Fundamentality and non-symmetric relations

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1 Introduction

Non-symmetric relations allow for differential application.[1] A binary relation R can hold of a and b in two different ways: 1. aRb and 2. bRa. Different states of affairs result from completing R by means of a and b, depending on the order in which a and b are combined with R. The extension of a binary non-symmetric relation is, accordingly, not to be understood in terms of a set of unordered pairs. One has to operate with a structured conception of the extension of a relation, for instance in terms of ordered pairs, that not only considers which things R relates, but also the order in which it relates them.

Differential applicability can straightforwardly be explained if relations have directions: R can then either go from a to b, or from b to a. Yet, if relations have directions, then non-symmetric relations would appear to have distinct converses. For instance, an asymmetric binary relation R, such as 'better than', seems to have a distinct converse R^{-1}, namely 'worse than', where R holds of x and y in a given order whenever R^{-1} holds of them in the opposite order.

Non-symmetric relations and their converses can fail to be interchangeable. This implies that these relations are distinct: $\Diamond \exists x \exists y (xRy \wedge \neg(xR'y)) \rightarrow R \neq R'$. If a stands in R to b, then b stands in the converse relation R^{-1} to a. Given that R is not symmetric, b can fail to stand in R to a. Yet, if b can stand in R^{-1} to a without standing in R to a, then R is distinct from R^{-1}.

$$
\begin{array}{ll}
1. & aRb \\
2. & \neg(bRa) \\
3. & bR^{-1}a \\
\hline
\therefore & R \neq R^{-1}
\end{array}
$$

The first part of this paper argues that there are no non-symmetric relations at the fundamental level (sections 2 and 3). The second part identifies different ways in which asymmetry and order can be introduced into a world that only contains symmetric but no non-symmetric fundamental relations (section 4). The

[1] Non-symmetric relations are those that are not symmetric. They are either asymmetric or neither symmetric nor asymmetric.

third part develops an account of derivative relations and puts forward identity criteria that establish that derivative non-symmetric relations do not have distinct converses. Instead of a plurality of relations, there are only different ways of picking out the same relation (section 5). The final part provides an account of how generative operations can induce order and argues for a reconceptualisation of grounding as an operation rather than as a relation (section 6).

2 Converse relations

Difficulties arise if non-symmetric relations have distinct converses.

1. REFERENTIAL INDETERMINACY

   If R is distinct from R^{-1}, then it is indeterminate whether a relational expression 'X' refers to R or its converse R^{-1} (cf. Williamson: 1985; van Inwagen: 2006). Williamson illustrates this point by means of the following languages:

language L	:	bXa = b stabs a
language L′	:	aXb = b stabs a
language L″	:	aXb = a is stabbed by b

   Language L uses the expression 'bXa' to refer to the fact that b stabs a. Language L′, by contrast, uses the expression 'aXb' to refer to this very same fact. L and L′ use the same relational expression 'X' to pick out the stabbing relation but employ different conventions. Language L″ uses the expression 'aXb' to refer to the fact that a is stabbed by b, i.e. that a stands in the converse of the stabbing relation to b. The problem now is that L′ and L″ are indistinguishable. These languages are for all extents and purposes the same. This implies that there is nothing that makes it the case that we are speaking L′ as opposed to L″. If we grant that relations are distinct from their converses, then it would seem to be indeterminate which relations our relational expressions refer to. Given all the facts about how we use relational expression 'X', it could just as well refer to R as to its converse R^{-1}. There is nothing to differentiate between these two candidates.[2] Distinguishing these languages is to draw a distinction without a difference. We should, accordingly, reject the idea that relations have distinct converses.

2. BRUTE NECESSITIES

   If R and R^{-1} are distinct relations, then we end up with brute necessities amongst distinct existences. Relations and their converses go together as a

[2]This indeterminacy is ineliminable since there is no way of picking out a relation rather than its converse in order to stipulatively fix reference (cf. footnote 31).

matter of necessity. It will necessarily be the case that x and y stand in R in a given order iff they stand in R^{-1} in the opposite order.

$$\Box\forall x\forall y(xRy \leftrightarrow yR^{-1}x)$$

This necessary biconditional would seem to involve a brute necessity that violates Humean strictures (cf. Dorr: 2004).

3. OVER-ABUNDANCE OF FACTS

The existence of both a relation R and its distinct converse R^{-1} leads to a proliferation of facts. If relations enter into relational facts and play a role in fixing the identity conditions of such facts, then distinct relations give rise to distinct facts. In addition to the fact [xRy], there would then also be a further fact, namely [$yR^{-1}x$]. For instance, the relation 'on top of' and its converse 'beneath' would enter into the two facts:

i. the cat is on top of the mat
ii. the mat is beneath the cat

Yet, intuitively there should only be one state of affairs, one fact about how the cat and the mat are related to each other (cf. Russell: 1913, pp. 85-89; Fine: 2000). The problem is not simply that recognising distinct converses leads to a non-parsimonious theory that countenances too many entities and thereby contravenes against Ockham's razor. Rather, the objection is that distinguishing these facts amounts to drawing a distinction without a difference. Intuitively, the fact that the cat is on top of the mat is the very same fact as the mat being beneath the cat. There is only one fact concerning the relative placement of these two objects.

The underlying problem is that relations and their converses are so closely and intimately connected that it is not plausible to consider them to be distinct.[3] Treating them as being distinct generates reference problems insofar as it is indeterminate which one we are picking out, bruteness problems insofar as they are not separable but of necessity go together, and a bloated ontology since they give rise to distinctions that do not make a difference. These problems multiply as the arity of the relation increases, since a relation of arity n has $n! - 1$ converses. Whereas binary relations have one converse, ternary relations have five converses, and quaternary relations have twenty-three converses. The greater the number of converses, the greater the amount of indeterminacy, the larger the number of brute necessities and the more extensive the over-abundance of states of affairs.

[3] Even though R and R^{-1} have different extensions across modal space when these are construed in terms of ordered n-tuples, they have the same extension understood in terms of unordered n-tuples.

2.1 Relational properties

It is often suggested that converse relations generate difficulties for the theory of relations and that there are no analogous difficulties in the theory of properties. The relevant contrast is taken to be one between properties and relations, between the monadic and the polyadic (e.g. van Inwagen: 2006). However, these problems do not only affect relations but also relational properties.

If a relation R is distinct from its converse R^{-1}, then the relational properties $\lambda x[xRa]$ and $\lambda x[aR^{-1}x]$ are also distinct. For instance, if x has the property of being taller than some particular object a, then x also has the converse property of being such that a is shorter than it. In the same way in which one complex cannot be the completion of two distinct relations (cf. the 'uniqueness' claim put forward by Fine: 2000, p. 5), one relational property cannot result when λ-abstracting one and the same object from two relational complexes involving distinct relations.[4] If the relations are distinct, then the corresponding relational properties are likewise distinct.[5]

The problems (of indeterminacy, brute necessities, and proliferation of facts) that derive from relations having distinct converses carry over to relational properties that have distinct converse relational properties. The source of the problem consists in relationality, not in polyadicity. In particular what generates the problems is the differential applicability of non-symmetric relations. This carries over to relational properties, since there will be two relational properties corresponding to the two different applications of a non-symmetric binary relation. The fundamental distinction is not between the polyadic and the monadic, but between the relational and the non-relational.

2.2 Converses: strict and loose

In order to address these problems, one has to deny that there are non-symmetric relations that have distinct converses.

1. The most radical approach denies that there are non-symmetric relations. Since symmetric relations are identical to their converses, there will not be any distinct converses if there are no non-symmetric relations. The denial of non-symmetric relations, however, is implausible since the world is full of asymmetry and order (though cf. Dorr: 2004, sections 8-9).

[4]If the relational properties were identical, then λ-conversion would not be functional, since it would then be one-many. One would get two distinct relational complexes from the same relational property.

[5]This means that those who consider relations to be distinct from their converses end up with hyperintensional commitments. The relational properties derived from these two different relations will be necessarily co-extensive: $\Box\forall x\forall y(\lambda x[yRx]x \leftrightarrow \lambda x[xR^{-1}y]x)$, yet distinct: $\lambda x[yRx] \neq \lambda x[xR^{-1}y]$. These properties differ merely hyperintensionally. They are distinct despite being necessarily co-extensive. Anyone who countenances distinct converses is thus committed to a hyperintensional theory of properties.

Alternatively, one can accept that there are non-symmetric relations, but deny that they have distinct converses.

2. Non-symmetric relations seem to have directions (what Russell: 1913, p. 86 calls their "from-and-to character"). This is what allows them to introduce order and to apply differentially to their relata. If relations have directions, then there is a meaningful notion of a converse relation. In order to ensure that relations do not have distinct converses, one has to adopt a sparse theory of relations and deny the existence of converses. Although the idea of the converse of a relation is intelligible and well-defined, no such relations exist according to this approach. As MacBride suggests: "Assuming universals are sparse, it doesn't follow from the fact that the *notion* of the converse of a relation is definable that there is a corresponding converse relation" (MacBride: 2014, p. 9).

   Adopting a sparse theory does not imply that one is committed to claiming that only one of 'taller than' and 'shorter than' refers to an existing relation whereas the other fails to refer. This is because one can follow MacBride and consider both of them to be impure referring terms that refer to the same relation (whichever of R and R^{-1} it is that happens to exist). Nevertheless, such a sparse theory would seem to involve a significant degree of arbitrariness. It recognises that there are two meaningful candidates that are on a par, yet privileges one of them.[6]

3. One can adopt a revisionist view about the nature of relations that does not consider them to have directions. By denying that relations have directions, one can reject the idea that there is a meaningful notion of a converse relation. The two main revisionist approaches are positionalism (cf. Williamson: 1985) and antipositionalism (cf. Fine: 2000). Positionalism attempts to explain differential application, not in terms of relations having directions, such that R can either go from a to b or from b to a, but in terms of relations having argument places. There are two ways in which

[6]Rather than arbitrarily privileging one of them, one can claim that there is a privileged relation yet that it is indeterminate which of them is privileged. This, however, implies a commitment to ontic indeterminacy. Moreover, sparseness has to be understood at the level of existence (rather than, say, naturalness) if one is to address all the problems deriving from the existence of distinct converses. It is not enough to accept an abundant conception of relations and then privilege one relation over its converse by taking the former to be more natural or fundamental than the latter. One has to deny that converses exist. Accordingly, one would be committed to existence being indeterminate if one were to attempt to mitigate arbitrariness by invoking indeterminacy. (Privileging one relation over its converse in terms of naturalness may well be sufficient for addressing the problems of uniqueness and redundancy (identified in section 3.1) and possibly also the problem of referential indeterminacy when invoking reference magnetism to secure determinate reference. Yet, it will neither address the problem of brute necessities nor the objectionable proliferation of states-of-affairs.)

