

Kant and the table of nothing

Ralf M. Bader

Merton College, University of Oxford

(N.B. this is a very rough and incomplete draft)

I The table of nothing

At the very end of the *Transcendental Analytic* Kant argues that the two concepts something/possible and nothing/impossible, which constitute the starting point of traditional ontology, are not in fact the highest concepts. Instead, these two opposed concepts stand under a higher concept, namely the concept of an object in general, and result from that concept's division.¹ Both the concept of something and that of nothing can be divided in accordance with the table of categories, resulting in two tables: 1. the table of the division of the concept of something, and 2. the table of the division of the concept of nothing.

Kant only provides the table of nothing, noting that the corresponding division of something follows straightforwardly.

Nothing,	
as	
1.	
Empty concept without object, <i>ens rationis.</i>	
2.	3.
Empty object of a concept, <i>nihil privativum.</i>	Empty intuition without object, <i>ens imaginarium.</i>
4.	
Empty object without concept, <i>nihil negativum.</i>	

*Thanks to Ian Proops for helpful discussions.

¹Whereas the highest concept of theoretical philosophy is the concept of an object in general, which is divided into possible and impossible, in the practical case the highest concept is that of an act of Willkür in general which is divided into Recht and Unrecht (*fas* and *nefas*), cf. 6:218 fn.

The table of nothing is usually either ignored altogether or rejected as an ill-motivated artefact of Kant's architectonic obsession.² This neglect is to some extent understandable, since Kant himself notes that this discussion is not "of special importance in itself" and is only required "for the completeness of the system" (A290/B346). This paper will establish that the table is systematic and well-motivated, explaining the different types of nothing and showing how they form a progression. Moreover, it will show that engaging with this table can be highly illuminating in numerous ways, most notably by shedding light on the categories of quality and the limits of cognition, as well as on Kant's understanding of negation and of the law of continuity.

1.1 A rule-governed and necessary table

The fact that the table of nothing is based on the table of categories is crucial for understanding the division of the concept of nothing.

First, the four types of nothing correspond to the four headings of the table of categories, insofar as each type of nothing corresponds to the negation of the respective heading. It is because one is negating the whole heading, rather than the particular categories under each heading, that there are only four types of nothing and not twelve, namely 1. *ens rationis* = negation of quantity, 2. *nihil privativum* = negation of quality, 3. *ens imaginarium* = negation of relation, and 4. *nihil negativum* = negation of modality. The different types of nothing, accordingly, have to be interpreted in such a way that none of them are to be identified with any of the categories, but instead with the negations of the respective headings.

Second, since it is based on the table of categories, the table of nothing is "a rule-governed and necessary table" (4:325) that inherits the architectonic structure of the former. In particular, there is a progression amongst the headings from quantity to quality to relation that culminates in modality, whereby each heading builds on the preceding ones such that one only arrives at a complete determination when one has reached the last heading. The interpretation of the corresponding types of nothing has to ensure that they likewise form a progression with the *nihil negativum* constituting the complete determination of nothingness.

Third, given that something and nothing are complementary concepts, the four-fold division of the concept of nothing is mirrored by a corresponding four-fold division of the concept of something. The different types of something and nothing need to be construed in such a way that they can be integrated into the construction of the system of ontology, constituting the initial levels of this system. In particular, the complementary pairs of something/nothing have to be interpreted such that they result from the first four divisions, starting with the division of the concept of an object in general.

²Cf. "As usual Kant's attempt to obtain parallels for the four classes of category breaks down" (Kemp Smith: 1918, p. 424).

2 Empty concept without object: ens rationis

An ens rationis is a mere thought-entity. It does not involve a logical contradiction and, as such, is something thinkable. Yet, it is not something that is given in intuition but instead something that is made up, i.e. a mere product of thought (= Erdichtung). The concept is, accordingly, empty on the grounds that it lacks a corresponding object that is given in intuition. “Die logische Möglichkeit ohne reale ist der leere Begriff ohne Inhalt, d.i. Beziehung auf object” (R4801; also cf. R4396 and A220/B267). The fact that the concept is empty ensures that, even though something is thought, nothing is cognised. Instead, we are dealing with a mere play of representations (cf. A155/B194-195).

Unlike in the case of the nihil negativum, we do have a concept though one that is empty because it does not have an object. This means that the concept lacks objective reality and that we cannot establish its real possibility. There is logical possibility since the thought is possible, yet real possibility cannot be established since there is nothing given in intuition that could establish the possibility of the object (cf. 20:325 and A596/B624 fn). Yet, this need not amount to it being impossible. Instead, the ens rationis is to be classified as non-possible.

Wir können kein object als Möglich annehmen als das, welches wir in der Anschauung darstellen, folglich seine Wirklichkeit darlegen können; sonst, wenn sich die Vorstellung nicht widerspricht, ist wohl der Gedanke, aber nicht die Sache möglich. Die logische innere Möglichkeit nach dem Satze des Widerspruchs ohne reale der keine anschauung correspondirt: leerer Begriff. (R5722)

This is one of Kant’s innovations that involves a crucial departure from the traditional Leibniz-Wolffian approach that considers everything that is not logically contradictory to be possible.³

By drawing a distinction between logical and real possibility, Kant replaces the traditional dichotomy with a trichotomy, whereby something can be either 1. logically impossible, 2. non-possible, or 3. really possible (cf. R3723). What contradicts itself is logically impossible. Logical contradiction is the only ground of cognition of impossibility.⁴ By contrast, the only ground of cognition of real possibility is being given in some way in intuition. What is neither given in intuition nor contradictory, is neither really possible nor impossible but is instead non-possible.

The third alternative opens up room for the ens rationis, since there can now be empty concepts that are not defective. This means that there is a notion of

³Cf. Caimi: 2005 for a helpful discussion of Kant’s departure from the Leibniz-Wolffian tradition.

⁴Restricted notions of impossibility, such as that which consists in being incompatible with the forms of experience, can be established even when there is no logical contradiction. However, importantly there is no ground of cognition of unrestricted real impossibility.

emptiness for concepts and thoughts that does not consist in the concept being contradictory. It is this notion which underlies Kant's famous claim that thoughts without content are empty (cf. section 2.2).

2.1 Two types of non-possibility

The non-possible includes two different cases. First, there is that which is really (phenomenally) impossible. In the case of such concepts, one can establish their impossibility a priori. In particular, one can establish a restricted sense of real impossibility, namely phenomenal impossibility, on the grounds of incompatibility with the forms of experience (= forms of intuition plus forms of thought). Second, there is that the real possibility of which simply has not been established. This kind of concept is neither such that one can establish its phenomenal real impossibility a priori, nor has its real possibility been established a posteriori. Instead, it simply has not been encountered in experience. This means that entia rationis encompass both concepts that cannot have corresponding objects in intuition as well as concepts for which no corresponding objects have been found.

A number of interpreters have adopted a more restrictive reading, whereby only objects that cannot be given in intuition are classified as entia rationis. For instance, Caimi understands an ens rationis as "etwas, das nicht bloß nicht gegeben ist, sondern auch nicht gegeben sein kann in der Anschauung" (Caimi: 2005, p. 141). Similarly: "concepts for which corresponding objects cannot be given in intuition (*ens rationis*, nothing)" (Stang: 2016, p. 167). Also Paimann, p. 792: "diesem korrespondiert keine mögliche Anschauung", and van Kirk, p. 137: "an object (the content of a concept) for which no intuition is possible". For these interpreters all entia rationis represent real phenomenal impossibilities. This amounts to identifying this type of nothing with the (schematised) modal category of impossibility, which is concerned with real impossibility as it applies to phenomena.

This type of non-possibility plays a central role in Kant's theory due to its role in delimiting the domain of possible experience. Moreover, it involves a definitive characterisation of a concept as being non-possible, since it can be established a priori, rather than a potentially merely provisional characterisation that can be overturned by further empirical information. Nevertheless, the ens rationis is not to be identified with real (phenomenal) impossibility, but merely includes it. That is, an ens rationis is something thinkable that is not really possible without having to be really (phenomenally) impossible.

Kant makes this clear by providing two examples of the ens rationis that correspond to the two types of non-possibility (also cf. R5726, 29:811-812, 29:961-962).

1. Noumena are entia rationis that cannot be counted amongst the possibilities ("nicht unter die Möglichkeiten gezählt werden können"). There is

something that rules them out, which ensures that they are in principle non-possible. This is because they are phenomenally impossible, due to being incompatible with the forms of intuition.

2. New fundamental forces are also entia rationis which, by contrast, are not to be counted amongst the possibilities (“nicht unter die Möglichkeiten gezählt werden müssen”). Although there is nothing that rules them out, i.e. there is no incompatibility with the forms of experience, and hence nothing in principle that speaks against their real possibility, there is nothing that speaks in favour of their real possibility. This is why Kant merely says that they “lack examples in experience” (A291/B347) rather than saying that they cannot be given in experience. As a result, even though it is not the case that they cannot be counted amongst the possibilities, one nevertheless has to not count them amongst the possibilities.

Kant also discusses the case of new fundamental forces in the Postulates of Empirical Thought. Such concepts are not derived from experience but are instead invented concepts. They are entia rationis and are not to be considered as being really possible unless their possibility can be established empirically.

Dergleichen gedichtete Begriffe können den Charakter ihrer Möglichkeit nicht so, wie die Kategorien, a priori als Bedingungen, von denen alle Erfahrung abhängt, sondern nur a posteriori als solche, die durch die Erfahrung selbst gegeben werden, bekommen, und ihre Möglichkeit muß entweder a posteriori und empirisch, oder sie kann gar nicht erkannt werden. (B269-270)

This line of reasoning finds further support in the Bxxvi footnote, where Kant argues that in order to cognise an object one needs to prove its objective reality. This can be done in two ways, namely (i) a priori by means of theoretical reason, insofar as it can be shown to be a necessary condition for the possibility of experience,⁵ as well as by means of practical reason, or (ii) a posteriori by being actually given in experience. Accordingly, a concept has objective reality (relative to a cognitive subject) if the real possibility thereof has been established (by that subject). If real possibility has not (yet) been proven, then one has not (yet) established the objective validity of the relevant concept and the concept is accordingly (at least for the time being) empty. That is, the contrast between cognition and empty thought is aligned with the contrast between real possibility having been proven and it not having been proven, where the latter encompasses both the case where it cannot in principle be established and where it simply has not been established as a matter of fact.

⁵Though cf. the General Note on the System of the Principles which argues for dependence on being exhibited in experience even in the case of the categories.

2.2 Thoughts without content are empty

We can now better understand Kant's famous dictum that "thoughts without content are empty" (A51/B75). In particular, it is to be construed as the claim that thoughts about *entia rationis* are empty and do not result in cognition.

