

The Notional Category of Modality

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It would be considered naïve today to attempt, as did Wegener (1885), to describe the semiotic stratification of human language with examples restricted to German, Greek and Latin. But it is remarkable how well Wegener's theory stands up now that the range of our evidence has been vastly broadened. It takes only a slightly more flexible calculus, I believe, to accommodate all the varieties of semiotic structure evident in ordinary discourse.

Uriel Weinreich.

Introduction

In this study, I want to explore the notional category of modality as reflected in certain expressions of German. I chose German since this is the language I know best. There is a number of very detailed investigations of the German modal system.¹ I profited from all of them.

In dealing with the semantics of modals, the main danger one is facing is to get utterly lost in the variety of interpretations one and the same expression can receive in different utterance situations. As a result, one may be tempted to develop sophisticated classifications and to study the characteristics of major types like alethic, epistemic or deontic uses of a modal expression. I am not primarily interested in such classifications. My main concern is to answer three questions:

What is the logical nature of these interpretations?

What is their variety due to?

How is this variety restricted by the vocabulary of German itself?

These questions are very much in the spirit of Gunnar Bech. I think, however, that I am in a better position today than he was: In modal logic, a semantic framework has been developed, which is more suitable for describing semantic relations between modal expressions than the tools available thirty years ago.

Traditionally, investigations of modality have concentrated on expressions like *necessarily*, *possibly*, *must*, *can*, *should* or *may*. Little attention has been paid to the fact that natural languages have means of grading and comparing possibilities. Furthermore, conditionals are usually not considered in connection with modality. Yet, *if*-clauses very often serve to restrict modals in an explicit or implicit way.

In what follows, I am trying to present a unified analysis of modality, which incorporates these facts.

Many insights gained in separate examinations of some of these phenomena will then come out as special cases of a few very general principles.

1. Expressing Modality in German

Modality has to do with necessity and possibility. In German, as in other languages, there are many ways of expressing these notions. Here is a selection:

1.1 Inherent modality

- (1) Niemand läuft in zehn Minuten von Andechs nach Aufhausen.
Nobody runs in ten minutes from Andechs to Aufhausen.
- (2) Dieses Auto fährt zwanzig Meilen pro Stunde.
This car goes twenty miles per hour.

(1) and (2) have a modalized reading:

- (1') Nobody is able to run from Andechs to Aufhausen in ten minutes.
- (2') This car can go twenty miles an hour.

Sentences (1') and (2') make explicit the modal element which seems to be inherent in the verb in the two original sentences.

1.2 Suffixes on adjectives

There are two suffixes in German which often have a modal meaning: *-lich* and *-bar*.

Consider the following lists, parts of which I borrowed from Hermann Paul.

<i>-lich</i>	
erblich	hereditary
umgänglich	sociable
zugänglich	accessible, approachable
käuflich	saleable, purchasable
zerbrechlich	fragile
sterblich	mortal
unsterblich	immortal
vergeßlich	forgetful

untröstlich	inconsolable
unvergeßlich	unforgettable
löslich	soluble
<i>-bar</i>	
zahlbar	payable
unfehlbar	infallible
brauchbar	useful, practicable
brennbar	combustible, inflammable
dehnbar	flexible, extensible
denkbar	conceivable
eßbar	eatable, edible
tragbar	portable, bearable
waschbar	washable

In general, the suffixes *-lich* and *-bar* express possibility. There are exceptions like *zahlbar*:

- (3) Die Miete für das Haus auf dem Leoni-Acker beträgt
 The rent for the house on the Leoni-Field amounts to
 zwanzig Gulden, zahlbar am ersten Januar.
 twenty guilders, payable on the first of January.

Here, it is not that the twenty guilders *can* be paid, they definitively *have* to be paid on the first of January.

1.3 Modal auxiliaries

The following auxiliaries are directly connected with the notions of necessity and possibility:

must	muß	müßte
can	kann	könnte
may	darf	dürfte
shall	soll	sollte
will	wird	würde
may	mag	möchte

The English translations are very rough approximations. The exact meaning of most of these auxiliaries will be discussed in detail as we go along. I included *wird*, as I was convinced by the arguments Heinz Vater gives in his article “*Werden* als Modalverb”.² *Müßte*, *könnte*, *dürfte*, *sollte*, *würde* and *möchte* are subjunctive forms of the corresponding verb on their left. They often have an independent meaning.