objects can be assigned to the argument places α and β of a binary relation R, each giving rise to a distinct completion: a can be assigned to α and b to β or, alternatively, a can be assigned to β and b to α. Antipositionalism, by contrast, attempts to explain differential application in terms of the idea of different completions of a relation being co-mannered. A completion of R by a and b can either be co-mannered or non-co-mannered to another completion of R by c and d. The relation of being co-mannered gives rise to equivalence classes of completions that correspond to the different applications of the relation in question. Differential application is then not understood in terms of the internal structure of a completion considered by itself, but in terms of how different completions relate to each other.[7]

On these approaches, there is no such thing as the converse of a relation, and, a fortiori, no such thing as a non-symmetric relation having a distinct converse. Problems arise, however, once we distinguish a strict from a loose sense of converse. Converses in the strict sense are characterised in terms of the notion of direction. A relation R^{-1} is the converse of a relation R iff these relations only differ in terms of their directions.[8] Converses in the loose sense (which we will also call 'duals'), by contrast, are characterised in terms of relations merely differing in terms of whatever it is that allows for differential application. Revisionist theories also need to account for differential application and hence need to find some substitute that plays the role that is played by directions. If a relation R^* involves the very same objects as R and differs only with respect to whatever it is that accounts for differential application, then these relations are mere duals. Such relations hold of the same things, i.e. they have the same extension (understood in an unstructured way) such that $\Box\forall x\forall y(xRy \leftrightarrow yR^*x)$, and only differ in terms of their argument places (in the case of positionalism) or in terms of their equivalence classes of co-mannered completions (in the case of anti-positionalism).

[7]The antipositionalist mirrors the relationalist approach to chiral properties which denies that objects are endowed with intrinsic handedness properties and only stand in same- or opposite-handed relations, on the basis of which one can partition them into equivalence classes. One important disanalogy is that for the relationalist there are no cross-world same- or opposite-handed relations, since one would otherwise end up with there being two worlds, each with a lonely hand, that differ merely in terms of their chiral properties. Yet, the antipositionalist is committed to completions in different worlds being co-mannered: cf. "we must be able to identify the manner of completion from one world to another" (Fine: 2000, p. 24). This, however, is deeply problematic since the relation of being co-mannered is an external rather than an internal relation, which contravenes the ban on cross-world external relations.

More generally, the parallels between the debate about chiral properties and the debate about relations are worth noting. For instance, the modal scenarios that Dorr invokes directly mirror those found in the chiral case.

[8]Cf. A converse relation in the strict sense "is one that differs from the given relation merely in the order of its arguments" (Fine: 2000, p. 3 fn. 1).

For the positionalist, for instance, a dual of R is such that necessarily for any objects x and y, x will be assigned to α and y to β in R iff y is assigned to γ and x to δ in R*. Whilst a binary relation can have one converse, since there are only two possible directions in which two relata can be related, it can have any number of duals. For instance, one can also have a further relation R** where x is assigned to ϵ and y to ζ. (van Inwagen seems to be invoking the loose notion of a converse when he questions why a binary relation has "just one converse" (van Inwagen: 2006, p. 474 endnote 4).) If one denies that relations have directions, then R* does not classify as the converse of R, i.e. as a relation that merely differs in terms of its direction from R. Nevertheless, R* gives rise to the very same difficulties of referential indeterminacy, brute necessities and a proliferation of states-of-affairs as the converse R^{-1} of R.

One cannot simply claim that R and R* are identical on the grounds that they are necessarily co-extensive. Given the differential applicability of relations, we need to distinguish aRb from bRa. This means that the extension of a binary relation cannot be understood as a set of unordered pairs: {{a,b}, {c,d} ...}. The extension has to specify not only the relata, but also their order. This, however, means that R and R* are not co-extensive. Although they hold of the same things, they do not hold of them in the same order (where the notion of order can be understood in terms of directions, assignments to argument places, or co-mannered equivalence classes).

The relevant sense of order is traditionally understood in terms of the direction of a relation and the extension is, correspondingly, construed in terms of ordered pairs: {⟨a,b⟩, ⟨c,d⟩ ...}. Two binary relations are then necessarily co-extensive iff they hold of the same ordered pairs across all of modal space. The positionalist, by contrast, invokes assignments of objects to argument places. The extension of a relation is understood in terms of assignments to argument places: $\{\{\langle a,\alpha\rangle,\langle b,\beta\rangle\}, \{\langle c,\alpha\rangle,\langle d,\beta\rangle\} \ldots\}$.[9] This, however, means that R and R* will fail to be co-extensive and hence turn out to be distinct. Given that R and R* have different argument places, they cannot be identical. Yet, since neither of them is privileged over the other, it would be arbitrary to adopt a sparse theory of relations and favour one of them, say, by claiming that it is only relation R which involves α and β that exists and that R* which involves γ and δ, though being perfectly well-defined, does not exist.

In order to consider R and R* to be co-extensive, one has to identify argument places across relations.[10] Yet, the intelligibility of such an iden-

[9] Similarly, for the antipositionalist extensions are understood in terms of sets of objects together with membership in an equivalence class of co-mannered completions.

[10] Similarly, in order to establish co-extensiveness the antipositionalist needs to accept that com-

tification is precisely what Fine denies in order to ensure that the notion of a converse is not definable for the positionalist. "We may indeed ask whether, for given argument-places α, β, α' and β', the relation R′ holds under the assignment of a to α' and b to β' just whenever R holds under the assigment [*sic*] of a to α and b to β. But this merely tells us whether the relations are coextensive under the given alignment of argument-places. To obtain the notion of converse, we also need to assume that $\alpha' = \beta$ and $\beta' = \alpha$. But I doubt that there is any reasonable basis, under positionalism, for identifying an argument-place of one relation with an argument-place of another" (Fine: 2000, p. 12). Fine's suggestion of denying that argument-places can be identified across relations allows the positionalist to deny the existence of distinct converses, however doing so opens up the possibility of distinct duals.

Revisionist approaches face a dilemma. If they deny the possibility of cross-relation comparisons,[11] they can deny that there is a meaningful notion of a converse relation that merely differs in terms of direction. Whilst successfully denying the intelligibility and hence existence of converses in the strict sense, such an approach cannot provide informative identity criteria for relations and hence cannot establish the identity of a relation with any of its duals. It cannot even provide an informative account of what it is for two relations to be co-extensive. This means that neither positionalism nor antipositionalism, construed in this way, is able to rule out distinct duals in a principled way.

The original difficulties, however, arise just as much in the case of duals. There is nothing to distinguish a relation from its dual in the same way as there is nothing to distinguish a relation from its converse. Given that there is nothing to privilege one of them, defenders of this approach likewise end up with a sparse theory of relations that only countenances some relations but rejects the existence of others (in a seemingly arbitrary manner). Rejecting duals and countenancing only one of R and R^* does not seem to be any less arbitrary and problematic than rejecting converses and only countenancing one of R and R^{-1}. In each case, there are a number of distinct yet intimately related relations that the theory deems to be intelligible, yet only one of which it deems to exist.[12]

Alternatively, if they accept the possibility of cross-relation comparisons,

pletions of different relations can be co-mannered, i.e. that a completion of R can be co-mannered to a completion of R^*.

[11] For the positionalist this amounts to rejecting the possibility of identifying argument places across relations. For the antipositionalist it amounts to rejecting cross-relation co-manneredness facts.

[12] In addition, we will see in section 5.5 that an unrestricted ban on cross-relation comparisons is problematic for independent reasons.

then they can meaningfully speak of different relations being co-extensive. This allows them to specify informative identity criteria for relations, for example by considering relations that are necessarily co-extensive to be identical. Distinct duals that merely differ in terms of the identity of their argument places can then be ruled out. However, one can also reintroduce the notion of a converse in the strict sense. A relation R* will classify as a converse of R for the positionalist if the assignment of objects to argument places in R is necessarily the reverse order of the assignment of objects to argument places in R*, whereas for the antipositionalist it will be the converse if necessarily there is a completion of R by x and y iff there is a completion of R* by the same objects such that the two completions fail to be co-mannered. This, however, means that revisionist approaches effectively collapse into the traditional approach and face the very same difficulties that are involved in either countenancing distinct converses or adopting a sparse theory of relations that arbitrarily privileges some relations over others.

A satisfactory account needs to not only rule out the existence of distinct converses in the strict sense but also in the loose sense. In order to do this, it needs to provide informative identity criteria for relations that preclude, in a principled way, any excessive proliferation of relations. This will be achieved, on the one hand, by rejecting non-symmetric relations at the fundamental level whilst individuating fundamental relations in terms of necessary co-extensiveness and, on the other hand, by developing an account of derivative non-symmetric relations that are individuated in terms of a theory of hyperintensional equivalence that does not allow for converses in either the strict or the loose sense.

3 Fundamental symmetric relations

Two further problems arise if fundamental non-symmetric relations have distinct converses, namely the problem of uniqueness and the problem of redundancy. These problems suggest that fundamental theorising is only allowed to invoke symmetric relations. They can be nicely illustrated by means of the difficulties that arise for Carnap's Aufbau due to the fact that he operates with an asymmetric basic relation.

In the Aufbau, Carnap puts forward the outlines of a logical construction of the world. Starting with an auto-psychological basis, consisting of 1. a domain of elementary experiences ('Elementarerlebnisse') and 2. a unique basic relation Rs (= recollected similarity), Carnap logically constructs the entire world: from the subject's inner mental life, to the intersubjective physical world, to cultural objects and ethics. This construction proceeds by giving pure structural descriptions of all items in the world. In virtue of being structural, such descriptions are objective.

They are communicable, scientific and can constitute knowledge, unlike what is merely subjective and can only be identified by ostension (cf. §16).

After having outlined his constructional system, Carnap returns in §153 to the problem of the basic relation. The goal of the Aufbau is to put forward a logical construction. Every scientific statement is to be transformed into a purely logical structural statement. Carnap notes that this goal has not been achieved at that point. The problem is that we only have a logical construction of the world if the construction proceeds on the basis of logical relations. The Aufbau, however, has not transformed anything into a purely logical statement. Instead it has produced constructions that contain one extra-logical primitive, namely the basic relation Rs. This relation is not a logical but a psychological relation. As a result, it undermines the purity of the system and prevents the Aufbau from classifying as a logical construction.