1. That there are two types of non-possibility implies that there can be empty thoughts even when dealing with things that are not supersensible. Accordingly, it is a mistake to consider Kant's dictum as being only concerned with claims, such as those of traditional metaphysics, that transcend possible experience. Friedman's reading turns out to be too restrictive: "pure concepts or categories have objective meaning only when applied to the spatiotemporal world of sense. (This, for Kant, is the meaning of the slogan 'thoughts without content are empty.')" (Friedman, p. 438) First, the dictum is about thoughts in general and not just about the categories. Second, thoughts about new fundamental forces, for instance, are equally empty, even though they concern the spatiotemporal phenomenal world.

2. In the case of the *ens rationis*, unlike the *nihil negativum*, we have something thinkable even though what is thought lacks objective reality. Thoughts about *entia rationis* are genuine thoughts contra McDowell: "for a thought to be empty would be for there to be nothing that one thinks when one thinks it; that is, for it to lack what I am calling 'representational content'. That would be for it not really to be a thought at all" (McDowell, pp. 3-4).

3. Intuition is required for establishing objective reality, yet it is not the case that every (non-empty) thought needs to involve an intuition (contra e.g. Allison's reading of the dictum). For a thought not to be empty it needs to involve concepts that have corresponding objects in intuition. Yet, it is not necessary that the objects are in fact given in intuition. In order to ensure non-emptiness, intuition is required to provide content that is objectively real, rather than an intuition being necessary to provide an immediate singular relation to some object. This means that content is provided but not constituted by intuitions. Put differently, intuitions provide data rather than themselves having to be the relevant contents. This is precisely, why Kant says that thoughts without content, rather than thoughts without intuitions are empty. (It is surprising how many interpreters misparaphrase (or even misquote) the passage as either stating 'concepts with intuitions are empty' or 'thoughts without intuitions are empty'.) The content (*Inhalt*) consists in the objectively real data provided by intuition/sensibility. Similarly, this explains the qualification at A50/B74 regarding concepts 'without intuition corresponding to them in some way'.

2.3 The limits of cognition

Entia rationis are empty concepts because they lack the requisite connection to intuition. As such they can figure in empty thoughts that, though not contra-

dictory, nevertheless lack content, i.e. lack objective reality. This means that nothing is cognised when making claims about entia rationis. By demarcating the limits of cognition the ens rationis helps to explain the location of the table of nothing at the very end of the Transcendental Analytic, insofar as it plays a role in introducing the transition from the Analytic to the Dialectic.

Corresponding to the two types of non-possibility, there are two ways in which cognition is limited:

1. Claims about those entia rationis that are to be classified as real phenomenal impossibilities go beyond possible experience, i.e. beyond what can be given in intuition. Making such assertion involves overstepping the boundaries of cognition.
2. Claims about entia rationis that are merely classified as non-possible due to not having been encountered in experience have no basis, i.e. they are unwarranted assertions for which one lacks a ground of cognition. Since one cannot cognise the real phenomenal impossibility of such entia rationis, one is merely making a claim that is not supported by experience rather than one that transcends possible experience.

Real phenomenal impossibility is the crucial notion for drawing the boundaries of experience, allowing us to demarcate the domain of possible experience from what transcends possible experience. In this way, it identifies the limits that are not to be transgressed, whereby the Dialectic is concerned precisely with the transgression of these limits that results from reason overstepping its bounds and making assertions that are not restricted to objects of a possible experience.

The limiting role of the ens rationis not only has a negative aspect, insofar as it restricts cognition, but also a positive aspect, by making room for practical grounds of cognition. As Kant famously said: he had to deny knowledge to make room for belief/Glaube (cf. Bxxx). The difference between being empty in the sense of the ens rationis and empty in the sense of the nihil negativum is crucial for making room for freedom, immortality and God, allowing practical grounds of cognition (as well as regulative ideas of theoretical reason) to fill the gap.

2.4 Negative cognition

Entia rationis are empty concepts that lack objective reality. As a result, nothing is cognised in the case of judgements involving such concepts. This, however, appears to conflict with the fact that those entia rationis that are real phenomenal impossibilities are cognised to fall outside the boundaries of possible experience. On the one hand, it seems that something is cognised, since we are able to establish that something does not belong to the domain of possible experience. Yet, on the other hand, there is the concern that one cannot cognise whatever falls outside these boundaries. The problem we are facing amounts to the puzzle whether the

limits of cognition can be cognised. In particular, can we cognise that something is beyond the boundaries of cognition and thereby classifies as an *ens rationis*? In short, can we cognise that something cannot be cognised? This question seems to pose a dilemma: if it is answered ‘yes’, then we can have cognition involving *entia rationis*; if ‘no’, then the category of impossibility does not result in cognition.

Resolving this puzzle is crucial for understanding the category of impossibility. In general, the categories are used to synthesise a manifold of intuition in order to result in cognition of objects. This characterisation of the role of the categories, however, is not applicable to the modal category of impossibility. Problems arise because there is no object that can be cognised when one is dealing with real phenomenal impossibilities. One does not have a manifold that is synthesised to result in a cognition of an object. In short, there is nothing there to be cognised. Since there is no object that can be cognised by means of the category of impossibility, this category, accordingly, cannot function in the way in which categories are usually meant to operate. It neither allows us to cognise an object, nor does it constitute a condition on the possibility of experience.

Instead of synthesising a manifold to yield cognition of an object, the category of impossibility allows us to identify ways in which a manifold cannot be synthesised. Even though no cognition of an object results, one is still operating on a manifold, insofar as one is identifying ways in which the manifold cannot be combined, i.e. what kinds of syntheses are not possible (or at least do not result in cognition of an object). This category can, accordingly, be used to represent the limits of synthesis (which, importantly, include the limits of a priori construction). It thereby allows us to draw boundaries that are not to be transgressed. These boundaries can be identified without violating the epistemic strictures of transcendental idealism, since they are identified by reflecting on our cognitive faculties, in particular by identifying what fails to be compatible with the forms of experience.

Although this category does not result in cognition of an object (and hence does not contribute towards reducing ignorance), it nevertheless plays an important role in avoiding error as a result of delimiting the domain of possible experience. Excluding something from the domain of possible experience is a form of negative cognition, whereby we cognise what a thing is not through negative marks (cf. Blomberg *Logik*, §117; Vienna *Logik*, 24:836-837; Jäsche *Logik*, 9:59). Accordingly, we need to understand the claim that nothing is cognised in the case of *entia rationis* as stating that nothing is positively cognised, i.e. nothing (corresponding to the concept) is to be placed inside the domain of possible experience. This is compatible with something being negatively cognised insofar as something can be excluded from this domain, which happens in the case of those *entia rationis* that are negatively cognised to be really (phenomenally) impossible. For instance, in the case of noumena, which constitute Kant’s example of the first kind of *ens rationis*, we can negatively cognise that they are not spatio-temporal

and hence fall outside the domain of possible experience, but cannot positively cognise anything about them.

This is, in fact, a general issue that applies to the correlates of all three modal categories, i.e. to impossibility, non-existence and contingency. None of them have a positive function. Instead, they are parasitic on the function of the categories of which they are the correlates. Each time, there is no positive condition that is satisfied, i.e. no distinctive kind of synthesis, but instead it is merely the non-satisfaction of the conditions corresponding to the positive modal categories. They do not embody distinctive forms of synthesis of their own, but are applicable when there is a failure of synthesis with respect to the positive modal category. All these negatively characterised categories only lead to negative cognition and do not constitute conditions of the possibility of experience. It is for this reason that no schemata are provided for the modal correlates.

3 Empty object of a concept: nihil privativum

The nihil privativum is the most difficult (and most important) type of nothing. The reasons for its difficulty are threefold.

1. Kant uses the term ‘nihil privativum’ in a number of different ways. For instance, some of the discussions of the ‘nihil privativum’ in the *Metaphysics Lectures* involve a notion that is radically distinct from the one at issue in the table of nothing. Accordingly, it becomes difficult to identify which texts are relevant and which ones are misleading and to be set aside.

2. There is a great deal of unclarity about the relation between this type of nothing and the categories of quality, which is (at least partly) due to the fact that Kant also uses the terms ‘reality’, ‘negation’ and ‘limitation’ in different senses. As a result of philosophical and terminological confusions, various interpreters have identified the nihil privativum with the category of negation and sometimes also with the category of limitation. “According to quality, [nothing] is the ‘absence of an object, such as shadow, cold’: this is the category of negation” (Longuenesse: p. 303). Similarly, Paimann holds that the nihil privativum “entspricht der Negationskategorie der zweiten Kategoriengruppe und bildet so einen Gegensatz zur ersten Kategorie der Qualitätskategorien, der Realität als Etwas” (p. 793). Aschenbrenner also identifies the nihil privativum with the category of negation (cf. p. 302).

3. As we will see shortly, the crucial resources for a satisfactory interpretation of the nihil privativum are only implicitly present in the *Critique* and were not explicitly worked out by Kant except for a series of *Reflexionen* from 1783-1788.

3.1 Conflicting usages

1. Traditionally the nihil privativum is contrasted with the nihil negativum, in terms of the latter being contradictory and hence impossible, whereas the former is non-contradictory and hence (logically) possible, yet not actual. For instance, in Adelung's Grammatisch-kritisches Wörterbuch der Hochdeutschen Mundart they are distinguished as follows:

1. Im schärfsten, engsten philosophischen Verstande, wo nur dasjenige nichts ist, was nicht nur nicht vorhanden ist, sondern auch nicht vorhanden seyn kann, nicht möglich ist; Nihilum negativum. . . . 2. In weiterer und gewöhnlicherer Bedeutung ist nichts nur dasjenige, was nicht vorhanden ist, nicht existiret, aber doch existiren oder wirklich werden kann, folglich möglich ist; Nihilum privativum. (pp. 486-487)

This usage can be found in the Metaphysik K₃.

a *nihil privativum*, although it can be thought, is one to which nothing existent corresponds; e.g. the aether in physics is an invented concept that has no reality *intuitu objecti*, but which can be thought without contradiction. Likewise a positive cold, as absolute cause of cold, contains nothing contradictory in thinking it, but it has no existence. All fictions belong here, since they are not to be found in nature *quoad objectum*, e.g. total contentment, – against its existence speaks the constant inclination of human beings to elevate themselves above their state and transplant themselves into another that they hold for better. Thus the *nihil negativum et privativum* differ among themselves as thought and object: one or the other does not correspond to the representation. (29:961)

2. The term 'nihil privativum' is also frequently used by Kant to refer to privatio, which refers to the cancelling of the consequences of two opposed grounds. In cases of real opposition, two opposed grounds cancel each other's consequence: $a - a = 0$, where this resulting lack is sometimes described as nihil privativum.

