 to their conjunction (e.g. lottery tickets being losers).
- Elegant story of belief updating by conditionalizing.
- Forms part of an elegant decision theory: e.g. maximize expected utility.

2 Considerations against sharpness

On the Bayesian view an agent's credal state is represented by a unique probability function p over W. But this requires assigning a sharp probability to each possible state of affairs in W.
Various problems with this, descriptive and normative.
Descriptive:

- Agents certainly don't seem to be aware of their sharp probabilities.
- Sharp probability not always necessary to make choices when maximizing expected utility.

Normative:

- Evidence doesn't seem to push one way or the other (even objectively). Principle of indifference only operational in even cases (e.g. Obama will win).
- Coin of unknown bias. (Note distinguish from coin with unknown bias represented by a uniform distribution.) Again POI would seem to suggest that we should just assign this .5, but then we have the problem of distinguishing this from our beliefs about a fair coin (or a coin with uniformly distributed possible biases).
- Suppose a positive integer has been selected at random. How can we assign a probability to this? (Note that we cannot assign the same probability to each natural number, since then the probability of any number being chosen would be greater than 1.)

3 Representations of unsharp credences

Intervals for each probability, high and low probabilities, for each event. So if we say $p_\uparrow(A) = .9$ we mean that the maximum probability we assign to A is .9. If $p_\downarrow(A) = .8$ then the minimum probability we assign to A is .8. The two of these values give us an an interval, and any value in the interval is fair game. Note certain consistency principle need to be met: for instance if $p_\uparrow(A) = .9$ then $p_\downarrow(\neg A) = .1$. When $p_\uparrow(A) = p_\uparrow$ and $p_\downarrow(A) = p_\downarrow$ we will write that $p(A) = [p_\uparrow, p_\downarrow]$, treating the probability as an interval.
A canonical way of ensuring the right sort of consistency while still getting interval values for the probability of any given value is by using CONVEX sets of probability functions over W to represent an agents beliefs with respect to W. (A set of probability functions C is convex iff for any p and $p' \in C$ and any real number $x \in [0, 1]$, $xp + (1-x)p' \in C$.) I will call such a set of probability functions a CREDAL SET.[1]
We think of these sets as akin to sets of truth-value assignments in a supervaluationist system. If someone's beliefs are presented by a credal set S than his beliefs only have some property if every member of C has that property. For example, suppose someone assigns somewhere between .6 and .7 probability to some proposition A. That can be presented by $S = \{p : p(A) > .6 \text{ and } p(A) > .7\}$. Every member p of this S satisfies the property that $p(A) > .6$ and $p(A) < .7$, so that is a property of the whole set. By contrast only some members are such that $p(A) > .65$ so that is not a property of the whole credal set. It is easy to show that for any credal set C and any proposition $A \subset W$, there is some minimal interval such that every member of C assigns a probability inside that interval. Thus, for any C we can discuss *the* interval that C assigns to A, which we write with $p_C(A)$, in case every p in C agrees on A then it is simply a number, rather than an interval.
How useful are such representations? Well, they answer to some extent the normative and descriptive problems with sharp credences. However, as with simple trivalent or supervaluationist vagueness, they replace one sharp value with two, thus perhaps not ideally answering *all* the worries about sharpness. (Similarly: if you're an epistemicist about vagueness, it's hard to see why you should be worried about sharp credences for lack-of-evidence-to-make-sharpness type reasons.)

4 Decision Problems

Adam Elga [2010] suggests a significant problem for the interval-valued credences view. The problem goes as follows. Consider some proposition H, like 'It is going to rain', or 'The Euro will fail'. Suppose that you are offered two bets:

Bet A If H is true you lose \$10. Otherwise you win \$15

Bet B If H is true you win \$15. Otherwise you lose \$10.

These bets, it is specified, are offered sequentially, one immediately after the other. Elga proposes that (given that you know these best are going to be offered sequentially) rationality requires for any H accepting one or both of the bets (you might accept just one if you have a strong credence in H or not-H).

[1]For most purposes the assumption of convexity will not be used.

He then argues that if we have a mushy-style theory we cannot explain why agents should act rationally. Suppose, that one's credence in H is the set $[.2, .8]$. Here are two theories about how to act given a credal set C

Conservative An action is allowed if and only if it maximizes expected utility on all probabilities in C.

Liberal An action is allowed if it maximizes expected utility on some probability in C.

Note that **Conservative** mandates rejecting both bets, while **Liberal** allows it. So neither explains or predicts that agents with mushy credences will behave rationally.
Elga considers and rejects various other options:

Midpoint Rule You can imagine a decision theory that forced one to bet as if one's probability was the midpoint of the interval. But this would destroy the whole point of mushy credences (according to Elga) as:

> So the midpoint rule robs unsharp probabilities of their point. With the midpoint rule in place, interval-valued probabilities yield exact "point-valued" constraints on rational betting odds. But if it is always OK to have point-valued constraints on betting odds, there is no good reason for objecting to point-valued probabilities in the first place.

However, Moss [tomorrow!] in the appendix points out that this is in fact wrong. She writes "the midpoint rule does not require agents to bet as if they had precise probabilities." She gives an interesting example where the midpoints of a credal set cannot be yielded by any probability function. In her example there are two events A and B. And the credal set consists of all probability functions p that assign between .2 and .6 to A, between .2 and .6 to B and 0 to $A \wedge B$. She notes that the midpoints for this credal set are: .4 for A, .4 for B, 0 for $A \wedge B$, and .6 for $A \vee B$. No single probability function could yield this.

Action forces sharpening We can imagine that whenever one acts, one's probabilities are constrained to rationalize the act. Elga argues this is ad hoc since it requires changing probabilities in the absence of evidence.

Planning On this proposal rational agents must plan a series of actions, and puts a coherence constraint on plans. Elga argues that there is no motivation for rational agents to follow through on plans, since they are stipulated not to care about plans (or can be).

Sequential rationality On this proposal sequences of actions themselves are evaluated for rationality. Elga considers second act in sequences, which is ruled irrational in some case and rational in other, and argues that no difference relevant to agent explains this:

> But in fact, SEQUENCE entails that rationality imposes different requirements on Sally in the two situations. For according to SEQUENCE, the following is true: In order to be perfectly rational it is not enough to avoid irrational actions. One must also avoid irrational *sequences* of actions. In particular, Sally, would be irrational if she were to reject Bet B in the second situation [when she has already rejected Bet A]. For her doing so would complete the irrational sequence of actions, "reject-Bet-A-then-reject-Bet-B." In contrast, her rejecting Bet B in the first situation [when she had already accepted bet A] would not complete any irrational sequences. So according to SEQUENCE, Sally's rejecting Bet B is consistent with her perfect rationality in the first situation, but not in the second.
> Bottom line: SEQUENCE entails that rationality imposes different requirements on Sally in the two situations. But Sally can see the her choices in the two situations are alike in every respect that she cares about. So it must be that rationality imposes on her the same constraints in the two situations. So SEQUENCE is incorrect.

Where, if anywhere, do we depart from Elga? How to understand holding of unsharp credences?
It seems to me that the prohibition on diachronic rules that Elga relies is not insurmountable. Perhaps though we need to interpret spread out mushy credences as in some way defective though, and diachronic rules as rules of thumb for not suffering from their defects.

References

Adam Elga. Subjective probabilities should be sharp. *Philosopher's Imprint*, 10(5): 1–11, 2010.

Sarah Moss. Credal dilemmas. Jowett Society Talk, tomorrow!