The basic relation Rs needs to be eliminated from the constructional basis. It itself has to be logically constructed. Only then will the Aufbau be a purely logical construction. The problem, however, is that any attempt to construct Rs would seem to require another extra-logical primitive, in which case the original problem would simply arise again. It is for this reason that Carnap proposes to provide, not an explicit definition of Rs in terms of some more basic concept, but an implicit definition in terms of the constructional system: Rs is the unique binary relation R such that R allows us to construct the Aufbau (more precisely, he picks a sufficiently high-level object in the constructional system). We use Rs to construct the system and then define this relation away as that relation which allows us to perform this logical construction of the world. Once we have constructed the Aufbau by means of Rs, we can kick away the ladder that got us there.

This suggestion runs into serious trouble. As Carnap notes in §154, it will trivially be the case that there is a very large number of relations that allow one to construct the Aufbau unless restrictions are imposed on what R can be, i.e. what R ranges over. If we have a domain D with an abstract structure in terms of Rs over it, then a permutation f of the members of the domain that is not an identity-mapping gives rise to a relation R* such that R*f(x)f(y) iff Rsxy. This means that if Rs allows one to construct the Aufbau, then so does R*. These two relations have the very same structure and hence can construct the same system. As a result, uniqueness will fail and Rs cannot be implicitly defined.[13]

Carnap attempts to resolve this problem in §155 by introducing a new primitive: foundedness. The failure of uniqueness is due to the fact that no restrictions were imposed on R, i.e. R can simply be understood in terms of a set of ordered pairs. Any such set of ordered pairs is as good as any other. This is what allows us to construct relations, such as R*, that have the same structure as Rs by means of

[13]This problem is closely related to the Newman problem for structural realism (cf. Newman: 1928), as well as to Putnam's model-theoretic argument (cf. Putnam: 1983).

arbitrary permutations. In order to ensure uniqueness, we need to rule out gerrymandered relations and narrow down the possible candidates that can satisfy the implicit definition. Foundedness is precisely meant to fill this role. Carnap suggests that Rs is the unique founded binary relation R such that R allows us to construct the Aufbau. The random permutations giving rise to R* and the like do preserve the structure that is required for constructing the Aufbau, however, they are not founded relations.[14]

For this suggestion to work, foundedness has to be a logical concept. Otherwise, the construction would not classify as being a purely logical construction. After all, the reason for trying to eliminate Rs by means of an implicit definition was that Rs is an extra-logical primitive that renders the constructional system impure. Replacing one extra-logical primitive (Rs) by another extra-logical primitive (foundedness) would not be an improvement. What needs to be done is to devise an implicit definition of Rs that only employs logical notions. For this reason, Carnap has to insist that foundedness is a purely logical primitive. It is this commitment that usually has been singled out as being the point where Carnap's project founders. "The idea that [the implicit definition of Rs] is a purely logical formula *is* absurd" (Demopoulos and Friedman: 1985, p. 637).

However, Carnap's construction fails even if one grants that foundedness is a purely logical primitive. The problem is that the basic relation Rs is an asymmetric relation (cf. §108). As such, it has a distinct converse Rs^{-1}. Foundedness cannot differentiate between a relation and its converse. It is implausible to think that one of them is privileged and more basic, say that 'worse than' is more fundamental than 'better than', or that 'taller than' is more fundamental than 'shorter than'. This means that Rs is founded iff Rs^{-1} is also founded. Foundedness, naturalness and the like do not differentiate between a relation and its converse – either both are founded/natural or both are gerrymandered.[15]

Foundedness thus does not rule out all permutations. It only allows us to exclude various gerrymandered permutations that do not preserve foundedness. Yet the permutation that maps a relation into its converse is precisely one that does preserve foundedness. By simply permuting the relata of Rs, we ensure that Rsxy iff $Rs^{-1}f(x)f(y)$, which is equivalent to $Rs^{-1}yx$. The resulting two structures are isomorphic and are equally founded. Even when the structure is augmented by

[14] This corresponds to the suggestion that Newman considers, namely that certain relations are 'important', and which he rejects as being absurd (cf. Newman: 1928, p. 147). It also corresponds to Lewis's proposal to appeal to 'naturalness' (cf. Lewis: 1984).

[15] Whilst many non-symmetric relations seem to be equally natural as their converses, there are some cases in which one can plausibly privilege a relation over its converse. For instance, when R is active and its converse R^{-1} is passive, it is plausible to consider R to be prior to R^{-1}. This difference, however, is not to be explained at the level of relations. These cases are best construed not in terms of one relation being more natural than or prior to another, but instead in terms of there being a generative operation that has an input-output structure (cf. section 6). The input of such an operation is prior to its output and actively generates it, whereas the converse operation is not generative. It is for this reason that the active is privileged over the passive.

means of foundedness, we are still left with a failure of uniqueness due to there being non-trivial structure-preserving permutations that are also foundedness-preserving. There is no unique founded relation that allows us to construct the Aufbau. As a result, uniqueness fails and it is not possible to implicitly define Rs even when appealing to foundedness.

3.1 Fundamental theorising

The lesson to be learnt from these difficulties for the Aufbau is that fundamental theorising is to invoke only symmetric and no non-symmetric relations (given that, if there were to be fundamental non-symmetric relations, they would have distinct converses or at least distinct duals).

4. FAILURES OF UNIQUENESS

Pinning down reference and ruling out unintended interpretations is a central role that the notion of fundamentality is meant to fulfil, not only in the Aufbau, but also in the context of the Newman problem and Putnam's model-theoretic argument. It can only fulfil this role if there are no fundamental non-symmetric relations that have distinct converses. Since fundamentality/naturalness cannot differentiate between a relation and its converse, it cannot achieve uniqueness when dealing with non-symmetric relations. If one is fundamental/natural, then so is the other. This means that the notion of fundamentality only succeeds in fulfilling one of the important roles that it is meant to fulfil if the fundamental level does not contain any non-symmetric relations that have distinct converses. Otherwise, the notion of fundamentality, though ruling out various gerrymandered candidates, will not enable us to achieve uniqueness.

How problematic one considers the failure of uniqueness to be depends on whether one considers implicit definitions to lack a denotation or to have an indeterminate denotation in cases in which there are multiple candidates.[16] Lewis, for instance, held each of these views at some point and ultimately settled on an intermediate position: "In cases where there is a unique x such that T[x], Lewis says that t denotes that x. What if there are many such x? Lewis's official view in the early papers is that in such a case t does not have a denotation. In 'Reduction of Mind', Lewis retracted this, and said that in such a case t is indeterminate between the many values. In 'Naming the Colours' he partially retracts the retraction, and says that t is indeterminate if the different values of x are sufficiently similar, and lacks a denotation otherwise" (Weatherson: 2014, section 4.1). Whilst outright

[16]Although Carnap insists on uniqueness (cf. §§13-14), Friedman and Demopoulos suggest that he could have put forward an existential claim and not incurred any commitments to uniqueness, cf. Demopoulos and Friedman: 1985, p. 637.

reference failure is particularly troubling, indeterminate reference is likewise not all that palatable. Either way, complete determinacy can only be achieved by denying that there are relations that have distinct converses at the fundamental level.

5. REDUNDANCY

If a fundamental relation R has a distinct converse R^{-1}, then the fundamental level will exhibit a radical form of redundancy. There will be two fundamental relations, but it will not be necessary to appeal to both of them. Since Rs and Rs^{-1} have the very same structural features, i.e. they satisfy the same Ramsey sentences, it follows that whatever can be constructed by means of the one can also be constructed by means of the other. As a result, one of them will suffice and the other will be redundant.

Fundamental relations should not be dispensable in this way. One should not be able to do without them when giving a fundamental characterisation of the world. Something has gone wrong if Rs^{-1} is deemed to be fundamental, yet the entire world can be constructed without recourse to this relation. Part of what it is to be a fundamental relation, one might think, is to play an ineliminable role in fundamental theorising. Fundamental theorising should satisfy a non-redundancy constraint and, accordingly, has to operate only with symmetric rather than non-symmetric relations.

This means that fundamental theorising (of which Aufbau-style theorising is a paradigmatic instance) has to proceed by means of symmetric relations. Non-symmetric relations have to be excluded from the fundamental level and instead have to be banished to derivative levels.[17]

4 Derivative non-symmetric relations

Fundamental theorising is to eschew non-symmetric relations. Restricting the fundamental level to symmetric relations makes it difficult to explain where order comes from. Symmetric relations seem to contain too little structure, or at any rate the wrong kind of structure, to account for all the order and asymmetry that is to be found in the world. This section explains how asymmetry and order can be introduced into a world that only contains symmetric but no non-symmetric fundamental relations.

[17]This means, for example, that one cannot posit a fundamental temporal arrow, but instead either has to explain the direction of time in terms of a generative operation, or consider it to be a derivative feature of the world that is grounded in the global distribution of property instantiations.

4.1 Asymmetric structures

The relata of a symmetric relation are not ordered. The relation goes in both directions: xRy and yRx. As a result, permuting the relata does not result in a different state of affairs. Nevertheless, symmetry can fail when operating only with symmetric relations. Even though one symmetric relation by itself does not order its relata, a network of such relations can do so. The pattern of instantiation of symmetric relations can give rise to an asymmetric structure.

If there is some permutation of the elements of the domain that does not preserve the structure, then there is some asymmetry and order. The three element structure xRy and yRz, though symmetric in x and z, fails to be symmetric in y, i.e. any permutation that non-trivially involves y will not be structure-preserving. This means that the ternary relation $R' = \lambda xyz[xRy \wedge yRz]$ is not fully symmetric, even though the relation R out of which it is constructed is symmetric.

If the structure does not allow for any non-trivial automorphisms, then it is possible to provide unique structural identifications of all elements in the structure. This can be achieved by combining different symmetric relations. For instance, the structure xRy and yR*z allows us to distinguish the different elements. Each element can be uniquely identified: x is the unique element that is R-related without being R*-related, whereas z is the unique element that is R*-related without being R-related, and y is the unique element that is both R and R*-related. This means that the polyadic property connecting x, y and z, namely the ternary relation $R'' = \lambda xyz[xRy \wedge yR^*z]$ is non-symmetric (if R and R* are incompatible, i.e. if xRy implies $\neg(xR^*y)$ then it will be asymmetric). This relation is internal to the structure, i.e. it is intrinsic to the triple. By derelativising the property, we can construct a non-symmetric binary property that is extrinsic to the pair, namely $R''' = \lambda xz[\exists y(xRy \wedge yR^*z)]$. This is a non-symmetric binary relation connecting x and z that is grounded in the symmetric relations in which x and z stand in to some y. The relative product of two symmetric relations can thus be non-symmetric.[18]

Symmetry can fail globally even when working with a single symmetric relation, as long as the structure is sufficiently complex. In particular, the structure has to form an asymmetric graph. In that case, one can uniquely identify each point in the structure in terms of the place that it occupies within the network of symmetric relations. For instance, an asymmetric connected graph that is based on only one binary symmetric relation will have to have at least six nodes (cf. Dipert: 1997, p. 347).[19]

[18] We can generate a binary non-symmetric relation that is intrinsic to the pair from a structure consisting of only two elements that stand in xRy and xR*x, i.e. $R'''' = \lambda xy[xRy \wedge xR^*x]$ (though it is not clear whether the reflexive relation R* is nothing but a monadic distinguishing feature had by x, in which case the failure of symmetry would not be due to how the relata are connected to each other, but due to the properties that they have).