1.4 Sentence adverbs and impersonal constructions

möglicherweise	possibly
notwendigerweise	necessarily
wahrscheinlich	probably

Phrases like:

es ist möglich daß	it is possible that
es ist notwendig daß	it is necessary that
es ist wahrscheinlich daß	it is probable that

are used in a similar function.

1.5 Adjectival phrases

imstande sein	to be able
in der Lage sein	to be in the position

What becomes obvious from this selection is that there is no syntactic category corresponding to the notional category of modality.

What then is modality?

The following sections are meant to shed some light on this question.

2 Basic Notions

Most of what I have to say in this section is found in more detail in my dissertation or in related articles listed in the bibliography. Anyone who is already familiar with my previous work on modals can skip whatever does not sound new to him or her.

In order to see what is involved in modality, let us look at the following example:

The murder

Much-Girgl has been murdered on his way home. The police start investigations. Certain conclusions may be drawn from what is known about the circumstances of the crime. Utterances of the following sentences are likely to have occurred in such a situation:

- (4) Der Kastenjakl kann der Mörder sein.
The Kastenjakl can the murderer be.
Kastenjakl may be the murderer.
- (5) Der Gauzner-Michl muß der Mörder sein.
The Gauzner-Michl must the murderer be.
Gauzner-Michl must be the murderer.

In uttering (4), a police inspector claims that it is possible in view of what is known about the murder, that Kastenjakl is the murderer. Some time later, when better evidence is available, the same inspector claims in uttering (5), that it is necessary in view of what is known about the murder, that Gauzner-Michl is the murderer.

The example shows that there are at least two ingredients involved in the interpretation of modals like *kann* or *muß*: A *conversational background* which contributes the premises from which conclusions are drawn. And a *modal relation* which determines the “force” of the conclusion. In his second utterance, the inspector drew a stronger conclusion than in his first.

In the example above, the conversational background was obvious from the context of the story. Modals are context-dependent expressions since their interpretation depends on a conversational background which usually has to be provided by the utterance situation. Only occasionally do we use phrases like *in view of what is known* ... for referring to conversational backgrounds in an explicit manner.

To make all this more precise, I have to introduce a few notions of what has been called “possible-worlds semantics”.

Propositions

When Lenz uttered the sentence

- (6) Bis jetzt hab' ich dir genug Bier weggesoffen.
Up to now have I you enough beer boozed away.

he thereby expressed a proposition. In possible-worlds semantics, a proposition is identified with the set of possible worlds in which it is true. The proposition expressed by Lenz's utterance of (6) would be the set of all those possible worlds where Lenz has drunk enough beer in Fink's pub up to the day of the utterance (roughly).

The meaning of a sentence is then described in specifying which proposition is expressed if the sentence is uttered in a situation.

Let W be the set of all possible worlds. A proposition is a subset of W .

Truth of a Proposition:

A proposition p is true in a world $w \in W$ if, and only if, $w \in p$. Otherwise, p is false in w .

Logical Consequence:

A proposition p follows from a set of propositions A if, and only if, p is true in all worlds of W where all propositions of A are true.

Consistency:

A set of propositions A is consistent if, and only if, there is a world in W where all propositions of A are true.

Logical Compatibility:

A proposition p is compatible with a set of propositions A if, and only if, $A \cup \{p\}$ is a consistent set of propositions.

Conversational Backgrounds

We know already that a conversational background is the kind of entity which might be referred to by the utterance of a phrase like *what is known* (we might ignore the *in view of*-bit). What is known is different from one possible world to another. And what is known in a possible world is a set of propositions. In our semantics, a conversational background will therefore be construed as a function which assigns sets of propositions to possible worlds. In particular, the meaning of *what is known* will be that function from W into the power set of W , which assigns to any world w of W the set of all those propositions which are known in w . This is an example of an epistemic conversational background. We will consider other kinds of conversational backgrounds later. First, I want to say something about modal relations.

The most familiar of these relations are simple necessity and possibility. Assume for the following that f is an arbitrary conversational background, that is a function from possible worlds into sets of propositions.

Simple Necessity:

A proposition is a simple necessity in a world w with respect to the conversational background f if, and only if, it follows from $f(w)$.

Simple Possibility:

A proposition is a simple possibility in a world w with respect to the conversational background f if, and only if, it is compatible with $f(w)$.