The consequence of the opposition is also nothing, but nothing in another sense to that in which it occurs in a contradiction (nihil privativum, repraesentabile). We shall, in future, call this nothing: zero = 0. Its meaning is the same as that of negation (negatio), lack, absence – notions which are in general use among philosophers – albeit with a more precise determination which will be specified later on. (2:172)

Relatedly, the nihil negativum and the nihil privativum are sometimes contrasted in terms of the former resulting from logical opposition, whereas the latter results from real opposition.

logical *opposita, simul sumta dant nihil negativum*, i.e. the impossible; *realiter opposita simul sumta dant nihil privativum*, i.e. lack, and it can be all

means be thought; if 2 opposing grounds are in the same subject, the result is zero, therefore if there are 2 logically opposing grounds, the result is the impossible (29:807).

Two real *opposita* do not cancel themselves, instead the consequences cancel themselves, and what arises through their combination is zero, null, *nihil privativum* (29:810).

Since the cancellation of consequences amongst grounds is represented in terms of the category of limitation in the Critical period, i.e. limitation amounts to the combination of reality and negation, insofar as a reality = a and a negation = -a oppose each other when combined to yield $a - a = 0$, these texts suggest that the nihil privativum is to be identified with limitation.

3. When introducing the qualitative kind of nothing Kant says: “reality is *something*, negation is *nothing*” (A291/B347). Some have interpreted this as saying that the category of reality corresponds to something and that the category of negation corresponds to nothing (to the nihil privativum). Similarly, in the Schematism chapter reality is understood as filled time whereas negation is understood as empty time (cf. A143/B182). Moreover, in the Prolegomena Kant refers to the table of nothing and notes that nothing corresponds to ‘full negation’: “in those for quantity and quality there is merely a progression from Unity to Totality, or from something to nothing (for this purpose the categories of quality must stand thus: Reality, Limitation, full Negation)” (4:325 footnote).

3.2 A fourth nihil privativum

NIHIL PRIVATIVUM ≠ POSSIBLE BUT NOT ACTUAL

We can straightforwardly dispense with usage 1, according to which the nihil privativum is something that is (logically) possible but not actual. This cannot be the correct interpretation since this role is already fulfilled by another type of nothing, namely the ens rationis.

NIHIL PRIVATIVUM ≠ CATEGORY OF NEGATION/LIMITATION

Identifying the nihil privativum with either the category of negation or that of limitation (in accordance with usage 2 or 3) is not acceptable. There are both general considerations that speak against identifying it with either of these categories, as well as specific considerations that only apply to negation or limitation.

First, since the nihil privativum results from negating the heading of quality it cannot be identified with any of the categories of quality. Such an identification would conflict with the requirement to construe the nothing as negating the heading, which rules out identifying it with a category falling under the heading.

Second, the examples that Kant gives of the nihil privativum are cold and shadow/darkness. These examples do not cohere with an interpretation that iden-

tifies the nihil privativum with either the category of negation or the category of limitation. As we will see in section 3.5, there are two problems: 1. neither interpretation can account for the fact that both shadow and darkness are given as examples. Although there is some prima facie plausibility in treating darkness as corresponding to negation and shadow to limitation, such an interpretation would not be able explain how it is possible for both shadow and darkness to be examples of the nihil privativum, and 2. Kant's substantive understanding of light as well as warmth and their corresponding opposites shadow/darkness and cold are incompatible with an identification with the categories of negation and limitation.

Third, at A292/B349 Kant notes that the nihil privativum classifies as empty data for concepts and illustrates this by pointing out that one would not be able to represent darkness if light had not been given to the senses. "Realitäten enthalten die *Data* und so zu sagen die Materie oder den transcendentalen Inhalt zu der Möglichkeit und durchgängigen Bestimmung aller Dinge" (B603; also cf. B609 and 2:87). The nihil privativum is not given in sensation. Instead, one has to form the concept by negating something that is given in this way. Accordingly, it presupposes a corresponding reality that has to be given first before the concept can be formed. There is thus an important asymmetry between reality and the nihil privativum. Yet, such an asymmetry is neither to be found in the case of the category of negation nor that of limitation.

NIHIL PRIVATIVUM ≠ CATEGORY OF NEGATION

First, the division into something and nothing is a dichotomy, yet the categories under a heading form a trichotomous division. This is problematic for two reasons. On the one hand, if one were to identify the category of reality with something and the category of negation with nothing, then reality and negation would form an exhaustive division. There would, accordingly, not be any room left for the category of limitation which is not to be identified with either. On the other hand, the third element in a trichotomous division results in a non-derivative way from the combination of the first two elements. As Kant notes in the letter to Schütz, the third category "enthält sie immer noch mehr, als die erste und zweyte für sich und zusammen genommen, nämlich die *Ableitung* der zweyten aus der ersten" (10:367). This means that limitation is to be understood as the combination of reality and negation. The combination of something and nothing, however, does not result in limitation but in a logical contradiction.

Second, the categories of quality are constitutive of objects, insofar as they are employed in synthesising a manifold of intuition to yield cognition of an object. However, negations in the sense of the nihil privativum cannot be constitutive of things but only of concepts. As Kant notes in R4301 "*Realitas* ist *materia cognitionis* (sind *constitutiva* der Dinge, negationen nur der Begriffe), sie mag affirmativ oder negativ gebraucht werden." Similarly, where (transcendental) negation

alone is thought “die Aufhebung alles Dinges vorgestellt wird” A574/B602-603. A nihil privativum is an empty object of a concept. This means that we only have a concept but that there is no object. Being an object requires reality (Sachheit), yet the nihil privativum is only constituted by negation. Whilst negations can be constitutive of concepts, i.e. we can have a concept of darkness or cold, there is nothing in reality that can correspond to such concepts. There is nothing of which these negations are constitutive, i.e. nothing the reality of which they constitute. In this sense, there are only negative concepts but not negative objects. Accordingly, in the case of the nihil privativum there is no object to be cognised but only a concept of an absence, which implies that the category of negation cannot correspond to the nihil privativum.

Moreover, there is no sensation in the case of the nihil privativum. This absence of sensory manifold ensures that there is no manifold for synthesis to which the categories can be applied. It is for this reason that absolute cold and darkness are not objects of experience and do not form part of experience. Yet the categories are involved in making experience possible. If the category of negation were to correspond to the nihil privativum, then this category would not be involved in the synthesis of a manifold resulting in the determination of an object and would not be a necessary condition of the possibility of experience.

Here, it is important to distinguish between a merely apparent absence and a genuine lack of a quality. The former is only a relative absence, which amounts to nothing but a low magnitude of the quality in question, whereas the latter is an absolute absence whereby the quality is not had at all. In the *Physical Geography*, for instance, Kant is concerned with the relative notions: “Warmth is thus something positive like light, and cold as well as darkness are simply names for their apparent absence” (9:251). A relative absence can be experienced insofar as it simply amounts to one thing being colder than the relevant object of comparison. Absolute cold (as well as darkness), by contrast, is not something that can be given in sensation and thus cannot be found anywhere in nature. “Nirgend fehlt sie [= die Wärme] ganz” (9:251 also cf. 1:378 “light . . . is the propagated pressure of the ether which is dispersed everywhere”). As a result, one can only form this concept via (qualitative) negation of warmth. The nihil privativum concerns absolute cold/darkness. Otherwise the concern raised by Kemp Smith that “[c]old is as real as the opposite with which it is contrasted” (Kemp Smith: 1918, p. 424) would be warranted.

Third, the category of negation should be understood as opposed reality, i.e. as $-a$.⁶ This is nicely illustrated by the *Metaphysical Foundations of Natural Science*, where reality and negation correspond to opposed forces. The former corresponds to repulsive force and the latter to attractive force (cf. 4:523). The reason for this is that the categories are concerned with the real rather than merely

⁶Cf. “Zwischen einer Realitaet die plus genant werden kan, und ihrer negation, minus ist, indeßen noch immer ein Mittelding, nemlich zero” (28:421).

logical use of the understanding. If the category of negation were construed in terms of ‘not a’ then reality and negation would be logically opposed. By contrast, by construing this category in terms of ‘-a’ we can ensure that it is really opposed to reality = a. Cf. R5823 “Nur realitet in der Erscheinung kan einer realität entgegengesetzt seyn und Negation zu einer realität zusammenstimmen.”

Moreover, the categories of quality are concerned with what corresponds to sensation, in particular with the ground of sensation. Whereas an opposed reality -a can be a ground, there is nothing that corresponds to a lack of sensation. As we have seen, the nihil privativum is not to be encountered in sensation and there is hence no corresponding ground of sensation which could be cognised by means of the categories of quality. Whilst sensations have grounds and result from the subject being affected, the absence of, say, light or warmth cannot be a ground and cannot cause anything.

Given that the category of negation is concerned with opposed realities, we can see that it differs fundamentally from the nihil privativum. A negative magnitude is not a nothing but instead something real. In fact, it is something that is in itself positive and that classifies as being negative only in relation to an opposed reality. As Kant makes clear in his essay: “negative magnitudes are not negations of magnitudes, as the similarity of expressions has suggested, but something truly positive in itself, albeit something opposed to the positive magnitude” (2:169; also cf. 2:172).

NIHIL PRIVATIVUM ≠ CATEGORY OF LIMITATION

First, the category of limitation is not limited to the case in which consequences of opposed grounds are cancelled. That is, $a - a = 0$ is a special case where the opposed grounds are of the same magnitude such that their consequences exactly cancel each other. If they are of different magnitudes, then they only partially cancel each other and the result will not = 0. This is clear in A265/B320-321 where real opposition is characterised such that “eines *die Folge des andern ganz oder zum Theil vernichten*”. Accordingly, “Sieht man bey der Realitaet allein auf die Quantitaet, so giebt die Limitation allein die Grade derselben an” (28:502). R5821 “Die limitation hat grade bis zum 0, also auch die realität.”

Second, $a - a = 0$ corresponds to $= 0$ due to the lack of a ground, which Kant sometimes represents as $a \times 0 = 0$, i.e. defectus rather than privatio which consists in cancelled grounds.

A negation, in so far as it is the consequence of a real opposition, will be designated a *deprivation (privatio)*. But any negation, in so far as it does not arise from this type of repugnancy, will be called a *lack (defectus, absentia)*. The latter does not require a positive ground, but merely the lack of such a ground. But the former involves a true ground of the positing and another ground which is opposed to it and which is of the same magnitude. (2:177-178)

This is usually illustrated in terms of the contrast between rest due to opposing moving forces that cancel the consequences of one another, and rest due to the absence of moving forces. “Das *nihil* der Beraubung $a - a = 0$ oder des *defectus* $a \times 0 = 0$ ” (11th Convolut, 22:433). These two ways of ending up with $= 0$ are in a similar sense nothing,⁷ yet Kant uses the term ‘nihil privativum’ only to describe privatio since only in that case is there a deprivation of reality but not defectus where there is a mere lack of reality.

