Adams’s Thesis is the claim that the probabilities of indicative conditionals equal the conditional probabilities of their consequents given their antecedents.\(^1\) It is strongly supported by both introspection and observation of the use of conditionals in hypothetical reasoning. But a series of ‘triviality results’ show that it is impossible to accommodate Adams’s Thesis within standard semantic and probability theory.\(^2\) The bottom-line is that there is no way of assigning truth-values to indicative conditionals so as to ensure that their probabilities of truth invariably satisfy Adams’s Thesis, unless the assignment itself varies in the right kind of way with the relevant conditional probability. It follows that either Adams’s Thesis is false or the probabilities to which it refers are not standard probabilities of truth.

What has not been generally recognized however is that triviality results (or something similar) can be established for a much wider class of conditionals than just those satisfying Adams’s Thesis. The class in question is identified by a very weak constraint on the interpretation of indicative conditionals to the effect that if someone considers it possible that \(A\) is true but not that \(B\), then they should regard the sentence ‘If \(A\) then \(B\)’ as false. I call this the Preservation Condition and I will argue that it is immediately and rationally compelling in a way that Adams’s Thesis is not. But it will be shown that the Preservation Condition is also incompatible with orthodox semantic and probability theory, raising the question as to whether there are not elements of the orthodoxy that require modification.

Throughout I will use italic capitals as sentence variables, reserving the symbols \(A\), \(B\) and \(C\) for variables that range over factual sentences only (these being sentences in which the conditional operator introduced below does not occur). The symbol \(\rightarrow\) will denote the sentential operation performed by the English words ‘If ... then ...’ in indicative conditional sentences. Without affecting the generality of the conclusions, our discussion will be confined to indicative conditionals with factual consequents and antecedents.

\(^1\) See Adams (1975) for the canonical statement and defence of this thesis. It was proposed prior to this by Jeffrey (1964) and perhaps Ramsey (1926).

We begin with a formal statement of the Preservation Condition. Let $L$ be a set of sentences and let $Pr$ be a normalised real-valued function on $L$ that measures the degree to which an agent believes that what each $L$-sentence says is true. Then, for all $L$-sentences $A$, $B$ and $A \rightarrow B$:

**Preservation Condition:**

If $Pr(A) > 0$, but $Pr(B) = 0$, then $Pr(A \rightarrow B) = 0$.

The Preservation Condition is so called because of its origins in the theory of belief revision. But its plausibility does not depend on whether or not conditionals are best understood in terms of such a theory. For all it says is that one cannot be certain that $B$ is not the case if one thinks that it is possible that if $A$ then $B$, unless one rules out the possibility that $A$ as well. You cannot, for instance, hold that we might go to the beach, but that we certainly won’t go swimming and at the same time consider it possible that if we go to the beach we will go swimming! To do so would reveal a misunderstanding of the indicative conditional (or just plain inconsistency).

Compelling though the Preservation Condition may be, there is in fact no Boolean semantic theory for indicative conditionals that can ensure that it is satisfied, on the reasonable assumption that neither indicative conditionals nor their antecedents generally imply their consequents. To apply the latter assumption, I will say that if a set of sentences, $L$, contains factual sentences, $A$ and $B$, and simple conditional $A \rightarrow B$ such that neither $A$ nor $A \rightarrow B$ implies that $B$, then $L$ constitutes a non-trivial conditional language (otherwise it’s trivial). Then the impossibility of non-trivial satisfaction of the Preservation Condition follows from the (rather banal) facts about probability measures established below.

**Lemma (1).** Let $B = \langle \Omega, \leq \rangle$ be a Boolean algebra, with $\Omega$ a set partially ordered by the relation $\leq$. Let $X$, $Y$ and $Z$ be any members of $\Omega$, such that $Y \not\leq Z$ and $X \not\leq Z$. Then there exists a probability measure, $Pr$, on $\Omega$, such that $Pr(Y) > 0$, $Pr(Z) = 0$ and $Pr(X) > 0$.

**Proof.** Since $Y \not\leq Z$ and $X \not\leq Z$, the probability of $Y$ and the probability of $X$ exceed that of $Z$, i.e. we can construct the probability measure, $Pr$. □

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3 It is not required that $Pr$ be a probability function, only that (a) for any sentences $X$ and $Y$, $Pr(X) \geq Pr(Y)$ iff the agent believes that $X$ to as great an extent as she believes that $Y$ and (b) $Pr$ be normalised in the sense that $Pr(X) = 0$ iff the agent is certain that it is not the case that $X$.