[19] Much of the Aufbau precisely consists in construing the world as such an asymmetric graph,

4.2 Internal relations

Derivative relations can fail to be symmetric if they are grounded in properties that are asymmetrically distributed amongst their relata. In this case, we are not concerned with the positions that the relata occupy in an asymmetric structure, but with the properties that the relata instantiate. The properties in which the relation is grounded can be intrinsic as well as extrinsic. Moreover, they can be monadic as well as polyadic, depending on whether the relata of the relation in question are individual objects or pluralities of objects. For instance, the asymmetric 'taller than' relation is grounded in the heights of its relata. What makes it the case that x is taller than y is nothing but the heights of x and y. The monadic properties instantiated by the relata, Fx and Gy, collectively ground xRy. Similarly, the 'further apart than' relation is grounded in the distances of its relata. What makes it the case that x and y are further apart from each other than are v and w is nothing but the distances between x and y as well as between v and w. The polyadic properties instantiated by the relata, namely xRy and vR′w, collectively ground xyR*vw.

These derivative relations, unlike those grounded in asymmetric structures, are internal relations in the sense that they can hold across worlds. That x is F whereas y is G grounds x's being taller than y, independently of whether x and y are to be found in the same world or not. Asymmetric structures, by contrast, are constructed out of external relations that can only hold amongst world-mates.[20]

This understanding of internal relations is sometimes questioned on the basis that the properties by themselves do not imply any sense of order. All that we get is a difference between the objects, yet the notion of a difference is symmetric. Russell, for instance, claims that "the mere fact that the adjectives are different will yield only a symmetrical relation" (Russell: 1903, §214). The asymmetric relation xRy, it is argued, can only be grounded in Fx and Gy because there exists an asymmetric higher-order relation R^* such that FR^*G (cf. Bigelow and Pargetter: 1990, pp. 55-56; MacBride: 2016, section 3). For instance, it is only because the heights F and G stand in an asymmetric second-order relation that it is possible for x and y to stand in the first-order 'taller than' relation in virtue of having these heights, i.e. being F and G respectively.[21] The properties instantiated by

i.e. a graph in which there are no homotopic points (cf. Carnap §14). Although Carnap's fundamental relation Rs is asymmetric, most of the constructions proceed via the symmetric closure of Rs, i.e. $Rs \cup Rs^{-1}$ (cf. Carnap §104).

[20]This construal of internal relations is a generalisation of the way in which this notion is usually understood, namely in terms of a relation that supervenes on the intrinsic properties of the relata. For our purposes there is no reason to restrict the grounds to intrinsic properties. Extrinsic betterness, for instance, can be treated as an internal relation in the same way as intrinsic betterness. That x is extrinsically better than y is grounded in the extrinsic values of x and y, i.e. it is grounded not only in the intrinsic but the extrinsic properties of the relata. This relation can unproblematically hold across worlds.

[21]For a higher-order theory of quantities cf. Mundy: 1987.

the relata, on this view, do not suffice but need to be supplemented with an asymmetric relation holding amongst those properties. This would mean that instead of explaining how asymmetry emerges, we would in fact be presupposing asymmetry.

This suggestion, however, is incorrect. The relation $xRy = \lambda xy[Fx \wedge Gy]$ is non-symmetric and allows for differential application (if F and G are incompatible, then R is asymmetric). Although the difference between F and G is symmetric in that these two properties differ from each other, the way in which these properties are distributed amongst x and y is asymmetric. Put differently, even though we only have a symmetric difference at the level of the properties, at the level of the property instantiations we have an asymmetric distribution.

When we are operating with numerical representations of, say, heights, then we are in fact presuposing an ordering over the real numbers by means of which we represent the members of the domain. This, however, is at the level of measurement theory, not at the level of metaphysics. The qualitative 'taller than' relation (which we are representing by means of the numerical 'greater than' relation) can be grounded in the intrinsic properties of the relata. This relation can be represented by means of a relation amongst numbers but is not to be identified with (nor is it grounded in) such a relation. The intrinsic properties of the relata, namely their heights, do not presuppose any ordering. As long as they are distributed asymmetrically, the resulting derivative relation will fail to be symmetric.

This is particularly clear when considering relations that are internal not to the relata but to the pair. Such relations are grounded not only in properties of the relata but also in external relations connecting the relata, where the failure of symmetry is due to the way in which the properties are asymmetrically distributed amongst the relata.[22] If a symmetric relation R^* is combined with a distinguishing intrinsic feature F that can be had by only one of the relata, then one ends up with a derivative non-symmetric relation R. The relation $xRy = \lambda xy[Fx \wedge R^*xy]$, where R^* is a symmetric relation, is a non-symmetric relation that is grounded in R^*xy together with Fx. For example, the relation that holds between x and y in case x is a brother of y is a non-symmetric relation that is grounded in the symmetric sibling relation and the intrinsic property of being male, i.e. x stands in the 'brother of relation' to y iff x and y are siblings and x is male.

This case clearly shows that one is not presupposing any asymmetric relations or orderings in the background. There is no need for any higher-order relation amongst the properties involved in grounding this non-symmetric relation. Instead, the failure of symmetry derives from the distinguishing feature F had by x. This property does not have to be related in any distinctive way to anything else, in particular it does not have to stand in any non-symmetric higher-order relations. All that is required is that it is not a trivial property relative to R^*, i.e. it has to be such that not all relata of R^* are automatically F. If it is such that only

[22] Accordingly, they cannot hold across possible worlds.

some of the relata of R* are F, then those will be asymmetrically related via R. In short, what is at issue is not any non-symmetric relation amongst properties but a non-symmetric distribution of property instantiations amongst the relata.[23]

4.3 Absolutism v. comparativism

This account of how order and asymmetry are introduced into a world that only contains symmetric relations at the fundamental level has important implications for the debate between absolutism and comparativism. This debate concerns the question as to whether comparative or absolute notions are more fundamental when it comes to providing an account of quantities/magnitudes. For instance, is goodness prior to betterness, or is betterness prior to goodness? Which of these is the more fundamental notion? Whereas comparativists take betterness to be basic and analyse goodness in terms of it (cf. Broome: 1993), absolutists take goodness to be the basic notion and consider betterness to be analysable.

An important problem for comparativism is that the relevant comparative notion is asymmetric. As a result, it has a distinct converse. If betterness is not grounded in goodness, but is instead taken to be prior to goodness, does this mean that worseness is likewise prior to goodness? Symmetry-reasoning would suggest so. All the arguments that speak in favour of betterness being prior to goodness also speak in favour of worseness being prior in this way. Privileging one of them would be arbitrary. This, however, means that there are two primitives: betterness and worseness. The resulting theory will be far from parsimonious and will give rise to an excessive proliferation of states of affairs. Its fundamental base, moreover, will be characterised by redundancy, since there will be two relations that can perform the very same work. In addition, the comparativist would seem

[23]To give a fully satisfactory account of derivative relations, one not only has to explain how they can fail to be symmetric but also give an explanation of their structure. It might be thought that, even though non-symmetric higher-order relations amongst properties are not in general required in order to ground non-symmetric relations, they are nevertheless required in order to adequately account for the structure of these relations, for instance when it comes to explaining the transitivity of the taller than relation. How can the monadic non-relational properties of x, y and z ensure that if x is taller than y and y is taller than z that x is also taller than z?

One way of explaining this is to operate at the level of the grounds of the relevant properties. In the case of extensive magnitudes (where these are construed in the traditional sense, i.e. as magnitudes that have a spatial or temporal extension) this can be done in terms of the parthood structure of the relevant inter-object grounds. If x is F, y is G and z is H, then this is because the xx's that compose x satisfy condition Γ, the yy's that compose y satisfy condition Δ and the zz's that compose z satisfy condition Λ. Now if the xx's can be partitioned into two sub-pluralities such that Γ(xx) consists in $\Gamma_1(x_1x_1)$ and $\Gamma_2(x_2x_2)$, whereby the condition satisfied by the x_2x_2's is the same as that satisfied by the yy's, i.e. $\Gamma_2 = \Delta$, and if in addition, the yy's can be partitioned into two sub-pluralities such that Δ(yy) consists in $\Delta_1(y_1y_1)$ and $\Delta_2(y_2y_2)$, whereby the condition satisfied by y_2y_2's is the same as that satisfied by the zz's, i.e. $\Delta_2 = \Lambda$, then we can explain why xRy (which is grounded in Fx and Gy) and yRz (which is grounded in Gy and Hz) implies xRz (which is grounded in Fx and Hz).

to incur brute necessary connections in order to ensure that betterness and worseness are co-ordinated in the right way.

The absolutist, by contrast, does not face these problems. Goodness is prior.[24] Both betterness and worseness are grounded in the properties of their relata. Given that they have the very same grounds, one can explain the identity of these two relations and can hence avoid ontological profligacy as well as redundancy. In short, the absolutist can simply appeal to the internal relations framework sketched above.

Not only do comparativists fail to give a satisfactory account, due to ignoring worseness and instead focusing exclusively on betterness, they are also unable to adequately deal with the 'equally good as' relation. Is this relation likewise prior to goodness? For the purposes of measurement theory, one usually operates with a single primitive, namely the weak betterness relation $\geq$. This allows one to define all the ordering relations, e.g. equally good can be defined in terms of weak betterness (i.e. $x = y$ iff $x \geq y \land y \geq x$). It is, however, implausible to consider $\geq$ to be a fundamental notion. This notion is used in measurement theory because it is the most convenient notion to work with. It is for this reason that it is used as a primitive in formal systems. Whilst convenient, it does not correspond to any metaphysical primitive. Instead, it is a disjunctive notion. We do not have any independent grasp on weak betterness except as the disjunction of strict betterness and equal goodness. (This is analogous to the situation in mereology where the notion of proper or improper parthood is used when constructing a formal mereology, yet where this notion is not understood to be metaphysically fundamental but to be a disjunction of identity and proper parthood.) The comparativist thus has to treat the 'equally good as' relation as yet another primitive alongside the 'better than' relation and its converse the 'worse than' relation.