The obvious thing to do now, is to link the meaning of the German modals corresponding to *must*, *necessarily*, *it is necessary that*, *can*, *possibly* or *it is possible that* to the notions I have defined above. We might want to say – for example – that a certain modal expresses simple necessity. I am going to spell out for one example what this would mean.

The meaning of Notwendigerweise

Consider an utterance of a sentence α of the form *notwendigerweise* β such that the proposition q is expressed by the utterance of the constituent sentence β . We have then:³

- (i) A proposition is expressed by the utterance of α only if there is one, and only one, conversational background for this utterance.
- (ii) If a proposition p is expressed by the utterance of α , and if f is the conversational background for this utterance, then p is that proposition which is true in exactly those worlds w of W , such that q is a simple necessity in w with respect to f .

Let us take this as a first approximation for a meaning rule for modals related to necessity.

One may wonder why there should be a unique conversational background for a modalized sentence to express a proposition. We'd better assume that in the case of several conversational backgrounds, there are several propositions expressed, one

relative to each background. It would then be part of the vagueness of modal expressions that sometimes, it remains unclear which proposition was intended. These considerations lead directly to the work Manfred Pinkal has done about definite descriptions.⁴

There is also a problem if the constituent sentence contains further modals, each requiring a conversational background of its own. To account for this, we would have to split up the utterance situation of α further and consider those parts where each modal is uttered. I elaborated this in Kratzer (1978) and I don't want to spend any more time on these kinds of refinements.

The analysis as it is, allows for one parameter to be fixed by the context of use. It implies that it is this parameter which is responsible for the variety of interpretations many modals can receive. In the murderer example, we had an epistemic conversational background. An epistemic conversational background leads to an epistemic interpretation of modal expressions. Other kinds of conversational backgrounds could lead to different interpretations. For further reference, I would like to draw attention to the following kinds of conversational backgrounds:

Realistic Conversational Backgrounds: In view of facts of such and such kind ...

A realistic conversational background is a function f which assigns sets of propositions to members of W , such that for any $w \in W$: $w \in \cap f(w)$.

That is, f assigns to every possible world a set of propositions which are true in it.

Totally Realistic Conversational Backgrounds: In view of what is the case ...

A totally realistic conversational background is a function f which assigns sets of propositions to members of W such that for all $w \in W$: $\cap f(w) = \{w\}$.

That is, f assigns to any world a set of propositions which characterize it in a unique way. For each world, there are many ways of characterizing it uniquely. This is the source of the vagueness of counterfactuals as we'll see in a later section.

Epistemic Conversational Backgrounds: In view of what is known ...

An epistemic conversational background is a function f which assigns sets of propositions to members of W such that for all $w \in W$:

$f(w)$ contains all those propositions which are established knowledge in w – for a group of people, a community etc.

Since only true propositions can be known, epistemic conversational backgrounds are special cases of realistic ones.

Of particular interest are:

Stereotypical Conversational Backgrounds: In view of the normal course of events ...

A stereotypical conversational background is a function f which assigns sets of propositions to members of W such that for any $w \in W$:

$f(w)$ contains all those propositions p such that it is the normal course of events in w that p – for someone, for a community etc.

Deontic Conversational Backgrounds: *In view of what is commanded*

A deontic conversational background is a function f which assigns sets of propositions to members of W such that for any $w \in W$:

$f(w)$ contains all those propositions p such that it is commanded in w that p – by someone, by the Law etc.

Teleological conversational backgrounds are related to aims and *buletic conversational backgrounds* have to do with wishes. An extreme case is the *empty conversational background*:

The Empty Conversational Background:

The empty conversational background is that function which assigns to any $w \in W$ the empty set.

We might think now that the “semantic field” of modal expressions could be described along two axes: One specifying a modal relation and the other one specifying restrictions for admissible conversational backgrounds.

For example:

muß

Modal relation: Simple necessity

Conversational backgrounds: No restrictions

darf

Modal relation: Simple possibility

Conversational backgrounds: Only deontic, buletic or teleological backgrounds are admitted.

The following sections will show that this view is too simple.

3 Grades of Possibility

I would like to take up the murderer example again.