3.3 Quantitative v. qualitative zero

Accordingly, we need an altogether different interpretation of the nihil privativum. In some Reflexion Kant draws a helpful distinction between quantitative and qualitative negation.

Die negation ist es entweder der quantitaet oder qualitaet nach. Im ersten Falle ist sie als ein verschwindendes *quantum* anzusehen und nichts als bloß Limitation und der realtaet nicht contradictorisch entgegengestzt, so daß eines von beyden statt fände, sondern *disparatum* oder *defectum*; im zweyten falle ist es *negatio oppositionis* (R5816; also cf. R5815).

By quantitatively negating a quality, such as motion, one ends up with a situation in which the quality is had to degree zero. Rest is the quantitative negation of motion. It is a state of motion that involves a quantitative lack. By contrast, qualitatively negating a quality amounts to denying the applicability of the quality to the object in question.⁸

Qualitative negation corresponds to transcendental negation, which Kant illustrates with various examples including darkness. This explains why the quality that is being negated needs to be presupposed, i.e. why data need to be given first that can then be negated.

Eine transcendente Verneinung bedeutet dagegen das Nichtsein an sich selbst, dem die transcendente Bejahung entgegengesetzt wird, welche ein Etwas ist, dessen Begriff an sich selbst schon ein Sein ausdrückt und daher Realität (Sachheit) genannt wird, weil durch sie allein, und so weit sie

⁷Though in the Negative Magnitudes essay Kant suggests that there can be important differences between them.

⁸Relatedly, there is a distinction between quantitative and qualitative opposition. In the former case, there is a third alternative (there will always be the alternative of not having the quality and sometimes also the alternative of having another quantity of the same quality), whereas in the latter case there is no third alternative, i.e. either the quality applies or it does not. It is for this reason that in the mathematical antinomies both thesis and antithesis can be false. They involve a quantitative opposition that allows for a third alternative (i.e. neither infinite nor finite but indefinite). Whereas the thesis and antithesis of the dynamical antinomies can both be true under different restricting conditions, i.e. they need to be attributed to different things (there is no freedom in the phenomenal world, i.e. the quality is denied of one domain, whereas there is freedom in the noumenal world, i.e. the quality is affirmed of another domain). Cf. R5817.

reicht, Gegenstände Etwas (Dinge) sind, die entgegenstehende Negation hingegen einen bloßen Mangel bedeutet und, wo diese allein gedacht wird, die Aufhebung alles Dinges vorgestellt wird. (B601-602)

Correspondingly, we need to distinguish two types of zero: the quantitative zero from the qualitative zero. In the case of a quantitative lack, the quality is had to degree zero, which can be either a case of defectus or privatio. (Sometimes the quantitative zero is also referred to as the diminishing zero.) By contrast, in the case of a qualitative lack, the quality is not had at all. Darkness/cold are qualitative lacks. This contrast comes out in Kant’s discussion of the different ways something can lack extension, namely by either being point-sized (i.e. an unextended object, namely a point), or by extension not being applicable to the object in question (e.g. the soul), cf. R5816, R5826 and R5828. Quantitative negation results in the former, i.e. being point-sized is the quantitative negation of extension, whereas qualitative negation yields the latter.

Rather than identifying the nihil privativum with one of the categories of quality, one needs to negate the quality altogether. Accordingly, it is distinct from the category of negation (which is to be understood as negative/opposed reality). It is also distinct from quantitative negation, whereby a quality is possessed to degree = 0 (i.e. neither privatio nor defectus). It is not that the object has the relevant quality to degree = 0, but that it does not have the quality at all. Contra Caimi: 2005, p. 141, who characterises it as “Grad Null der Realität”, the nihil privativum is not a zero degree of reality. Instead, it needs to be understood in terms of qualitative negation, i.e. as lack of the quality such that the property is not had to any degree, not even degree = 0 (cf. also 29:792). We are dealing with a complete lack of a quality, i.e. a qualitative zero.

We need to distinguish the nihil privativum understood in terms of the qualitative zero from the quantitative zero, which involves either privatio whereby the consequences of opposed grounds of equal magnitude exactly cancel each, i.e. $a - a = 0$, or defectus where the ground is not posited at all i.e. $a \times 0 = 0$. The nihil privativum amounts to a lack of a quality, rather than a lack of a consequence due to either cancellation of grounds or due to lack of a ground.

LACK OF QUALITY	QUALITY
	+a reality
qualitative zero (nihil privativum)	= 0 quantitative zero
	-a negation

$$\left\{ \begin{array}{l} a - a = 0 \text{ privatio} \\ a \times 0 = 0 \text{ defectus} \end{array} \right.$$

We can illustrate some of the implications of this account by briefly considering Warren’s treatment of the relation between reality and negation.

For Kant, reality can be thought of as shading off into negation. The latter is simply one end of a continuum. And since it is by means of mere

diminution or augmentation that we run through this continuum, we regard reality and negation as differing only quantitatively (i.e., only by degree), and as being otherwise homogeneous with one another. Negation simply corresponds to one of the values that the intensive magnitude can take on, namely, the value zero (Warren, p. 42; also cf. p. 179).

This account is problematic in a number of respects. First, the category of negation is to be understood in terms of opposed reality $-a$, and neither corresponds to negative properties (= nihil privativum) which are transcendental negations, such as cold (contra Warren p. 9), nor to the quantitative zero where a quality is had to degree zero. Second, whilst quantitative negation preserves the quality and results in a quantitative zero that is homogenous with reality, some qualities do not include a quantitative zero but instead fade out of existence, i.e. the limit is the qualitative zero which is not homogenous with reality. That is, in some cases the limit is contained within the quality, i.e. diminution results in a mere quantitative negation, whereas in other cases it is not contained therein, i.e. diminution results in a qualitative negation such that one ends up with something that is specifically different (cf. section 3.7). Third, the quantitative zero is not always to be understood in terms of being the end of a continuum. In particular, in cases where $= 0$ is the mid-point between opposed realities the very notion of an end of a continuum is inapplicable, as happens for instance in the case of pleasure $+a$, indifference $= 0$, and pain $-a$.⁹

3.4 Two senses of reality

We have seen that the contrast between reality = something v. negation/nihil privativum = nothing cannot be understood in such a way that the former is identified with the category of reality and the latter with the category of negation. Instead, reality is used in a broad sense that includes all categories of quality. Kant frequently uses 'reality' and 'quality' interchangeably. 'Reality' thus need not pick out the particular category of reality but can refer to the whole class of categories of quality.

There is good reason for this usage. As we have seen, negative magnitudes are likewise realities, namely opposed realities, and what results from their interaction, namely limitation, is also a reality. This means that all categories of qualities represent realities. Moreover, the category of reality enjoys a certain primacy since in the case of each heading the first category is primary. On the one hand, the negative is what is opposed to the positive real, i.e. we need to start with something real in order for there to be something that can be opposed to it and hence negative relative to it. On the other hand, one only ends up with limitation when there is something real being opposed by something else such that one then has

⁹Though one might think of this as being a case where there are two continua, namely the pleasure and the pain continuum, that share the same end-point.

a limitation of the real, which can take the form of a full or partial cancellation of its effects.

Likewise, the schematisation in terms of the contrast between filled v. empty time should be understood, not as schematising particular categories but rather the categories of quality as a class. That is, filling time consists in having quality to some degree or other, whereas time being empty amounts to lacking quality. The contrast between time as filled and time as empty is the schematisation of the contrast between reality in the broad sense and the nihil privativum.

This also helps to explain something that has puzzled a number of interpreters, namely the fact that Kant does not provide a schema for the category of limitation. If Kant were only to provide schemata for categories 1 and 2 but not 3, then this would indeed be puzzling. However, it is not puzzling if he identifies a schema for the heading of quality which is then contrasted with its negation. Moreover, it is difficult to make sense of what the schema of limitation would look like, since the contrast between empty and filled time is exhaustive, thereby not leaving room for a third option.

Correspondingly, there are two senses of negation that oppose the two senses of reality.

TRANSCENDENTAL NEGATION: Negation as what is opposed to quality, namely the nihil privativum.

CATEGORIAL NEGATION: Negation as what is opposed to the category of reality, namely the category of negation (= opposed reality).

Moreover, we can also distinguish two senses of limitation that correspond to combinations of the different notions of reality and negation.

TRANSCENDENTAL LIMITATION: A thing is limited insofar as it is both characterised by transcendental affirmations and transcendental negations.

Here, one is not combining realities with their corresponding negations. Instead, some realities are combined with some different negations. Otherwise, if one were to combine transcendental affirmations and their corresponding transcendental negations, one would end up with the nihil negativum, since they are logically opposed and thus cancel each other out. This means that this notion of limitation is not the application of the category of limitation, given that limitation arises from the combination of the first two categories, where the third category “da nicht anwendbar ist, wo die zwey erste gelten” (10:366). Since realities can be understood as perfections, the resulting limited being lacks some perfections and is hence an imperfect being. “Diese werden limitata genannt, weil gewisse Realitäten removirt sind, mangeln” (28:635). (Every thing other than the omnitudes realitatis is limited in this sense.)

CATEGORIAL LIMITATION: The category of limitation is concerned with the interaction, in particular the real opposition, of opposing grounds that correspond to the categories of reality and negation.

In this case, the realities and negations that are combined are of the same quality. For this reason, one ends up with real opposition, which involves the cancellation of the consequences of opposed grounds, rather than a logical contradiction, which amounts to a cancellation of the grounds themselves. It is for this reason that the categories of quality can form a synthetic unity (cf. B110) which involves “1) a condition, 2) a conditioned, 3) the concept which arises out of the combination of the conditioned with its condition” (5: 197).

Reality in the broad sense corresponds to quality = transcendental affirmation and is qualitatively opposed to the nihil privativum = transcendental negation (cf. 28:1148), whereby a limited or imperfect being results from the combination of realities and different negations (whereas a combination of realities and their corresponding negations leads to logical opposition and hence contradiction). Reality in the narrow sense, by contrast, corresponds to the category of reality. It is quantitatively opposed to the categories of negation and limitation, whereby a combination of a reality and its corresponding categorial negation yields a categorial limitation due to the real opposition of these opposing grounds.

3.5 Shadow and darkness

A potentially puzzling feature of Kant’s discussion of the nihil privativum is that he illustrates it at A291/B347 by means of shadow, yet at A292/B349 he switches to darkness. This is particularly puzzling since shadow sometimes seems to be construed as a form of limitation that arises from the interaction of light and darkness.

Limitation, i.e. the representation of a thing, whose being is affected by its non-being, thereby whose concept contains a being combined with non-being. E.g. light is reality; darkness is negation; shadow is limitation, since it is a darkness which is bounded by light. (29:998)

The fact that Kant uses both shadow and darkness poses a serious problem for interpretations of the nihil privativum that try to identify this type of nothing with either the category of negation or the category of limitation. Such interpretations can, at best, account for either the shadow example or the darkness example, but not for both examples.