Given completeness, one can define equal goodness in terms of strict betterness/worseness, i.e. $x = y$ iff $\neg(x > y) \land \neg(y > x)$. Or alternatively, in terms of $x = y$ iff $\forall z[(z > x \leftrightarrow z > y) \land (x > z \leftrightarrow y > z)]$. Completeness, however, cannot simply be assumed in this way. On the one hand, there are plausible cases of incompleteness. On the other, it should be a substantive question whether the axiological ordering is complete, not one that is settled by stipulation. Difficulties arise, in addition, for the comparativist once we note that completeness is domain-relative. When we say that a relation induces a complete ordering, this is always an ordering of a given domain. The question now, however, is how one is to specify the domain in question. This domain has to be restricted to objects that are value-bearers. Otherwise, completeness will be guaranteed to fail since objects that are not value-apt will end up being classified as

[24]One might worry that in the same way that focusing on betterness leads one to ignore worseness, focusing on goodness likewise leads to a problematic neglect of badness. However, goodness is understood in terms of the entire value field, not just its positive segment, i.e. goodness refers to absolutist value properties whether they be positive, neutral or negative.

being equally good as each other (given that they will vacuously satisfy the condition) and as non-comparable to all those things that are value-apt. Restricting the domain to value-bearers, however, is problematic for the comparativist since the most natural characterisation of an object as being a value-bearer is an absolutist characterisation.[25]

Measurement theory takes a qualitative relational structure as basic and then establishes a homomorphism into a suitable numerical structure. Comparative notions, however, are metaphysically derivative. Monadic non-relational properties are metaphysically prior and give rise to asymmetric comparative relations. (When dealing with distance functions and the like, the relata of the comparative ordering are not individual objects but n-tuples. In that case it is absolutist symmetric relations connecting the members of n-tuples that are prior and that give rise to asymmetric comparative relations amongst the n-tuples. E.g. that the distance between x and y is greater than that between v and w is grounded in the absolutist distances, i.e. xRy and vR′w ground xyR*vw, rather than R* being a fundamental comparative relation that connects the pairs.)

Although comparativism is very natural from a measurement-theoretic point of view, we should not take measurement theory to be a guide to metaphysics. Metaphysical commitments should instead be decided on the basis of metaphysical considerations. These considerations strongly favour an absolutist approach. Whilst the comparativist runs into serious difficulties due to operating with non-symmetric comparative relations, the absolutist is working within the framework of internal relations and does not have any of these difficulties. There is no redundancy in the base, no indeterminacy of reference, no brute necessities and no overabundance of states of affairs. Accordingly, we should consider goodness to be prior to betterness.

5 Identity criteria for derivative relations

Order and asymmetry can arise at the derivative level, even when all fundamental relations are symmetric. Derivative non-symmetric relations, however, would seem to have distinct converses, or at least distinct duals. As a result, the original problems of indeterminacy, brute necessities and the proliferation of states of affairs still seem to loom large and it is only the problems of uniqueness and non-redundancy that are avoided, given that they apply only to fundamental relations.

We can address these problems by showing that derivative relations neither

[25] If one were to treat 'equally good as' as a primitive, then one could characterise value-bearers as things that stand in the 'equally good as' relation to themselves. (However, even this is questionable since one might well construe reflexive instantiations of 'equally good as' in absolutist terms, i.e. as simply amounting to the instantiation of a monadic non-relational property, since the reflexive restriction of this relation is not an external relation and hence not in the requisite sense a comparative primitive.)

have distinct converses nor distinct duals. Instead of there being distinct relations, we simply have different ways of picking out one and the same relation. In order to do this, we have to provide identity criteria for derivative relations. These criteria have to establish that whenever there is a derivative relation R, there is no distinct relation R* that is intimately connected to R in the way that a converse or dual is connected to R. By establishing that it is not possible for distinct derivative relations to coincide in this way, we can address all the original problems.

Derivative properties and relations (i.e. both monadic and polyadic properties) can be individuated in terms of a criterion of hyperintensional equivalence.[26] What it is to be a given property is to be grounded in certain ways in certain things. This is what makes a derivative property the particular property that it is. Accordingly, sameness of grounds implies sameness of property. Derivative properties, whether monadic or polyadic, are identical iff they have the same grounds, i.e. they are grounded in the same ways in the same things across all of modal space.

If we understand the identity criteria for derivative relations in terms of hyperintensional equivalence, then there can neither be distinct converses nor distinct duals. In order for two relations to be connected in the relevant way as to classify as converses/duals, they would have to derive from the same base. Yet, in that case the criterion of hyperintensional equivalence classifies them as being identical. For instance, there is only one relation that holds between x and y on the grounds that x is F and y is G, namely $xRy = \lambda xy[Fx \wedge Gy] = yR^{-1}x = \lambda yx[Gy \wedge Fx]$. Both are grounded in x being F and y being G. Whatever grounds xRy also grounds $yR^{-1}x$ and vice versa. Fx plays the very same grounding-role in xRy as it does in $yR^{-1}x$, and likewise for Gy. A derivative relation and its converses thus have the very same grounds. They are grounded in the same ways in the same things. Accordingly, there is only one relation that is picked out in different ways.

Given that there is only one relation, there is no proliferation of facts. For example, what it is for x to be taller than y is the very same thing as what it is for y to be shorter than x. There is no difference in terms of which aspects of the world make it the case that the fact [x is taller than y] obtains and those that make it the case that [y is shorter than x] obtains. These facts involve the same objects standing in the same relation and are grounded in the same distribution of underlying monadic non-relational properties amongst x and y. They are hence identical. There is only one fact about the relative heights of x and y that can be represented in two ways.

These hyperintensional identity criteria allow us to establish that there is only

[26] Cf. "Hyperintensional equivalence" (Bader: manuscript) for the resulting hyperintensional logic.

one relation in any given case.[27,28] Instead of there being two intimately connected binary relations, there are two different ways of picking out one and the same relation. The distinction between a relation and its converse, accordingly, is a merely nominal rather than a real distinction. Relations and their converses are distinguished only in language and thought but not in the world.[29]

5.1 Individuating fundamental relations

By individuating relations in terms of their possible grounds, one can establish that derivative non-symmetric relations do no not have distinct converses or duals. Since they have the same grounds, they are identical. This strategy for avoiding an excessive proliferation of derivative relations cannot be applied to fundamental relations. Since they do not have any grounds, they cannot be individuated in terms of their grounds.

In order to avoid a proliferation of relations at the fundamental level, one has to deny the existence of fundamental non-symmetric relations. Unless one is willing to endorse a sparse theory that privileges some relations in a seemingly arbitrary manner, one has to reject non-symmetric relations at the fundamental level. Since symmetric relations cannot have distinct converses/duals, there will then not be any distinct converses or duals at the fundamental level.

Moreover, once non-symmetric relations are ruled out at the fundamental level, one can reject the differential applicability of fundamental relations. This allows one to individuate fundamental relations in terms of their unstructured extensions across modal space. A structured conception of extensions, for instance in terms of ordered pairs or assignments to argument places, is only required when relations allow for differential applicability. If there are no non-symmetric fundamental relations, then the extension of a fundamental relation can be understood in terms of unordered n-tuples. In that case fundamental relations cannot be had in different ways. Differential applicability does not apply to them. It is only when dealing with derivative relations that we need to consider differential applicability and bring in a structured construal of extensions that takes into consideration not only which objects instantiate the relation but also which grounding-roles are played by which of the relata. Fundamental relations R and

[27] This does not mean that we end up with a sparse theory of relations, where this is understood relative to an abundant background of possibilities that we can make sense of. It is not the case that we recognise a number of relations and deem them to be intelligible, yet only consider some of them to exist. Instead, we have a theory according to which there is no room for distinct converses/duals of derivative relations. There is hence no need for arbitrarily privileging some relations over others.

[28] Whilst ruling out distinct converses and duals, the theory is nevertheless fine-grained and allows for hyperintensional differences, distinguishing relations that hold of the very same things across modal space yet nevertheless differ in terms of their grounds.

[29] Cf. Ben-Yami: 2004, pp. 90-91 for a proposal as to why we have names for both relations and their converses in natural language.

R* are thus identical iff they are necessarily co-extensive. Fundamental relations that apply to the very same things cannot differ merely in the way in which they apply to those things and hence are identical. As a result, one can avoid any over-abundance of relations at the fundamental level. There is no proliferation of relations and no need to invoke a sparse theory of relations that draws invidious distinctions.

5.2 Relational expressions and conventions

Relations do not have distinct converses. Instead of two relations, there are two ways of picking out one and the same relation. 'R' and 'R^{-1}' co-refer. This, however, conflicts with the original argument for distinctness:

$$
\begin{array}{ll}
1. & aRb \\
2. & \neg(bRa) \\
3. & bR^{-1}a \\
\hline
\therefore & R \neq R^{-1}
\end{array}
$$

Given that 'R' and 'R^{-1}' are not interchangeable, it would seem that these relational expressions do not refer to the same relation after all.

This conflict can be resolved by appealing to Williamson's approach to relational expressions and the role that conventions play in establishing reference to relations (as well as MacBride's related proposal in terms of impure referring terms). The reason why the relational expressions 'R' and 'R^{-1}' are not interchangeable is due to the fact that they only achieve reference in combination with a convention: "one must know not just which relation [the relational expression] stands for, but which way round its flanking terms are to be fed into that relation" (Williamson: 1985, p. 249; also cf. p. 257). Although these two expressions refer to the very same relation, they do so via different conventions. Accordingly, one needs to adjust for differing conventions when substituting co-referring relational expressions. Since the convention associated with the converse relational expression is the converse of the convention of the relational expression, substitution requires permuting the flanking terms of the expression. When substituting 'R^{-1}' for 'R', one has to reverse the order of the flanking terms. This defuses the original argument for distinctness and avoids referential indeterminacy.

Although there are two relational expressions, namely 'R' and 'R^{-1}', there is only one relation. Both expressions refer to the very same relation, where this is possible on the grounds that they achieve reference in combination with different conventions. The role of the convention is to indicate which flanking term plays which role in grounding the derivative relation.[30] For instance, in the case of the relation that holds between x and y on the grounds that Fx and Gy these relational expressions differ in that the relatum that plays the F-role is mentioned first in

[30] For the positionalist the role of conventions is to assign the flanking terms to argument places.

the case of 'R' but second in the case of 'R^{-1}', whereas the G-role is played by the second flanking term in the case of 'R' yet by the first in the case of 'R^{-1}'.[31]

5.3 Reflecting differences v. inducing order

A relation that allows for differential application can hold in different ways: e.g. xRy as well as yRx. If there were to be fundamental non-symmetric relations, then it would be a primitive fact that such a relation holds of the relata in a given way. There is nothing about the relata that makes it hold one way rather than the other. The relata by themselves do not fix the way in which the relation holds, the manner in which R applies. They do not order themselves. The relation is an external relation. It is added to the relata and thereby induces order. Order does not derive from the relata but is imposed on them.