Instead of (4) or (5), the police inspector might have uttered one or several of the following sentences:

- (7) Es kann gut sein, daß der Gauzner-Michl der Mörder war.
It can well be that the Gauzner-Michl the murderer was.
There is a good possibility, that Gauzner-Michl was the murderer.
- (8) Es besteht aber immer noch eine geringe Möglichkeit,
There is however still a slight possibility
daß der Kastenjakl der Mörder war.
that the Kastenjakl the murderer was.
There is, however, still a slight possibility that Kastenjakl was the murderer.

- (9) Der Gauzner-Michl kann eher der Mörder sein als der
 The Gauzner-Michl can rather the murderer be than the
 Kastenjakl.
 Kastenjakl.
 Gauzner-Michl is more likely to be the murderer than Kastenjakl.
- (10) Es ist wahrscheinlich, daß der Gauzner-Michl der Mörder war.
 It is probable that the Gauzner-Michl the murderer was.
 It is probable that Gauzner-Michl was the murderer.

The police inspector does not know what the real world is like. But he can draw conclusions from the growing evidence available to him.

At any time, this evidence is compatible with a set of worlds which “could” be the real world. These are the *epistemically accessible* worlds.

There is a straightforward connection between conversational backgrounds and accessibility relations as used in modal logic: If f is a conversational background, then the set of worlds which are accessible in a world w with respect to f is simply $\cap f(w)$. That is, the set of worlds where all propositions of $f(w)$ are true.

There are certain worlds among the accessible worlds which are more far-fetched than others. A world where Kastenjakl is the murderer is more far-fetched than a world where Gauzner-Michl has killed Girgl. Gauzner-Michl couldn't stand Girgl, but Kastenjakl got along very well with him. Even more far-fetched are worlds where someone from the other end of the world committed the crime. Far-fetched in respect to what? In respect to what is the case in the real world? This can't be true, since it seems quite natural to say that something which was almost impossible, turned out to be the case. Actually, it is things like this which usually happen in detective stories. The most unlikely candidate is the murderer. What is far-fetched about someone from the other end of the world having killed Girgl, is that things like that do not correspond to the normal course of events. Normally, you don't meet people from the antipodes in that village. And should someone show up who does not actually live in the neighbourhood, he wouldn't just go and kill Girgl. Normally people need a motive for killing someone. It couldn't have been for money since Girgl wasn't robbed: all his money was found on him. In view of the normal course of events, it is far-fetched that someone from the other end of the world has killed Girgl. And in view of the normal course of events it is more far-fetched for Kastenjakl to be the murderer than for Gauzner-Michl.

Worlds in which the normal course of events is realized are a complete bore, there are no adventures or surprises. The concept of a normal course of events is analogous to the concept of “frame” which plays an important role in psychology and artificial intelligence.

In our example, the epistemic conversational background determines for every world the set of worlds which are accessible from it. It forms the *modal base*.

There is a second conversational background involved in the above uses of modals, a stereotypical background. It induces an ordering on the set of accessible worlds, thereby functioning as *ordering source*.⁵

Quite generally, a set of propositions A can induce an ordering \leq_A on the set of all possible worlds in the following way: (The idea is taken from David Lewis' work on ordering semantics, personal communication.)

The Ordering \leq_A :

For all worlds w and $z \in W$:

$w \leq_A z$ if and only if $\{p : p \in A \text{ and } z \in p\} \subseteq \{p : p \in A \text{ and } w \in p\}$

The intuitive idea is this: A world w is at least as close to the ideal A as a world z if, and only if, all propositions of A which are true in z , are true in w as well.

It can be shown that the relation \leq_A is reflexive and transitive.

We are now in the position to define some additional modal relations:

Human Necessity:

A proposition p is a human necessity in a world w with respect to a modal base f and an ordering source g if, and only if, the following condition is fulfilled:

For all $u \in \cap f(w)$ there is a $v \in \cap f(w)$ such that

(i) $v \leq_{g(w)} u$

and

(ii) for all $z \in \cap f(w)$: If $z \leq_{g(w)} v$, then $z \in p$.

Roughly speaking, a proposition is a human necessity in view of a modal base and an ordering source if, and only if, it is true in all those accessible worlds which come closest to the ideal. The accessible worlds are determined by the modal base and the ideal is provided by the ordering source.

As we can't assume that there have to be such things as *closest* worlds, the definition is rather complicated. It resembles the one David Lewis gives for counterfactuals.⁶

Human Possibility:

A proposition is a human possibility in a world w with respect to a modal base f and an ordering source g if, and only if, its negation (that is its complement) is not a human necessity in w with respect to f and g .