Darkness is not to be understood in terms of the category of negation. If darkness were a negation and light a reality, then their combination should result in a limitation. However, since darkness is understood as not brightness (cf. 29:792), one does not end up with a limitation when light and darkness are taken together but with a contradiction. That is, light and darkness are logical rather than real opposites.

Nor should shadows be understood in terms of the category of limitation. The real opposition of opposed grounds of the same magnitude results in quantity = 0, where this quantitative zero is the mid-point between the opposed magnitudes. Shadow, however is not located between two opposing realities. There is no such thing as negative brightness. Darkness does not constitute an opposed reality that can be combined with brightness, to result in the cancellation of consequences. Since we do not have two opposing grounds, we do not end up with a real opposition leading to quantity = 0. Instead, there is only the lack of brightness.¹⁰

Darkness does not have a positive ground but is a mere absence of light. (Likewise for coldness.) It is a lack of light due to the absence (rather than the cancellation) of a ground of light. For Kant, light is the result of light-matter (“Lichtstoff”) and warmth that of fire-matter (“Wärmestoff”), such that both (absolute) darkness and (absolute) cold consist in the absence or lack of some matter.¹¹ This is why darkness and cold are instances of nihil privativum.

To make sense of shadows one needs to appeal to causal blocking/deprivation rather than real opposition. A shadow arises when a potential ground is being blocked rather than when a ground is present in the object but then opposed by another ground. At 2:185 Kant explains how one body can deprive another of warmth and makes clear that this involves a mechanism that is fundamentally distinct from real opposition and that coldness should, accordingly, not be called negative warmth. In the case of shadows, one object prevents something else from being illuminated and hence deprives that object of light. The resulting darkness is a shadow that is limited to that region that is blocked from illumination.

Shadow thus turns out to be a particular type of darkness that is produced in a certain way, namely via an obstacle blocking light. Since both shadow and darkness consist in the lack of light, they are both instances of the nihil privativum. The puzzle as to why Kant uses both of them as illustrations of this type of nothing can, accordingly, be resolved.

Light, darkness and shadow are thus not to be understood in terms of the categories of reality, negation and limitation, but in terms of transcendental realities, negations and limitations. All degrees of light are qualities whereas darkness and shadow are absences of light and hence classify as nihil privativum. In particular, shadows only classify as limitations in the limited sense of combining realities

¹⁰Moreover, real opposition requires that “the determinations which conflict with each other must exist in the same subject” (2:176). Yet, when it comes to shadows it is not the case that the (lack of) consequence is in the thing in which the opposing grounds meet and cancel one another. Strictly speaking, in the case of a shadow we do not even have an object, given that a shadow is a non-illuminated two-dimensional surface, where surfaces, like lines and points, are mere boundaries but not objects.

¹¹The two are related in Prop. VIII of the Meditations on Fire: “The matter of heat is nothing but the ether (the matter of light) compressed by a strong attractive (adhesive) force of bodies into their interstices” (1:377).

(= transcendental affirmations) and negations (= transcendental negations), since they are localised lacks of light.

In R5580 Kant gives a list of various triples consisting of realities, their opposing contraries, and the limitations resulting from their combination.

Einer jeden Realitaet a correspondirt ihr Gegentheil = o und ihr Widerspiel (*contrarium*) –a, d.i. ein Grund, der mit a verknüpft = o wird. Vergnügen, Schmerz, Gleichgültigkeit; Wahrheit, Irrthum, Unwissenheit. Aber Licht, finsternis und Beraubung des Lichts durch dunkle Körper, d.i. Schatten. Tugend, Laster, *adiaphoron*. Überhaupt: Ursach, Unwirk-samkeit (Trägheit) und Hindernis. Nutzen, Schaden, gleichgültig seyn.

Interestingly, the triple involving shadow is prefixed by ‘Aber’. This seems to have puzzled the editors of the Reflexionen, who inserted an editorial note in which they excluded the possibility that Kant’s handwriting should be read as starting this triple with ‘Oder’, which would simply indicate a further illustration rather than an exception: “Aber, *völlig sicher*, Oder *unmöglich*” (18:239). Our discussion shows that there is no need for puzzlement. Light, darkness and shadow do not constitute a triple consisting of reality, opposed reality and limitation, which is precisely why Kant sets it apart by means of an ‘Aber’. Darkness is not an opposed reality but rather a nihil privativum, and shadow is not a limitation resulting from the real opposition of realities but merely a limited absence of light, i.e. a localised nihil privativum.

3.6 Four case studies

Unfortunately, Kant only developed the resources necessary for distinguishing the different types of negations after the A-Edition. The most detailed discussions of qualitative v. quantitative negation and opposition can be found in Reflexionen R5815 to R5831, all of which are dated 1783-1788. He also never explicitly applies them to the case of zero – yet, it is clear that this distinction plays a crucial role in a number of Kant’s arguments, as we can see in the following case studies.

I. THE VANISHING OF THE REAL

In the Anticipations, Kant claims that there is a stepwise transformation (*stufenartige Veränderung*) from empirical consciousness to merely formal consciousness of the manifold in space and time, where “das Reale desselben ganz verschwindet” (B208) leaving us with “reine Anschauung = o”. This transition has been criticised on the grounds that “surely this *a priori* argument for the principle of degree proceeds only by committing the very error with which Kant was always taxing his predecessors, namely that of confusing a difference of *kind* with a difference of *degree*” (Guyer: 1987, p. 203).

This objection can be understood as stating that there is a difference in kind between the real, on the one hand, and pure intuition, on the other, yet that this

is incompatible with representing it as a difference between zero v. non-zero since this is merely a difference in degree. However, we can note in response that if the $= 0$ is understood as a qualitative zero, then this transformation amounts to going from matter/reality in form to nothing (= qualitative negation of reality) in form, such that one is left with mere form (= pure intuition). Since qualitative negation is at issue, we are dealing with a genuine difference in kind and not merely one in degree, i.e. what results is the absence of the quality and not merely a situation where the quality is possessed to degree zero.

This response though raises a further question, namely how can a difference in kind result from a stepwise transformation that would only seem to involve differences in degree. (This will be addressed in the next section.)

2. REFUTING MENDELSSOHN

A closely related issue arises in the context of Kant's refutation of Mendelssohn's proof of the permanence of the soul (cf. B413-415). Mendelssohn had argued that the soul could not go out of existence without violating the law of continuity, since the transition from existence to non-existence would involve a leap. Kant responds that the evanescence of the soul could take place by means of a continuous diminishing of the soul's intensive magnitude that would result in its non-existence. Since non-existence is not a state of being, the zero that is reached as a result of this diminution must be a qualitative zero.

3. THE BALANCING ARGUMENT

In the balancing argument, Kant attempts to establish that, if there is to be matter, there must be an attractive force that can counter-balance the repulsive force of matter. This is because matter would otherwise disperse infinitely. As a result, the density of matter would everywhere end up being $= 0$, such that the repulsive force exerted in any region of space would $= 0$. This, in turn, would amount to space not being filled, i.e. there not being any matter. "Folglich würden bei bloss repellirenden Kräften der Materie alle Räume leer, mithin eigentlich gar keine Materie dasein" (4:508). This means that the transition involved in the infinite dispersal of matter would amount to a switch from there being matter to there not being any matter at all. As a result, the zero has to be a qualitative zero for the argument to work.

4. REST AS A STATE OF MOTION

In the *Metaphysical Foundations of Natural Science*, Kant argues that "rest cannot be explicated as lack of motion, which, as $= 0$, can in no way be constructed, but must rather be explicated as perduring presence at the same place, since this concept can also be constructed, through the representation of a motion with infinitely small speed throughout a finite time, and can therefore be used for the ensuing application of mathematics to natural science" (4:486).

This passage might suggest that Kant is distinguishing between zero motion and a diminishingly small quantity of motion. However, what is really going on is that Kant is distinguishing between rest understood as the lack of motion where this amounts to a qualitative zero, and rest as perduring presence where this amounts to a quantitative zero, i.e. a situation in which the quality is had.¹²

The important thing for Kant is that rest is a state of motion. In particular, it is that state of motion in which the quantity of motion is zero. This plays a central role in Kant's argument in favour of the metaphysical-dynamical theory of matter over the mathematical-mechanical approach, which involves absolutely impenetrable bodies. When dealing with such bodies, motion is not defined at the turning point or the point of impact. This means that rest cannot be constructed on this approach and consequently classifies as a qualitative rather than quantitative lack of motion.

Such a lack of motion is problematic for two reasons. First, it leads to violations of continuity: "There is a *lex logica continui*, if something *general* is said of an object, which is in motion, then it also holds of the same body when it is at rest; (one can view it as diminishing – infinitely small – motion)" (28:662). For Kant, the law of continuity requires that rest belongs to the same quality and only differs quantitatively from other states of motion such that it is governed by the very same laws of motion.

Secondly, there is the problem that a lack of motion cannot be constructed (cf. 4:486). This is problematic because qualities can only be combined when they are comparable: "nothing can be combined with a motion, which diminishes it or destroys it, except another motion of precisely the same movable in the opposite direction" (4:497). Whilst one can combine a quantitative zero with a positive quantity, one cannot combine a qualitative zero with a positive quantity. Rest has to be understood as a state of motion rather than as a lack of motion if motions are to be combined and if a mathematical treatment of motion is to be possible.

Unlike the mathematical-mechanical approach, the dynamical approach is compatible with a thoroughgoing mathematisation of nature, since it treats rest as a state of motion, whereby the quantity of motion = 0. As a result, rest is governed by the laws of motion, can be mathematically constructed, and can be combined with other motions.

¹²For a similar assessment cf. "This certainly makes it look as if Kant is attributing a literally infinitesimal but non-zero speed to the decelerating body (at point B). Note, however, that Kant's notation ' $= 0$ ' is used to denote a complete lack of motion (at B) rather than what we would call a well-defined state of rest (with speed equal to zero). We represent the latter, in particular, by a well-defined tangent to the motion (at B) with zero slope, and Kant (not surprisingly) is simply not clearly distinguishing this notion from a literally infinitesimal but non-zero state of motion" (Friedman: 2013, p. 51 fn 25). Though Friedman is mistaken in claiming that "Kant's notation ' $= 0$ ' is used to denote a complete lack of motion", since Kant uses $= 0$ for representing both quantitative and qualitative zero and in many other places explicitly characterised rest as $= 0$ (and likewise for other diminishing quantities).

3.7 Qualitative negation and the law of continuity

There is thus an important difference between cases which involve a qualitative zero (e.g. the case of the evanescence of the soul), and cases that merely involve a quantitative zero (e.g. the case of motion). The question now is how this difference is to be explained. Why is it that in the case of motion we end up with the quantity being = 0, whereas in the other cases we end up with a lack of the quality? Moreover, how can one end up with a qualitative zero as a result of a gradual transformation, without violating the law of continuity?