If there were to be fundamental non-symmetric relations, then they would impose order externally. It would then be possible to add, not just one such relation to a given set of relata, but several such relations that merely differ in terms of their directions. If x and y do not order themselves but have their order imposed upon them, then inducing order on them by means of R, such that R goes from x to y, is perfectly compatible with there being another fundamental relation, namely the converse relation R^{-1}, that goes from y to x.[32] Since these relations order their relata differently, they are distinct despite holding of the very same things. Moreover, since they are distinct relations, there is no conflict in them ordering the same relata in opposite ways. That way one would end up with distinct fundamental relations that merely differ in terms of how they order the same relata.

By contrast, this cannot happen if order is not externally imposed by a relation that is superadded to the relata, but instead derives from the relata. In that case,

[31] When it comes to fundamental relations one cannot specify conventions in terms of the grounding-role that the various flanking terms of a relational expression play. This is unproblematic in the case of fundamental symmetric relations. However, it means that those who believe in fundamental non-symmetric relations will not be able to specify what conventions are operative in our language. For instance, the positionalist would have to be able to identify and differentiate the various argument places of fundamental relations in order to specify how conventions operate. This does not seem to be possible. The problem is particularly pressing when working with a sparse theory of relations of the kind proposed by MacBride. In that case, conventions specify which flanking term refers to the relatum from which the relation proceeds and which one refers to the one to which it proceeds. Yet, which of the two possible relations, namely R or its converse R^{-1}, exists would seem to be an inaccessible fact about the sparse realm of relations. Accordingly, we cannot identify which conventions are operative, i.e. whether the first flanking term refers to the relatum that plays the 'from-role' or to the one that plays the 'to-role'. This means that if there were to be fundamental non-symmetric relations, then the conventions that would be operative in the case of these relations would not be transparent to us.

[32] Or if there is a relation R with assignment x to α and y to β, then this is likewise compatible with there being another fundamental relation, namely a dual relation R*, with assignment x to γ and y to δ (or for that matter any number of dual relations with distinct argument places).

one cannot add different relations that merely differ in direction. Rather than inducing order, the derivative relation only reflects differences amongst the relata that are antecedently given. The relation is then internal. In such cases we do not need to induce order. There is no need to add a fundamental relation that applies one way rather than another. The relata (or the system of fundamental symmetric relations) already take care of this. Instead of order being externally imposed, the relation is fixed by the differences amongst the relata.

The derivative relation R derives from x and y instantiating certain properties and/or standing in certain relations, such that any relation that derives from the same basis will be identical to R. Put differently, R reflects certain underlying differences, such that any relation that reflects the same differences will be identical to R. For each way of grounding order amongst the relata, there is only one relation that is grounded in this way, where this relation can be picked out in different ways, namely by means of different relational expressions. As a result, one does not end up with a proliferation of relations.

Differential application is then not to be explained in terms of the possibility of inducing order in different ways. Instead, differential application is to be explained in terms of there being different ways in which the underlying properties and relations can be distributed. By changing which relata play which roles in grounding the derivative relation, one generates different applications of the same derivative relation. One and the same binary relation can be instantiated in different ways by the same objects, i.e. aRb and bRa, since there are different possibilities as to which object plays which grounding role. For instance, whereas Fa and Gb ground that a is taller than b, Fb and Ga ground that b is taller than a. We can explain differential application without having to invoke the idea that relations have directions or argument places. Nor do we need to consider differential applicability to be an unexplained primitive (as is done by the antipositionalist). Instead, we can explain it in terms of there being different grounding roles that can be played by the relata of derivative non-symmetric relations.

On this approach, relations can be understood as being completely unstructured. Differential application is not explained in terms of the internal structure of non-symmetric relations. In particular, it is not explained in terms of the idea that relations have directions, argument places or the like, which means that there is no need for a reification of directions or argument places. Unlike antipositionalism, which also rejects a conception of relations according to which they have internal structure, the grounding account does not locate the complexity in the external connections amongst different instantiations of a relation. Instead, it locates the complexity that accounts for differential applicability in the grounds of the relation, thereby ensuring that the way in which a relation applies to its relata is an internal rather than external matter. That which makes it the case that the relation is instantiated explains the particular manner in which the relation is instantiated. The structure is not in the relation but in its grounds.

Since fundamental relations are ungrounded, the extension of such relations is to be understood in terms of a set of unordered n-tuples. (As we have seen, this does not leave any room for distinct converses or duals, given that the fundamental level is restricted to symmetric relations.) The extension of a derivative relation, by contrast, needs to be understood in terms of assignments to grounding roles: $\{\langle a,F\rangle,\langle b,G\rangle\}$. Given the possibility of differential application, it is not enough to be told which objects are related by the relation in question. In addition, we need to be told in which way the objects are related. This amounts to being told which object plays which grounding role. Accordingly, the extension is a set of assignments to grounding roles. This ensures that relations and their converses are necessarily co-extensive (and, in fact, even hyperintensionally equivalent) since they involve the very same objects fulfilling the same grounding roles.

5.4 Symmetric instantiations

The grounding account construes extensions in terms of assignments to grounding roles. This is somewhat analogous to positionalism, which operates with assignments to argument places $\{\langle a,\alpha\rangle,\langle b,\beta\rangle\}$. There is, however, an important difference when it comes to symmetric relations. A major problem for positionalism is that $\{\langle a,\alpha\rangle,\langle b,\beta\rangle\}$ will be distinct from $\{\langle b,\alpha\rangle,\langle a,\beta\rangle\}$ even when R is symmetric (cf. Fine: 2000, pp. 17-18). A binary symmetric relation will have two distinct argument places and hence will allow for two distinct completions.

The problem for positionalist is, in fact, more general. Difficulties arise not only in the case of symmetric relations. Positionalism is also unable to provide a satisfactory account of symmetric instantiations of relations that are neither symmetric nor asymmetric. Such non-symmetric relations allow for two different kinds of symmetric instantiations.

On the one hand, a symmetric instantiation of a binary relation can consist in a single two-way instantiation. For example, the weak betterness relation which is the disjunction of the symmetric 'equally good as' relation and the asymmetric 'strictly better than' relation can hold symmetrically of objects that are equally good. In this case there is only one state of affairs. When a and b are equally good, the fact that a is weakly better than b and the fact that b is weakly better than a are the very same fact, i.e. [aRb] is identical to [bRa]. Although there are two ways in which [aRb] can obtain (i.e. two ways in which R can be instantiated by a and b in that order) and two ways in which [bRa] can obtain, there is only one way in which both [aRb] and [bRa] can co-obtain, namely the one that is shared by both of them. Even though it is possible to have [aRb] without [bRa] (and vice versa), they are identical when both of them obtain together. Such a two-way instantiation involves the same objects instantiating the same relation on the basis of one and the same ground. There is hence only one fact to the effect that a and b are symmetrically R-related.[33]

(This means that facts are to be individuated in terms of their constituents and their grounds, not in terms of the possible ways in which they can be grounded or the different ways in which they can obtain. Facts can be identical even though they admit of different possible grounds, as long as they have the same constituents and the same actual grounds.[34] Object-individuation and property-individuation are, accordingly prior. Facts are constructed entities that are individuated in terms of their constituents and grounds.)

On the other hand, a symmetric instantiation of a binary relation can consist in two one-way instantiations. For example, the 'brother of' relation is grounded in the symmetric sibling relation together with the distinguishing feature of being male that can but need not be had by only one of the relata. If the distinguishing feature is had by both relata, then this relation holds in both directions. The fact that a stands in the brother of relation to b and the fact that b stands in the brother of relation to a are two different facts. Although they involve the same objects and the same relation, [aRb] and [bRa] have different grounds and hence are distinct. Even though both are partially grounded in a and b instantiating the symmetric sibling relation R*, [aRb] is partially grounded in Fa whereas [bRa] is partially grounded in Fb.

The positionalist is unable to account for single two-way instantiations. Since R has two argument places and holds symmetrically of a and b, there will be two completions corresponding to the two assignments of a and b to the two argument places α and β. Accordingly, it treats all symmetric instantiations as two one-way instantiations.[35]

This problem is particularly clear when there is a change from a single-two way instantiation, e.g. from a $\geq$ b and b $\geq$ a, to an asymmetric instantiation of R, i.e. to a $\geq$ b yet $\neg$(b $\geq$ a). Positionalism is unable to make sense of the way in which a $\geq$ b differs across the two cases. It construes both situations as involving an assignment of a to α and b to β. The only difference is that a is in

[33]Accordingly, it is not only possible for there to be different instantiations of the same 'fact' insofar as one and the same property can be instantiated multiple times over by the same object on the basis of different grounds, but also for different 'facts' to share one and the same instantiation.

[34]Both sameness of constituents and sameness of grounds is required for identity of facts. On the one hand, [(F $\vee$ G)x] in virtue of [Fx] is distinct from [(F $\vee$ G)x] in virtue of [Gx], even though they involve the same constituents, due to obtaining in virtue of different grounds. On the other, [(F $\vee$ G)x] and [(F $\vee$ H)x] are distinct due to involving different constituents, even though they can obtain in virtue of the very same ground [Fx].

[35]It has been suggested that the positionalist can address the problem of symmetric relations by claiming that such relations do not involve two argument places but instead only one argument place that is filled by both relata. (Cf. MacBride: 2007, pp. 41-43 and Fine: 2007, p. 59 for critiques of this response.) The problem of symmetric instantiations that we have identified clearly shows that this approach cannot succeed. A non-symmetric binary relation has to have two distinct argument places, yet can be symmetrically instantiated in such a way as to allow for only one two-way rather than two one-way completions.

addition assigned to β and b to α in the former situation, whilst this is no longer the case in the latter situation. Yet, intuitively these two situations do not share anything in common. In the former case there is a single ground for the two-way instantiation, whereas there is a different ground for the asymmetric one-way instantiation in the latter case. Since there is no ground that persists across the two cases, the way in which a $\geq$ b is instantiated in these cases is importantly different.

Whereas positionalism mistakenly classifies states of affairs as being distinct when they are identical and thus leads to an excessive multiplication of completions, antipositionalism faces the opposite problem. It treats all symmetric instantiations as single two-way instantiations and is unable to adequately account for the possibility of two one-way instantiations.