Slight Possibility:

A proposition p is a slight possibility in a world w with respect to a modal base f and an ordering source g if, and only if,

(i) p is compatible with $f(w)$

(ii) the negation of p is a human necessity in w with respect to f and g .

Human possibility is just the dual of human necessity.

If a proposition is a human necessity in a world with respect to a modal base and an ordering source, there may still be accessible worlds in which it is not true. In this case, its negation will be a slight possibility in the same world with respect to the same parameters.

Comparative Possibility:

A proposition p is more possible than a proposition q in a world w in view of a modal base f and an ordering source g if, and only if, the following conditions are satisfied:

- (i) For all $u \in \cap f(w)$:
If $u \in q$, then there is a world $v \in \cap f(w)$ such that $v \leq_{g(w)} u$ and $v \in p$.
- (ii) There is a world $u \in \cap f(w)$ such that:
 $u \in p$ and there is no world $v \in \cap f(w)$ such that $v \in q$ and $v \leq_{g(w)} u$.

In simple words, these conditions say roughly this:

- (i) For every accessible q -world there is an accessible p -world which is at least as close to the ideal.
- (ii) There is an accessible p -world for which there is no accessible q -world which is at least as close to the ideal.

It can be shown that the relation of comparative possibility is transitive and asymmetric. The proof is left to the reader.

I think that these four modal relations can be linked to the four modal expressions in the sentences (7) to (10) in the following way:

Es kann gut sein daß ...	Human Possibility
Es besteht eine geringe Möglichkeit daß ...	Slight Possibility
Es kann eher sein daß ... als daß ...	Comparative Possibility
Es ist wahrscheinlich daß ...	Human Necessity

The English equivalents would be roughly:

There is a good possibility that ...	Human Possibility
There is a slight possibility that ...	Slight Possibility
It is more likely that ... than that ...	Comparative Possibility
It is probable that ...	Human Necessity

I think it is clear enough how these linkages are to be understood: The four modals require a pair of parameters to be fixed by the context of use. The rest of the corresponding meaning definitions would be as for *notwendigerweise* with the necessary adjustments made.

To check the correctness of this analysis, imagine utterances of sentences of the following form, where modal base, ordering source and the propositions expressed by α stay the same for all examples:

- (11) Es ist wahrscheinlich, daß α
It is probable that α
- (12) Es ist wahrscheinlich, daß nicht α
It is probable that not α
- (13) Es kann gut sein, daß α
It can well be that α
- (14) Es kann gut sein, daß, nicht α
It can well be that not α

- (15) Es kann nicht gut sein, daß α
It can not well be that α
- (16) Es kann nicht gut sein, daß nicht α
It can not well be that not α
- (17) Es besteht eine geringe Möglichkeit, daß α
There is a slight possibility that α
- (18) Es ist eher möglich, daß nicht α , als daß α
It is more likely that not α than that α

We may suppose further that the modal base for these utterances is *normal*. A conversational background f is normal if, and only if, for all w of W , $f(w)$ is a consistent set of propositions. Any realistic conversational background is normal. The above analysis of the four modal expressions gives us now for example the following predictions:

- (a) The propositions expressed by the utterances of (11) and (12) are not compatible with each other.⁷
- (b) The propositions expressed by the utterances of (13) and (14) are compatible with each other.
- (c) The proposition expressed by the utterances of (13) follows from the proposition expressed by the utterance of (11).
- (d) The proposition expressed by the utterance of (14) follows from the proposition expressed by the utterance of (12).
- (e) The propositions expressed by the utterances of (11) and (14) are not compatible with each other.
- (f) The propositions expressed by the utterances of (12) and (13) are not compatible with each other.
- (g) The propositions expressed by the utterances (12) and (15) follow from each other.
- (h) The propositions expressed by the utterances of (11) and (16) follow from each other.
- (i) The proposition expressed by the utterance of (18) follows from the propositions expressed by the utterances of (12), (15) and (17).

The interpretations of the four modal expressions in sentences (7) to (10) depend on a pair of conversational backgrounds. In our example, it was an epistemic modal base and a stereotypical ordering source. Does this mean that for different types of modals, a different number of parameters has to be fixed by the utterance context? Would it be one parameter for modals like *muß*, *kann*, *es ist notwendig daß* etc. and a pair of parameters for expressions of the kind we have discussed in this chapter?