4 See Gärdenfors (1988), chapter 5, for a discussion of the preservation condition in belief revision and Döring (1997) for its relevance to the semantics of conditionals.

5 The Preservation Condition does not, of course, hold for the material conditional in the sense that it may be possible that $A$ and impossible that $B$, but still possible that either not $A$ or $AB$. But it is precisely cases like this that show that the probability of truth of the material conditional is a poor guide to our belief attitudes to indicative conditionals.
**Triviality Theorem.** Let \( \langle L, \vdash \rangle \) be a Boolean algebra, with \( L \) a set of sentences closed under conjunction, disjunction and negation and partially ordered by the implication relation \( \vdash \). Assume that the Preservation Condition holds for all probability measures on \( L \). Then \( L \) is a trivial conditional language.

Proof. By Lemma (1) if \( L \) contains factual sentences \( A \) and \( B \), and simple conditional \( A \rightarrow B \) such that \( A \not\vdash B \) and \( A \rightarrow B \not\vdash \) then there exists a probability measure, \( \Pr^* \), on \( L \), such that \( \Pr^*(A) > 0 \) and \( \Pr^*(B) = 0 \), but \( \Pr^*(A \rightarrow B) \neq 0 \). But contrary to assumption, \( \Pr^* \) does not satisfy the Preservation Condition. So \( L \) cannot contain such \( A, B \) and \( A \rightarrow B \). Then by definition \( L \) is a trivial conditional language. \( \Box \)

This triviality result is based on little more than the observation that if two sentences, \( X \) and \( Y \), belong to the same Boolean algebra of sentences, then, unless \( X \) implies \( Y \), \( X \) can have greater probability than \( Y \). (By ‘can’ here is meant that there is no semantic restriction to \( X \) having greater probability than \( Y \).) As with the triviality results for Adams’s Thesis, the trick is then to assume that the conditional belongs to the same Boolean algebra as the sentences from which it is compounded. The assumption is automatically satisfied if one is looking for a truth-conditional characterisation of indicative conditionals, but it might seem like a natural enough condition on any semantic theory. Nonetheless, the Triviality Theorem would seem to show that it must be denied, if the Preservation Condition is to be retained.

This is the conclusion that I am inclined to draw, despite the ‘costly’ implication that the logic of conditionals is non-classical. But the Triviality Theorem does leave open the possibility that it is a quite different part of the orthodoxy that requires revision. I have been taking it for granted that one’s attitude to a sentence is determined by one’s attitude to what it says; that, for instance, the degree to which one believes that the sentence ‘The door is closed’ is true is determined by the degree to which one believes that the door is closed. On this view the truth of the Preservation Condition is to be explained by what indicative conditionals say. It is because I assumed it to be true that I posed the problem in terms of the constraints that the semantic ordering of conditional sentences imposes on rational belief. And why I take the Triviality Theorem to show that either the Preservation Condition is false (which I doubt) or that the right semantic theory for conditionals will be non-Boolean.

One might, however, take the Triviality Theorem to show something quite different: namely that the truth of the Preservation Condition is at least partially to be explained by non-semantic features of our attitudes to conditionals. It might be that the requirement that partial belief conforms to the Preservation Condition derives not just from what conditionals mean, but from some property of rational partial belief not already ex-
pressed in the standard axioms of probability. Indeed two recent theories
of conditionals – Jeffrey and Stalnaker (1994) and McGee (1989) – implicit-
ly assume that this is the case, because they adopt Adams’s Thesis as an
exogenous constraint on the semantic content of conditionals (without,
however, offering an explanation as to why rational belief should have this
property). If this position is to be substantiated, the relevant independent
constraint on rationality should be stated and something along the lines of
a Dutch Book argument given for it. To my knowledge nothing like this is
currently on offer and it’s difficult to judge the prospects of producing one.
But until such an argument is produced, intuition favours the thesis that
the validity of the Preservation Condition is to be explained by what indica-
tive conditionals say or mean.6

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6 This paper has benefited from comments by Robert Stalnaker and Frank Döring on
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