If aRb and bRa yet cRd though ¬(dRc), then one cannot consider the application of R to c and d to be co-mannered with that of R to a and b, since co-manneredness is an equivalence relation (cf. Fine: 2000, p. 24 fn 13). This, however, means that the antipositionalist is unable to account for cases involving two one-way instantiations. All symmetric instantiations will be treated as single two-way instantiations. Relations that can but need not hold in both ways are thus mistakenly treated in the same way as relations that are disjunctions of symmetric and asymmetric relations.

Moreover, a change from a situation in which aRb and ¬(bRa) to one in which both aRb and bRa will involve a change from aRb being co-mannered with cRd to them no longer being co-mannered. However, aRb is unchanged and should remain in the same equivalence class. None of the facts that are involved in grounding aRb have changed: it is still grounded in aR*b together with Fa. All that has changed is that b has changed from ¬F to F, thereby making it the case that the second partial ground of bRa also obtains. This, however, is not relevant to aRb but only to bRa. Put differently, in the case of relations that allow for two one-way instantiations, the way in which aRb holds should be independent of whether bRa holds as well.[36] This requirement cannot be satisfied by the antipositionalist.[37]

The grounding account has none of these difficulties. It can distinguish between cases in which there is one ground for a symmetric two-way instantiation and cases in which there are two different grounds for two one-way instantiations. The grounding approach neither gives rise to an objectionable multiplication nor to an objectionable collapse of completions. If the grounds of aRb and bRa are

[36]Two one-way instantiations differ crucially from a single two-way instantiation, where aRb has to change when there is a change from ¬(bRa) to bRa. E.g. in the case of the weak betterness relation, there is a change from a $>$ b when ¬(bRa) to a $=$ b when bRa.

[37]Similarly, a case in which aRb, bRa, aRc yet ¬(cRa) will be treated by the anti-positionalist in such a way that aRb and aRc are not co-mannered. This, however, means for instance that the way in which a is a brother of b (who is also a brother of a) will be fundamentally different than the way in which a is a brother of c (who is not a brother but a sister of a).

the same, then one is dealing with a single two-way instantiation. In that case the relata play the same grounding role: {⟨a,F⟩,⟨b,F⟩}. Permuting the relata does not generate a distinct completion of this relation. There is only one grounding role that is played by both relata and hence only one assignment of relata to grounding roles. By contrast, if they have different grounds, then there are two one-way instantiations of R by a and b. In that case the relata play multiple grounding roles such that there are two distinct completions of the same relation: {⟨a,F⟩,⟨b,G⟩} and {⟨a,G⟩,⟨b,F⟩}.[38]

5.5 Cross-relation comparisons

A further advantage of the grounding account is that it provides a satisfactory account of cross-relation comparisons. It neither accepts such comparisons across the board, nor deems them to always be unintelligible, but instead accepts cross-relation comparisons in a restricted range of cases.

At one extreme, one can accept what MacBride: 2014 calls the third degree of relatedness and countenance comparisons across arbitrary relations. For any relation R there will be a fact of the matter whether it orders some objects a and b in the same way that a different relation R′ orders c and d, e.g. that the shorter than relation applies to a and b in the same way that the brighter than relation applies to c and d. This approach is based on the problematic idea that there is an absolute ordering of the way in which a relation applies to its relata, allowing for comparability of these absolute positions across arbitrary relations. This approach is implausible.

At the other extreme, one can reject all cross-relation comparisons and hold that one can only compare the way in which a relation R applies to a and b with the way in which that very same relation applies to c and d. Positionalism and antipositionalism are committed to this approach. They have to preclude the possibility of cross-relation comparisons in order to ensure that the notion of a converse in the strict sense is not well-defined. This, however, is also troubling.

Even though unrestricted comparability is implausible, there are a number of important cross-relation comparisons and implications that any satisfactory theory of relations will have to make room for. First, constructed disjunctive relations such as the weak betterness relation should be comparable to the relations out of which they are constructed. If a and b are ordered by the strict betterness relation in a given way, then they are also ordered in that way by the weak betterness relation. Second, relations that merely differ in terms of their adicity should be comparable. For instance, we have to make sense of the idea that objects are

[38]Symmetric relations are those that are always instantiated symmetrically. Since there are two ways in which a relation can be symmetrically instantiated, namely via a single two-way instantiation or via two one-way instantiations, there are important hyperintensional differences amongst symmetric relations that are not recognised when working with ordered n-tuples and the like.

related in the same way by the binary betterness relation as by the ternary betterness relation: a and b are ordered in the same way in the case of $>$a,b as in the case of $>$a,b,c.

In each case, one has to explain both 1. the way in which the instantiation of R implies the instantiation of R′, and 2. the way in which these two relations order certain objects in the same way. On the one hand, we need to account for the fact that $a > b$ implies $a \geq b$. Similarly, that the ternary betterness relation holds: $>$a,b,c implies that the binary relation holds: $>$a,b (as well as $>$b,c and $>$a,c). On the other hand, we need to make sense of the idea that if $a > b$, then a and b are ordered by $\geq$ in the same way as by $>$. Likewise, if $>$a,b,c, then a and b are ordered by the ternary betterness relation in the same way as by the binary betterness relation.

Both desiderata are satisfied by the grounding account. It explains implication across relations in terms of the notion of hyperintensional implication.[39] A (monadic or polyadic) property hyperintensionally implies another if for every ground Γ of the former there is a ground Δ of the latter such that Δ is either identical to Γ or is a partial ground of Γ. The strict betterness relation, accordingly, implies the weak betterness relation, since the possible grounds of the former form a subset of those of the latter: every ground of $>$ is identical to some ground of $\geq$. Similarly, the ternary betterness relation implies the binary betterness relation, since the grounds of the binary are amongst the grounds of the ternary: the grounds for $>$a,b,c form a sub-plurality of the grounds for $>$a,b.

The grounding account can also explain the way in which the different relations order objects in the same way in terms of the objects playing the same grounding role in each case. Even though the two relations $>$ and $\geq$ are distinct, it is nevertheless the case that a and b play the same role in grounding $a > b$ as in grounding $a \geq b$. The binary and ternary betterness relations are, likewise, distinct yet nevertheless allow for a and b to play the same grounding role: $>$a,b,c and $>$a,b do not differ in terms of how a and b are implicated but only differ in that the ternary ordering involves an additional object c, where the grounding role played by c does not have any effect on those played by a and b.

The grounding account can, accordingly, make sense of both cross-relation comparisons and cross-relation implications amongst relations that involve the same grounding roles (whereby the grounds of the one are either amongst the grounds of the other or are partial grounds thereof), without invoking the problematic idea of an absolute ordering of the relata that would give rise to comparability across completely unrelated relations.

[39] This notion is characterised in detail in "Hyperintensional equivalence" (Bader: manuscript).

6 Generative operations

Derivative non-symmetric relations can be grounded in a fundamental level that does not contain any non-symmetric relations. The proposed account makes use of grounding both to generate derivative asymmetry and to specify the hyperintensional identity criteria for derivative relations. The problem now is that the notion of grounding is asymmetric. Even if one can account for asymmetry at the derivative level by developing a theory of derivative relations that does not countenance distinct converses or duals, the very relation that connects the fundamental to the derivative and by means of which one can explain how order arises at the derivative level, namely the grounding relation, is itself asymmetric. Since grounding is not part of the derivative but what gets us to the derivative, the asymmetry of grounding itself cannot be merely derivative. As a result, it would appear that failures of symmetry cannot be restricted to the derivative level.

More generally, there seem to be a number of other asymmetric relations besides grounding, such as causation and composition, that play an indispensable role in fundamental theorising that would similarly constitute counter-examples to the claim that there are no non-symmetric relations at the fundamental level. Whilst one might be able to adopt a Humean approach to, say, causation and consider it to be a derivative matter that can be accounted for in terms of the internal relations model, non-reductive approaches will have to consider these asymmetric relations to be fundamental.

This problem can be addressed by arguing that grounding, causation, composition and the like are not relations but are instead generative operations.[40] Whereas relations presuppose the existence of their relata, generative operations generate outputs from inputs and are thus prior to their outputs. Such operations do not merely reflect underlying differences but induce order.

The relata of a relation are in an important sense prior to the instantiation of the relation. In the same way that individual objects are prior to the instantiation of monadic properties, relata are likewise prior to the instantiation of polyadic properties. A property instantiation [Fa] results from combining an object a and a property F. Similarly, a relation instantiation [aRb] results from putting together the objects a and b (= the relata) and the relation R. We start out with the relata and then connect them by means of the relation to form a relational complex.

Generative operations, by contrast, only presuppose their inputs. The outputs are not presupposed by the operation. They are instead generated by applying the operation to the inputs. We do not start out with both the outputs and the inputs and then connect them by means of a relation. Instead of inducing order by

[40] This is not to be confused with the idea that a logic of ground should be formulated by means of a sentential operator. For arguments in favour of operationalism in the case of composition cf. Fine: 2010. Fine also uses the term 'generative operation', though in a somewhat different way and for different purposes.

connecting up pre-existing objects,[41] generative operations induce order by generating outputs from the independently given inputs. The input is presupposed and has to be independently given.[42] For example, in the case of grounding, we start out with what is fundamental. That is all that we have to begin with and hence the only thing that can enter as input into the operation. We then generate the derivative out of the fundamental. Since we only get the output from the input by applying the operation, it is not possible to start with the output. The input must come first, whereas the output that is generated comes second.

Although operations can have converses that invert the inputs and outputs, it is not the case that the converse of a generative operation is likewise generative. Once the output has been generated, one can ask from what it was generated. We can retrace our steps back to the input. This, however, does not amount to generating the input, i.e. the converse operation is not generative. For instance, if some parts compose a whole then the whole can be decomposed into the parts, yet if composition is generative then the whole owes its existence to the parts that compose it but not vice versa.[43] The parts are independently given. They are not generated by the application of the decomposition operation to the whole. The whole, by contrast, only exists because it has been generated via an application of the composition operation to the independently given parts. This ensures that the application of the converse of a generative operation is not independent but parasitic on the generative operation. We first need to generate the output in order to then be able to (non-generatively) go back to the input via the converse operation. This is what allows us to privilege one operation, namely the generative one, over its converse. The fact that we are dealing with operations rather than with relations is, accordingly, not sufficient for addressing the various problems concerning converses. What is crucial is that the operations in question are generative operations.

6.1 Causation as a generative operation

The difference between relations and operations can be illustrated by considering presentism. On a presentist approach, the causal unfolding of the world amounts to a series of successive replacements. Given that only the present exists, cause and effect cannot both co-exist. Instead, the one gets replaced by the other. There is no time when both of them are present, which implies that they cannot co-exist.

[41] The notion of a 'pre-existing object' does not have to be understood temporally but can also be understood in terms of metaphysical priority.