The dispute with Mendelssohn crucially involves the notion of continuity. Mendelssohn's argument was that the soul cannot go out of existence without violating the law of continuity, since the change from existence to non-existence involves a leap. Kant, by contrast, argues that the self can go out of existence without any leap as a result of a gradual diminution of its intensive magnitude. On the face of it, this is rather puzzling since existence and non-existence differ qualitatively. When the self goes out of existence, we do not just end up with a quantitative zero. Instead, we are dealing with a transition that has as its limit the complete lack of any quality.

The difference involved in going from quality to the lack of quality is a specific difference that would seem to be incompatible with the law of continuity. This is particularly clear when considering what Kant calls the logical law of continuity, according to which what holds for the members of a series also holds for its limit. "*Logical law.* What holds in general for a certain magnitude that can become smaller, this also holds for it if it is vanishingly small" (28:41). In the argument against Mendelssohn, non-existence is meant to be the limit of the series, yet non-existence is radically heterogeneous from all the members of the series.

A helpful passage from the *Metaphysik Mrongovius* clarifies the problem of continuity.

With respect to quality the simple is certainly something positive, but not with respect to quantity. One cannot obtain a complete concept of the *lege continui* if one does not take the first state in which a thing is for the infinitely smallest part of the state into which a thing is yet to come. Thus one must consider the zero to be the infinitely smallest part of the following state. Rest, e.g., as the infinitely smallest part of motion. One cannot consider the first state to merely be 0 with respect to the following, for 0 is entirely different from every state. (29:921)

This passage nicely illustrates the two notions of zero, since it distinguishes the zero understood quantitatively as being infinitely or diminishingly small from the qualitative zero which involves the lack of the quality. Yet, it also accentuates the puzzle that we are faced with, insofar as it seems to tie the law of continuity to the quantitative zero. Given that the qualitative zero is 'entirely different from every state', how can transitions such as the evanescence of the soul be compatible with the law of continuity?

The problem can be resolved by noting that a different understanding of continuity is at issue. In particular, we need to distinguish between two different laws of continuity.

1. LIMIT PRINCIPLE: what holds of the members of a series also holds of its limit, such that the members do not differ qualitatively but only quantitatively from the limit. This renders the limit comparable with the members of the series and ensures that they are governed by the same laws.
2. NO LEAP PRINCIPLE: alteration is continuous in the sense that when a thing changes from state *a* to state *b*, there is a densely ordered sequence of intermediary states connecting *a* and *b* that is isomorphic to the time-interval connecting these states (cf. A208-209/B253-254).

The debate with Mendelssohn is not concerned with the LIMIT PRINCIPLE version, since both consider non-existence to differ qualitatively from existence. What is at issue instead is the NO LEAP PRINCIPLE. Mendelssohn argues that there would be a leap if the soul were to go out of existence, i.e. that the transition from its existence to non-existence cannot be continuous. It is the existence of such a leap that is disputed by Kant. The crucial point in Kant's refutation of Mendelssohn's proof is that a continuous intensive magnitude that does not have a smallest element, even though it is bounded by a limit, allows for there to be an open interval of diminishing intensive magnitudes followed by non-being. Since there is no smallest element, there will not be a moment in which the soul exists and then another moment in which it does not exist without there being any intermediary states connecting them. Instead, for every state of being, no matter how small the intensive magnitude is, there will be a further state where the intensive magnitude is yet lower. Accordingly, there can be a gradual diminution that has as its limit non-existence yet that does not involve any leap, and hence proceeds in accordance with the law of continuity.

Mendelssohn held this proof not to be adequate: he says the substance would perish if it were in one moment and not in the other; between two moments there is always a time. Its being would thus be in the one moment, its non-being in the other; now what is supposed to be between these two moments? This proof is not stringent. The soul cannot perish through division, but clearly through remission, through remission of powers (just as consciousness has various degrees of clarity, which become ever weaker, e.g., in falling asleep). The extinguishing of the human soul until complete evanescence can therefore be quite easily thought. There will also be no *saltus* here, but rather all can go according to the laws of continuity. With one degree of power the soul is there in one time; between this and the moment where it wholly disappears, there are a multitude of moments where the degrees are various. (28:763-764)

Just as the clarity of a representation can gradually become obscure so that finally the soul slumbers in it and thus its consciousness is lost little by little, so can all degrees of the powers of the human soul give way little by little, and when they have been diminished through all degrees, finally pass over into a nothing. Here is no *saltus*, but rather it observes the laws of continuity by descending through ever smaller degrees, between which there is always again a time. (29:1037-1038)

In order to comply with the law of the continuity of alteration, it is thus not required that the two states that are connected by the series in question are of the same quality. In particular, it is not necessary for the final state, i.e. for the limit of the series, to be of the same quality as the members of the series. Instead, they can differ qualitatively, allowing for the possibility of a transition that terminates in a qualitative zero.

The crucial question then is: which magnitudes are governed by the LIMIT PRINCIPLE version of the law of continuity that has as its limit the quantitative zero, and which magnitudes diminish in a way that is only governed by the law of the continuity of alterations and have as its limit the qualitative zero? This is, in fact, the question with which we started, since the question as to whether a magnitude terminates in a quantitative zero or a qualitative zero corresponds to the question whether it is governed by the LIMIT PRINCIPLE version or only the NO LEAP PRINCIPLE.

The distinguishing feature of those cases that include a zero quantity (e.g. motion) and those that do not but instead fade out into non-existence (e.g. warmth) is that the former are states of the object whereas the latter are powers of the object. The distinction between the quantitative and the qualitative zero thus maps onto the distinction between powers/grounds v. states/consequences.

Kant repeatedly points out that intensive magnitudes (understood as powers/grounds) are continuous and do not have a smallest degree. An intensive magnitude “can always be lessened” and “has a degree that, no matter how small it may be, is never the smallest” (A169/B211). Also: “The property of magnitudes whereby no part in them is the smallest possible (i.e. no part is simple) is called their continuity” (A169/B211). Although such magnitudes do not have a minimal element, they are nevertheless bounded below by zero. The zero is hence not part of the magnitude, since otherwise there would be a smallest degree. Given that the zero is not part of the magnitude, it classifies as a qualitative zero, which means that powers go out of existence when the quality diminishes to zero. Powers thus have to have a non-zero magnitude in order for them to exist.

By contrast, when it comes to a state of an object, the relevant quality can be had even when the quantity is zero. This is particularly clear when concerned with states/consequences that are the results of grounds that can stand in real opposition. In such cases, there will be a mid-point zero which is generated by opposing grounds of equal magnitudes. There will then be a state = 0 which con-

stitutes the mid-point between the positive and negative quantities of the relevant quality, even though the grounds themselves do not have a mid-point.¹³

For instance, moving force is the reality/ground. It can diminish to zero but it cannot be zero. Moving forces like all powers need to have a non-zero magnitude. Motion, by contrast, is the consequence. It can be zero = rest, either as a result of a real opposition of moving forces or due to the absence of moving forces.

The fact that powers have to be non-zero explains why the balancing argument works. If there were no attractive force to stop matter from dispersing infinitely, then repulsive force would diminish down to = 0. Repulsive force, like all forces, needs to be non-zero, which means that infinite dispersal would result in there not being any repulsive force at all. Since matter is what fills space, which it does by being (relatively) impenetrable, i.e. by having repulsive force by means of which it resists penetration (cf. 4:496). That is, nothing fills space when there is no repulsive force since there is then no resistance and hence no matter.¹⁴

Similarly, Kant's refutation of Mendelssohn's proof succeeds because he is concerned with mental powers that are gradually diminishing (cf. B414-415). These powers need to have non-zero magnitude, which means that they will go out of existence as a result of the diminution of their intensive magnitudes. In the same way that matter disappears when repulsive force fades out of existence (i.e. space is only filled when there are non-zero repulsive forces), the mind disappears when its mental powers fade out of existence.

A worry remains though. Why is it the case that if the qualities go out of existence (rather than diminishing to the quantitative zero), that the soul itself also goes out of existence? Why does the subject have to disappear when all its qualities disappear? This might be thought to be particularly problematic because the subject does not have a degree. "Alle Eigenschaften der Dinge haben einen Grad, das Ding selbst aber (substantz) nicht" (R5590). Accordingly, the subject itself cannot undergo a continuous diminution, which might suggest that it cannot fade out of existence but instead only disappear by means of a leap.

First, we can note that there is no violation of continuity. Continuity of change is only defined for alterations of an object's states, not for objects themselves, and as we have seen there is a continuous series of states of the object terminating in its non-existence.

¹³Interestingly, the limit version of the law of continuity that Kant specifies in R6317 only holds for cases where there is a mid-point: "Zwischen a und -a (Anziehung und zurückstoßung z. B. bey dem Magnetisirten Stabe) giebt es einen Punct, wo das Prädicat des Dinges verschwindet, = 0 wird."

¹⁴Contra Friedman: 2013, pp. 191-192 who understands the argument as follows: 1. for there to be matter there needs to be a specifiable quantity of matter in any given volume of space, yet 2. no such quantity can be specified when a finite quantity of matter is spread out over infinite space. However, instead of the issue being about the quantity of matter not being specifiable, the issue is rather that infinite dispersal results in there not being matter at all, i.e. non-existence. What needs to be explained is how matter goes out of existence rather than simply spreading out in a way that one is no longer left with a specifiable quantity in any finite region.

Second, even though substance is not to be identified with power (cf. 8:224 footnote), the absence of powers nevertheless implies the absence of a substance. This is because power is the relation between substance and accidens insofar as the former contains the ground of the latter. “Die Kraft ist nicht das, was den Grund der Existenz der Accidenzen enthält (denn den enthält die Substanz): sondern ist der Begriff von dem bloßen Verhältnisse der Substanz zu den letzteren, so fern sie den Grund derselben enthält, und dieses Verhältniß ist von dem der Inhärenz gänzlich unterschieden.” (8:224 footnote). This means that no Kraft implies no accidens, which in turn implies no substance.