[42] What is independently given can either be understood in an absolute sense, i.e. independent of any application of the operation, as happens in the case of that which is fundamental, or in a relative sense independent, i.e. relative to a particular application of the operation.

[43] Which operation is generative is not settled by adopting an operational approach, i.e. operationalism is neutral between holism, which privileges decomposition, and atomism, which privileges composition.

Since relations (at least external relations) are existence-entailing, one cannot have a relation without its relata: aRb cannot obtain unless both a and b exist. If R is a trans-temporal relation, aRb cannot obtain unless one countenances the existence of objects that are located at different times. This contradicts the presentist's thesis that only the present exists. The problem is thus that if causation is a trans-temporal relation that necessitates its relata, then both relata have to exist. Yet at most one of them is present, which means that at most one of them exists, according to the presentist. Since it is never the case that both cause and effect exist, they cannot stand in a trans-temporal relation to each other.

Whilst the impossibility of trans-temporal causal relations has traditionally been considered to be a problem for presentism (cf. Bigelow: 1996, pp. 39-40), we can resolve this difficulty by adopting an operational approach. Causation, on this approach, is not a trans-temporal relation that connects things existing at different times, but a trans-temporal operation that has inputs and outputs that exist at different times. Unlike a causal relation that presupposes the existence of both relata, an operation takes inputs and gives rise to outputs. Accordingly, it is not necessary for the inputs and outputs to co-exist. There is no need for both the cause and effect to co-exist. Instead, the input can exist at time t and give rise to the output that exists at a subsequent time t'. Reality, on this approach, is considered to evolve dynamically, whereby one state of the world generates and is replaced by the next. This presentist approach illustrates that causation, construed as a generative operation, can be operative, even when it is not possible for there to be causal relations.[44]

6.2 Grounding as a generative operation

A related problem arises in the context of grounding. Here the problem is not that of connecting things across time, but across levels of the fundamentality hierarchy. The problem is that it is impossible to account for the idea that the derivative owes its existence to the fundamental in terms of a grounding relation. Since the existence of the derivative is presupposed in order for a grounding relation connecting the fundamental to the derivative to hold, one cannot make sense of the derivative owing its existence to the fundamental when operating with grounding relations. That y owes its existence to x cannot consist in x and y standing in a grounding relation. The instantiation of the relation by x and y would be dependent on the relata, in particular it would be dependent on the relatum y, given that the relation presupposes its relata. Yet, y's existence is meant to be explained in terms of and hence be posterior to the holding of this grounding

[44]This approach to causation naturally gives rise to a construal of persistence in terms of immanent causation. Rather than adopting the traditional 'only x and y' rule that treats persistence as an intrinsic trans-temporal relation connecting up relata existing at different times, one should adopt the 'only x' rule, which specifies the intrinsic conditions that x has to satisfy at t in order for it to persist until t'. Cf. "The fundamental and the brute" (Bader: manuscript, section 3.2).

relation.[45]

This problem can be addressed by adopting an operational approach. Instead of presupposing the output, the operation generates the output from the input. All that there is to begin with is the fundamental input, which then generates the derivative output. The application of the operation is, accordingly, prior to the output, allowing fundamental inputs to generate derivative outputs. We can thus make sense of the idea that the derivative owes its existence to the fundamental.

On this approach, there are no grounding relations and hence no grounding facts. Grounding is not a relation that connects different items in the world, but something that generates the derivative from the fundamental. The operation itself is neither fundamental nor derivative. It is not part of the fundamentality hierarchy but stands outside it. It is what gets us from the fundamental to the derivative. It does not occupy a position in the grounding order but instead gives rise to this order. It generates the hierarchy and induces its structure.

In the same way that the instantiation 'relation' cannot be instantiated (for that way lies Bradley's regress), so the grounding 'relation' cannot be grounded. The only things that are apt to be grounds or be grounded are instantiations of monadic as well as polyadic properties. That the fundamental is the input into an operation that has a certain derivative output (and correspondingly that the derivative was generated from a certain input) is, accordingly, not a fact in the world, but at best a fact about the world.[46] As such, it is to be located outside

[45]A further problem is that any connection between the fundamental and the derivative would seem to violate the purity constraint identified by Sider (cf. Sider: 2011, sections 7.2-3 & 8.2.1). If a fundamental fact stands in a grounding relation to a derivative fact, then the question arises whether this grounding fact is a fundamental or a derivative fact. For instance, it would seem that the fact [[Γ] grounds [Δ]] is either derivative or fundamental. Since fundamental facts only contain fundamental constituents (= purity constraint), it cannot be a fundamental fact. After all, it involves a derivative fact, namely [Δ], as one of the relata of the grounding relation, i.e. the relational fact cannot be more fundamental than its relata. It would thus have to be a derivative fact. Yet, in that case one has to find a ground of this fact. This, in turn, would constitute a further grounding fact that would likewise need to be grounded and so on.

Superinternalists attempt to ground all these grounding facts in the fundamental fact that one started out with: [Γ] not only grounds [Δ] but also [[Γ] grounds [Δ]], as well as [[Γ] grounds [[Γ] grounds [Δ]]] and so on (cf. Bennett: 2011; deRosset: 2013). This, however, conflicts with our account of hyperintensional equivalence (i.e. if [Δ] and [[Γ] grounds [Δ]] have the very same grounds, namely [Γ], then they are identical). Moreover, it does not explain how the derivative derives from the fundamental. If the grounding fact is a derivative fact, then we have not yet been given a satisfactory account of how the grounding relation connects the fundamental to the derivative. Positing another derivative fact, namely a grounding fact, that itself is in need of explanation does not amount to an explanation of the derivative in terms of the fundamental. Grounding is meant to be the connection that gets us from the fundamental to the derivative. Grounding relations (on the superinternalist account), however, do not establish a bridge between the fundamental and the derivative but instead only generate additional derivative facts.

[46]This is analogous to the way in which the 'fact' that a certain collection of facts is the totality of facts is not itself a fact in the world but only a fact about the world. (Otherwise, the relevant collection of facts would not in fact be the totality of facts.)

the grounding hierarchy. The question of what grounds the grounding facts can, accordingly, be circumvented altogether.[47]

7 Conclusion

All fundamental relations are symmetric. Nevertheless, there is order and asymmetry in the world. On the one hand, there is order at the derivative level. Non-symmetric derivative relations can be grounded in asymmetric networks of symmetric relations as well as in asymmetric distributions of properties amongst objects. Such derivative relations are individuated in terms of their grounds, thereby precluding the existence of distinct converses as well as distinct duals. On the other hand, order arises in the world as a result of the application of various generative operations. The derivative level is generated from the fundamental level by means of grounding, later times are dynamically generated from earlier ones by means of causation, and complex objects are generated from simple ones by means of composition.[48]

[47] This does not mean that one cannot explain grounding connections. One can explain why Fx grounds Gx in terms of the possible basic grounds of F forming a subset of those of G. Alternatively, one can explain grounding connections in terms of laws that govern grounding operations. These explanations of grounding connections are not themselves grounding explanations. One explains why Γ grounds Δ, yet one does so without identifying a ground of the supposed grounding fact that [[Γ] grounds [Δ]].

[48] For helpful comments and discussions I would like to thank audiences at Edinburgh, Princeton, the Foundation of Reality conference at Oxford, the Objects and Properties conference at Cambridge, the Metaphysics and the Theory of Content conference at Tübingen, and Nick Jones, Christoph Michel, Katherine Hong, Gonzalo Rodriguez-Pereyra, Martin Pickup and Alex Roberts, as well as an anonymous referee.

References

[1] Ben-Yami, H. *Logic & Natural Language: On Plural Reference and its Semantic and Logical Significance*. Ashgate, 2004.

[2] Bennett, K. By our bootstraps. *Philosophical Perspectives 25* (2011), 27–41.

[3] Bigelow, J. Presentism and properties. *Philosophical Perspectives 10* (1996), 35–52.

[4] Bigelow, J., and Pargetter, R. *Science and Necessity*. Cambridge University Press, 1990.

[5] Broome, J. Goodness is reducible to betterness: the evil of death is the value of life. In *The Good and the Economical: Ethical Choices in Economics and Management*, P. Koslowski and Y. Shionoya, Eds. Springer, 1993, pp. 70–84.

[6] Carnap, R. *Der logische Aufbau der Welt*. Felix Meiner Verlag, 1928.

[7] Demopoulos, W., and Friedman, M. Bertrand Russell's *The Analysis of Matter*: Its Historical Context and Contemporary Interest. *Philosophy of Science 52*, 4 (1985), 621–639.

[8] deRosset, L. Grounding explanations. *Philosopher's Imprint 13*, 7 (2013), 1–26.

[9] Dipert, R. The mathematical structure of the world: the world as graph. *The Journal of Philosophy 94*, 7 (1997), 329–358.

[10] Dorr, C. Non-symmetric relations. *Oxford Studies in Metaphysics 1* (2004), 155–192.

[11] Fine, K. Neutral relations. *The Philosophical Review 109*, 1 (2000), 1–33.

[12] Fine, K. Response to Fraser MacBride. *Dialectica 61*, 1 (2007), 57–62.

[13] Fine, K. Towards a theory of part. *The Journal of Philosophy 107*, 11 (2010), 559–589.

[14] Lewis, D. Putnam's paradox. *Australasian Journal of Philosophy 62*, 3 (1984), 221–236.

[15] MacBride, F. Neutral relations revisited. *Dialectica 61*, 1 (2007), 25–56.

[16] MacBride, F. How involved do you want to be in a non-symmetric relationship? *Australasian Journal of Philosophy 92*, 1 (2014), 1–16.

[17] MacBride, F. Relations. *Stanford Encyclopedia of Philosophy* (2016).

[18] Mundy, B. The metaphysics of quantity. *Philosophical Studies 51*, 1 (1987), 29–54.

[19] Newman, M. H. A. Mr. Russell's 'Causal Theory of Perception'. *Mind 37*, 146 (1928), 137–148.

[20] Putnam, H. *Reason, Truth, and History*. Cambridge University Press, 1983.

[21] Russell, B. *The Principles of Mathematics*. George Allen & Unwin, 1903.

[22] Russell, B. *Theory of Knowledge – The 1913 Manuscript*. Routledge, 1913.

[23] Sider, T. *Writing the Book of the World*. Oxford University Press, 2011.

[24] van Inwagen, P. Names for relations. *Philosophical Perspectives 20* (2006), 453–477.

[25] Weatherson, B. David Lewis. *Stanford Encyclopedia of Philosophy* (2014).

[26] Williamson, T. Converse relations. *The Philosophical Review 94*, 2 (1985), 249–262.