“Der sinnliche Begriff der sustentation (der Träger) ist Misverstand. accidentien sind nur die Art zu existiren der substantz nach dem Positiven” (R5861; also cf. A187/B230). Without accidens there is no way in which the substance exists, i.e. it is not to be understood as a separate entity in which the accidens inhere and that could exist without them. R5650 “Substantz ist das letzte Subject der Realitaet. Ihr Verhältniß zum Daseyn dieser heißt Kraft, und diese ist es allein, wodurch die Existenz der Substanz bezeichnet wird und worin ihre Existenz auch selbst besteht.” There have to be qualities/accidens, in particular realities/powers for there to be a thing. They constitute a thing’s *Sachheit*. Accordingly, there cannot be a substance without powers. R4056 “Eine jede substantz hat eine Kraft, ist ein identischer Satz.” It is for this reason that Kant says that the soul has “a degree of reality in regard to all its powers, indeed in regard to everything that constitutes existence” (B414). This means that if a thing’s powers go out of existence by continuously diminishing to the qualitative zero, then that thing will thereby also go out of existence.¹⁵

Accordingly, we can answer the concern that Friedman raises: “Why can we not distinguish the substance of the soul from its realities or determinations and conclude from the principle of the permanence of substance that the former certainly cannot disappear even if the latter can be continuously decreased? Perhaps the relevant degree of reality can be continuously decreased but never actually vanish – or, more to the point, perhaps it can take on a value of zero (during sleep in the case of degree of consciousness, for example) without the underlying substance also needing to disappear” (Friedman: 2013, pp. 318-319).

¹⁵Watkins distinguishes between a weak and a strong sense of conservation. “Substance is conserved in one, relatively weak sense simply if it cannot perish (i.e., if it is necessarily permanent). If something is conserved in this sense, it can be the substratum of all time-determination. Substance is conserved in another, much stronger sense if its *quantity* remains unchanged over time. Physical monads or substances having intensive magnitudes are conserved in the weak, but not the strong sense. Only substances with extensive magnitudes are necessarily conserved in the strong sense” (Watkins: 1998, p. 549).

If the quantity is not conserved insofar as the intensive magnitude can diminish, then the problem arises that it can also diminish to zero. In that case, however, the thing in question would no longer exist, i.e. it would perish, which means that it is not permanent and hence cannot be a substance. Conservation in the weak sense thus cannot be separated from conservation in the strong sense.

There are two questions here: 1. why do the qualities disappear when they diminish to zero rather than simply take on a quantity of degree = zero? and 2. why does the self go out of existence when the qualities disappear?¹⁶ These questions can be answered in terms of all powers having to have non-zero magnitudes for them to exist, together with the claim that a substance cannot exist without having powers. The suggestion that the soul can persist even when consciousness has a degree = 0 as during sleep is explicitly rejected by Kant.

die Lebenskraft, wenn sie im Schlafe nicht durch Träume immer rege erhalten würde, erlöschen und der tiefste Schlaf zugleich den Tod mit sich führen müßte (8:175).

Das Träumen scheint zum Schlafen so nothwendig zu gehören, daß Schlafen und Sterben einerlei sein würde, wenn der Traum nicht als eine natürliche, obzwar unwillkürliche Agitation der inneren Lebensorgane durch die Einbildungskraft hinzukäme (8:190; also cf. 7:111-112 and 29:885).

For Kant, existence and *Lebenskraft* are inseparable. A person's *Lebenskraft* needs to be constantly exerted, i.e. kept active, which happens during sleep by means of dreams.¹⁷ If a person's *Lebenskraft* would diminish to zero, that person would no longer have *Lebenskraft* and hence go out of existence.

4 Empty intuition without object: *ens imaginarium*

Kant introduces the *ens imaginarium* as follows:¹⁸

Die bloße Form der Anschauung ohne Substanz ist an sich kein Gegenstand, sondern die bloß formale Bedingung desselben (als Erscheinung), wie der reine Raum und die reine Zeit, die zwar etwas sind als Formen anzuschauen, aber selbst keine Gegenstände sind, die angeschaut werden (*ens imaginarium*). (B347)

We find similar claims in the Reflexionen and lecture notes:

“Bloße Form ohne realität als Ding an sich gedacht ist *ens imaginarium*. Raum.” (R5577)

“eine bloße Form, Dinge vorzustellen, die nicht ein Ding selbst ist: *ens imaginarium*.” (R5725)

¹⁶Friedman tries to address these questions in terms of considerations regarding time-determination, arguing that only conserved quantity can function as the substratum of time-determination and hence classify as permanent substance in the sense of the First Analogy. However, he does not explain how it is that the self can go out of existence.

¹⁷It is important to distinguish between forces/powers and capacities. The former need to be constantly active (e.g. *Lebenskraft* needs to be operative in same way), whereas the latter can remain unexercised.

¹⁸Occasionally, Kant also uses ‘*ens imaginarium*’ in a different way, namely in the way in which ‘*ens rationis*’ is used in the table (cf. 28:426, 28:543-544, 28:555).

“Raum und Zeit sind wahre Entia imaginaria, bloße Bestimmungen und Formen unsrer Vorstellungen.” (28:495)

Space and time by themselves are empty intuitions in the sense that they merely consist in form without any matter/content. An intuition of space or time, as opposed to an intuition of something real that fills space and time, is an empty intuition that does not have an object but merely represents empty form. Entia imaginaria are without object because space and time themselves are not objects. They are not something real, but are merely the forms of the (phenomenal) real. As Kant notes in §23 of the B-Deduction, space and time by themselves are nothing: “sie sind nur in den Sinnen und haben außer ihnen keine Wirklichkeit” (B148; also cf. 8:153).

Whilst not being objects, they can nevertheless be intuited as forms. This means that space and time are something qua forms of intuition. “Ein blos formeller Begriff ist das nichts, insoweit der Mangel alles Materiellen in der Anschauung sich dabei findet. Er ist also nur ein nihilum in sensu materiali, aber allerdings ein Etwas in sensu formali” (29:962). Given that an ens imaginarium, such as space, can be intuited as form, we are in that respect not dealing with an empty concept. “Den Begriff den wir nichts nennen, dem zwar eine Anschauung, nur nicht die eines existirenden Dinges correspondirt. Der Begriff vom Raum gehört hieher, er ist kein conceptus inanis, wir können seine Dimension angeben” (28:624). This means that space is not something that is made up. It is not a mere Gedankending that is arbitrarily invented by us, but something that has fixed features/determinations, such as its dimensionality, that we can identify by means of (pure) intuition. Yet, it is not an object. In short, it is a real form though not a real thing. It is for this reason that Kant says of space that it “ist relativ also ein Nichts” (29:962). Space is a relative nothing, insofar as it is only something relative to appearances (namely a determination of appearances and hence has objective reality only vis-à-vis appearances), yet in itself is nothing. This means that when considered as a thing, rather than as a form, the concept of an ens imaginarium is a conceptus inanis, since no object corresponds to it.

As in the case of the nihil privativum, entia imaginaria are empty data for concepts. That is, concepts of these kinds of nothing are not concepts of things that are given to us, but are concepts that are formed negatively in the case of the nihil privativum, or by abstracting the formal element in the case of the ens imaginarium. In each case, data of real objects need to be given to the senses in order for the relevant concepts to be formed. This is because these concepts can only be formed indirectly insofar as they do not have objects that can be given to us. In particular, this applies to space and time since we can only identify the forms of intuition by abstracting them from empirical intuitions: “wenn nicht ausgedehnte Wesen wahrgenommen worden, keinen Raum vorstellen” (A292/B349). That is, we need matter that is provided by intuition and that ends up in the form in order to be able to then abstract the matter from the resulting matter-form

compound and thereby identify the purely formal element that is contributed by sensibility. The representations of space and time, like the categories, are a priori yet not innate, which means that we need something that is given in sensation to which the forms of intuition (as well as forms of thought) apply, such that the representations of these forms can then be abstracted (cf. R3930 where Kant develops this parallel between the categories and space).

4.1 Entia imaginaria v. Undinge

The *ens imaginarium* represents another way in which the table of nothing introduces the transition to the Transcendental Dialectic, insofar as one of the crucial mistakes made by transcendental realists is that they consider space and time to not merely be forms of intuitions but features of a mind-independent reality. This amounts to considering a nothing as a something. As a result, transcendental realists run into various contradictions, most notably the mathematical antinomies. Since considering space and time as having absolute reality amounts to positing Undinge (cf. A39/B56, B70-71), the only way of saving space and time from being classified as Undinge consists in considering them to be *entia imaginaria*. This means that this third type of nothing is indispensable for avoiding the contradictions of transcendental realism.

5 Empty object without concept: nihil negativum

The nihil negativum results when the principle of non-contradiction is violated, i.e. when one is dealing with a self-contradictory concept. In that case, there is nothing not only in the sense that no object corresponds to the concept in question but in the sense that, strictly speaking, the concept does not even exist. “Der Gegenstand eines Begriffs, der sich selbst widerspricht, ist Nichts, weil der Begriff nichts ist” (A291/B348). Instead of having a concept, one only has a word (or some other linguistic item) to which no concept corresponds, which is why Kant describes this type of nothing as an empty object without concept. This is because the combination of contradictory concepts does not result in a further concept, due to the fact that they cancel each other out, leaving us with the “nihil negativum irrepraesentabile” (2:171). In that sense, nothing is thought by means of a nihil negativum, i.e. thought annihilates itself.¹⁹

¹⁹The fact that thought annihilates itself when dealing with self-contradictory concepts explains why we cannot even have a problematic judgements involving a nihil negativum, cf. 29:966, which might seem problematic because the modality of a judgement is meant to abstract entirely from the content that is being judged. In particular, this is explained because in the case of a nihil negativum we do not even have a thought, i.e. we do not have any content that can even be considered problematically. In that sense, it abstracts from all content as long as it has a content, i.e. as long as it is logically possible and hence not a nihil negativum.

Some interpreters have identified this type of nothing with the category of impossibility. For instance, Aschenbrenner p. 303 claims that the nihil negativum involves categorial impossibility, i.e. it is the correlate of the tenth category. Similarly: “entspricht der Kategorie der Unmöglichkeit, also dem untergeordneten Bestandteil der Modalitätskategorie der Möglichkeit” (Paimann, p. 797; also cf. Motta p. 58 footnote 172). However, this is a mistake. The category of impossibility is concerned with real rather than logical impossibility – after all, the categories are real and not merely logical concepts. The nihil negativum, by contrast, involves logical impossibility. As such, the nihil negativum negates the heading of modality, rather than negating the category of possibility. Logical impossibility amounts to a negation of all real modality, insofar as logical possibility is a precondition of determining something by means of the modal categories.

At B349 we are told that both in the case of the ens rationis and the nihil negativum we are dealing with empty concepts, yet the latter is characterised in the table of nothing as ‘empty object without concept’. This might seem to be problematic, since having an empty concept requires one to have a concept, yet in the case of the nihil negativum we lack a concept (all we have are the component concepts that when combined do not result in a further concept but in nothing). However, all this shows is that there are two ways of being empty for Kant, namely lacking an object and lacking content such that nothing is thought. R5726 “Entweder so fern dem object kein Gedanke correspondirt, d.i. das Denken selber nichts ist, d.i. sich widerspricht, oder dem Gedanken (der sich nicht widerspricht) kein object (keines in der Anschauung) correspondirt”. This means that, strictly speaking, whereas in the case of the ens rationis the concept is empty, in the case of the nihil negativum the thought is empty (also cf. R5722).

Relatedly, Kant distinguishes between an object not being possible (which does not mean that it is impossible), which happens in the case of the ens rationis, and a concept not being possible (and in fact being impossible), which happens in the case of the nihil negativum. R5688 “Die Möglichkeit eines Begriffs beruht darauf, daß er sich nicht widerspricht; die Möglichkeit eines Dinges, daß der Begriff objective realität habe, daß davon ein Beispiel gegeben werden könne, d.i. ihm ein object correspondire”. Logical possibility attaches in the first place to concepts and thoughts, whereas real possibility applies to objects.

5.1 Contradictorie opposita v. disparata

We are dealing with a nihil negativum when a concept contradicts itself. A difficulty arises because there are two ways in which a contradiction can come about, i.e. two understandings of logical opposition, namely a narrow sense that applies when dealing with contradictories, which contain exactly as much as is needed for a contradiction, and a broad sense that includes the combination of contraries/disparata, which contain more than what is needed for a contradiction.

The question now is: are we to understand the nihil negativum as the combination only of contradictories or also of contraries?

Kant distinguishes logical opposition from real opposition: e.g. “Logically opposed is that quo posito tollitur aliud” (29:807). This holds equally in the case of contraries as in the case of contradictories. However, we are then told that: “Logical opposition is contradictoria. A cornered circle is a contradiction. Two logical opposita completely cancel themselves and nothing remains (nihil negativum)” (29:810). This would suggest that only the combination of contradictories generates the nihil negativum. Yet only a few lines later we are told that “logical opposita are either contradictorie opposita, like e.g., A et non A, or disparata, e.g., every body is either red or green.”

The issue is further complicated by the fact that Kant is not entirely consistent in his terminology and sometimes even labels cases of contraries as involving contradictorie opposita, e.g.: “Der Tugend = +a ist die negative Untugend (moralische Schwäche) = o als logisches Gegentheil (contradictorie oppositum), das Laster aber = -a als Widerspiel (contrarie s. realiter oppositum) entgegen gesetzt” (6:384; also cf. 7:230).

When combining disparata there is conflict and cancelation of content, but there is also something left over. “Disparata contain besides the contradictio opposita still something which is added, e.g. something is either red – the contradictorie oppositum would be not red. In the case of opposition of disparaten there is something additional, namely green” (29:810). This suggests that not red is somehow implicated in being green, without exhausting the concept of being green.²⁰ To classify something as not red is to posit it outside the sphere of red (cf. Jäsche Logik, §22). To classify it as green, by contrast, is to posit it within a certain sphere (that is contained within the sphere of not red). Being green is thus a more determinate way of being not red and, in this way, amounts to something more than being not red which is added to the contradictio opposita of red.

As a result, disparata contradict parts of one another but there is a remainder. Opposing disparates can thus always be factored into a contradiction plus a remainder. This is because it is only by containing (without being exhausted by) a logical contradiction that they can be logically opposed, since logical opposition cannot be based on a substantial metaphysical incompatibility that is due to the nature of the conflicting entities but has to be based on a formal incompatibility.

Accordingly, there is a remainder that consists of the shared and compatible

²⁰How exactly to make sense of this is rather difficult, since there is a concern that one will end up in a circle if green is (partially) specified in terms of not red and red is (partially) specified in terms of not green. (Such a circle can be avoided if the analysis of green ensures that things satisfying certain conditions are not green, whereby what is red satisfies these conditions, and vice versa, such that green does not make reference to not red but instead excludes conditions that red things would satisfy.) Making sense of the supposed logical contradiction between red and green amounts to explaining why the fact that nothing can be both red all over and green all over is not an a priori synthetic truth, but is instead an analytic truth.

marks of the dispartes being combined. As such, their combination only leads to partial cancellation. For instance, if A is analysed into F and G whereas B is analysed into not F and H, then the combination of A and B involves a contradiction due to the cancellation of F and not F but the components G and H do not cancel each other. This means that although nothing can be ‘A and B’, since this would require it to be both F and not F, it is nevertheless not the case that nothing is thought when A and B are combined, i.e. G and H are still left over. If there is a remainder, then we do not end up without a concept, which means that it is not the case that the opposites ‘completely cancel themselves and nothing remains’. Accordingly, we need the strict sense of logical opposition involving contradictories. Only then do we end up with an empty object without a concept.²¹

5.2 Bilineum rectilineum

An important difficulty arises when considering Kant’s example of a nihil negativum, namely ‘a rectilinear figure of two sides’ (gradlinige Figur von zwei Seiten). The bilineum rectilineum constituted the standard example of a self-contradictory concept in Wolffian thought.

Impossibile dicitur, quicquid contradictionem involvit. E.gr. Bilineum rectilineum contradictionem involvit, cum supponat duas lineas rectas inter eadem puncta contineri posse, quod contradicit propositioni verae, inter duo puncta nonnisi rectam unicam contineri. Est igitur bilineum rectilineum impossibile. (Wolff: *Ontologia*, §79)

Yet, Kant had argued both in the *Transcendental Aesthetic* and in the *Postulates* that there is no contradiction in the concept of such a figure and that its impossibility could only be established on the basis that it cannot be constructed in intuition.

So ist in dem Begriffe einer Figur, die in zwei geraden Linien eingeschlossen ist, kein Widerspruch, denn die Begriffe von zwei geraden Linien und deren Zusammenstoßung enthalten keine Verneinung einer Figur; sondern die Unmöglichkeit beruht nicht auf dem Begriffe an sich selbst, sondern der Konstruktion desselben im Raume, d.i. den Bedingungen des Raumes und der Bestimmung desselben. (A220/B268, also cf. A47/B64-65 and 2:404)

Thus, in the table of nothing the bilineum rectilineum is meant to function as an example of a logical/analytic impossibility, yet in the *Aesthetic* and the *Postulates* the bilineum rectilineum is treated as a real/synthetic impossibility that is not

²¹It might be asked how one is to classify the impossibility involved in the combination of *disparata*, i.e. where this impossibility ends up in the table. The answer is that one can always factor out the contradiction and the remainder, such that the impossible part consists in the nihil negativum and the remainder is simply an incomplete concept.

based on the concept alone. Kant would accordingly seem to be contradicting himself (cf. Martin: 1967).²²

There are three interpretative strategies for resolving this contradiction, none of which is particularly satisfactory.²³

1. One can argue that the nihil negativum in the table of nothing is to be identified not with logical but real impossibility. However, as we have seen, such an interpretation cannot be correct, since (i) it conflicts with the relation between the table of nothing and the table of categories, (ii) it cannot make sense of the idea that the concept is nothing, and (iii) it conflicts with Kant's consistent usage of 'nihil negativum' throughout all of his writings.
2. Kant can be interpreted as simply giving the bilineum rectilineum as an example to help the reader (familiar with traditional Wolffian thought), without intending to endorse the claim that it constitutes a satisfactory example of the nihil negativum. However, this is a rather unhappy solution, since there is no textual basis for this and since Kant could have easily used an unproblematic example instead (in various places he uses the round square, what he calls the four-cornered circle, to illustrate the nihil negativum, e.g. 28:479, 28:624, 29:811)²⁴.
3. If Kant were dealing, in the context of the table of nothing, with an enriched concept that builds in the axioms of geometry, e.g. the concept of a 'Euclidean bilineum rectilineum', then the example would succeed. In particular, this strategy would work if Kant should have in mind, not the concept of a mere geometrical object, but of an empirical object in space. This is because the concept of a 'bilineum rectilineum that is an object of experience' is self-contradictory, since the axioms of intuition are included in this concept, which importantly include the axiom that "zwei gerade

²²In addition, the example is unfortunate since, even if it were to involve a logical contradiction, it would not involve contradictorie opposita but disparata. (The same holds for the other example, discussed below, that Kant frequently uses to illustrate the nihil negativum, namely the four-cornered circle.)

²³Martin: 1967, pp. 234-235 claims that the contradiction cannot be removed and suggests (with hesitation) that it might be explained in terms of the table of nothing being a remnant of pre-Critical thinking. This patchwork hypothesis, however, is not plausible since the Reflexionen relating to the table of nothing are all dated after the mid-1770s (e.g. R5552 is from 1778-79 or possibly 1780-83). Moreover, the different types of nothing are distinguished in a large range of lecture notes, none of which contain the bilineum rectilineum but instead examples that involve straightforward logical contradictions.

²⁴Since a circle is a curved line every point of which is equidistant from a single point, and since it is analytic of a curved line (i.e. one that is such that none of its parts are straight, cf. A732/B760) that it does not have any corners, we have a logical contradiction between having corners (due to being four-corned) and not having corners (due to being a circle).

Linien schließen keinen Raum ein” (A163/B204). The concept of an object that is subject to the axioms of intuition and that is enclosed by two straight lines involves a straightforward logical contradiction.

This kind of concept enrichment can be found at the end of the Negative Magnitudes essay. There Kant claims that analysing the concept of God’s willing does not give us any understanding of the existence of the world, yet that through analysis of God’s omnipotent will one can distinctly understand the existence of the world (cf. 2:202-203). God’s willing is the real ground of the existence of the world, so that the latter cannot be understood through analysis of the former. However, enriching the concept of God’s willing by including God’s omnipotence leads to a logical connection between God’s omnipotent willing and the existence of the world. Internalising the real ground into the concept in this way brings about a logical connection, analogously to how enriching the concept of a bilineum rectilineum by means of the axioms of intuition turns a real impossibility into a logical impossibility.²⁵

(Importantly, internalising does not in any way explain the real connection but simply takes it for granted. Kant makes this clear when noting that the mark by means of which the concept is enriched, namely ‘power’ in the form of omnipotence, “bezeichnet schon die Beziehung eines Realgrundes auf die Folge, die ich mir gerne möchte erklären lassen” (2:203). Likewise, in order to properly understand the impossibility of a ‘bilineum rectilineum that is an object of experience’ one needs to understand the axioms of intuition and see why it is that such a figure cannot be constructed in intuition. That is, rather than engaging in conceptual analysis, one needs to identify the real ground, which is to be found in the form of outer intuition.)

This interpretation renders Kant’s various claims about ‘the’ bilineum rectilineum consistent. Unfortunately, there is no textual evidence to the effect that this is what is going on.²⁶

6 The progression of nothing

[...]

²⁵This is analogous to the transformation of hypothetical into absolute necessities. “Eine jede hypothetische Nothwendigkeit kann in absolutam verändert werden, wenn die hypothesis dem Begriff der Sache selbst zugesetzt wird: z.E. Alle gefallne Menschen sündigen” (28:18).

²⁶Since the discussion of the bilineum rectilineum in the Postulates occurs after the Axioms of Intuition, one cannot simply hold that the concept is automatically enriched once the axioms are in place.

7 The table of something

[...]

8 The system of ontology

[...]