We shall see in chapters yet to come that modals of the first kind may express graded notions of modality too. And grading involves an ordering source as well as a modal base. So a better view would be to assume that the interpretation of modals in general depends on a modal base and an ordering source where either parameter may be filled by the empty conversational background. Further support for this view will come from the analysis of practical inferences and conditionals. Simple necessity and possibility might now be seen as special cases of human necessity and possibility respectively. The

reader may convince him- or herself that the following equivalences are true for any modal base f and the empty ordering source g :

Simple and Human Necessity:

A proposition is a simple necessity in a world w with respect to f if, and only if, it is a human necessity in w with respect to f and g .

Simple and Human Possibility:

A proposition is a simple possibility in a world w with respect to f if, and only if, it is a human possibility in w with respect to f and g .

As a new start, we may try now to describe the semantic field of modal expressions along three axes specifying:

- (i) a modal relation
- (ii) conditions for the modal base
- (iii) conditions for the ordering source.

In the following section I will begin with a discussion of the two major types of modal bases which are realized in German.

4 Two Basic Kinds of Modal Reasoning

We have seen that in modal reasoning, a conversational background may play the role of a modal base or an ordering source. The modal base determines the set of accessible worlds and the ordering source determines an ordering on it.

In this chapter, I want to investigate the two major types of modal bases which are relevant for German.

Some examples will be useful:

Root or Circumstantial Modal Bases:

- (19) Sie wollte schreien und konnte nicht, gewann aber
 She wanted to scream and could not, regained however
 endlich die Herrschaft über ihre erlahmten Glieder.
 finally the control over her paralyzed limbs.

Genovev was so terrified that she was unable to move.

- (20) Der Jani-Hans schimpfte nie, fluchen konnte er gar nicht.
 The Jani-Hans scolded never, curse could he at all not.

Jani-Hans had such a mild character that he just wasn't capable of getting angry.

- (21) Hier können die Tomaten gedeihen.
 Here can the tomatoes prosper.

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- (22) Wer nichts hat, dem kann man auch nichts nehmen.
Who nothing has, from whom can one also nothing take away.

Epistemic Modal Bases:

- (23) Es kann nur einer gewesen sein, der sich im Haus
It can only someone been have, who (refl.) in the house
auskennt hat'.
been at home has.

The Heimrath's have been burgled and Girgl tries to find out who might have been the thief. It must have been someone who was familiar with the house.

- (24) Sie hatten den Befehl, den jungen König zu suchen, der sich
They had order the young king to look for, who (refl.)
in einer seiner Jagdhütten aufhalten mußte.
in one of his hunting huts stay must (past).

The young king has disappeared and in view of what is known, he must be hiding in one of his hunting huts.

Unlike the English *must*, the German *muß* has a past tense form *mußte*.

- (25) Soweit wir wissen, muß es für sie nie etwas
As far as we know, must there for them never anything
anderes gegeben haben als Geborenwerden, Aufwachsen,
else been have but being born, growing up,
unermüdliche Arbeit und Sterben.
tireless work and dying.

Oskar Maria Graf draws this conclusion from the historical sources about the life of the Heimrath family some centuries ago.

The term “epistemic modality” is familiar in linguistics and philosophy. The term “root modality” is usual in the tradition of generative grammar. “Circumstantial modality” is in the spirit of Terence Horgan (1977).

There is a clear intuitive difference between the two kinds of occurrences of modals which I grouped under the two headings. It is a difference in the kind of premises from which we reason. If we use an epistemic modal, we are interested in what else may or must be the case in our world, *given everything we know already*. And if we use a circumstantial modal, we are interested in what can or must happen, given *circumstances of a certain kind*. Circumstances of a certain kind are facts of a certain kind. Facts concerning the outside world, our bodies or our mind, for example. Usually, circumstances permit or exclude that certain things happen. Only sometimes do they necessitate an event or an action: We have to die, to cough, to vomit, to laugh, to cry or to realize that we are lost.

Epistemic modality and circumstantial modality involve a different categorization of the facts. The problem is now to find out some more details about this partition.

I shall present a few observations towards this goal. Consider the following pairs of sentences:

- (26) (a) Aus dieser Kanne Milch kann die Kathl ein Pfund
From this can of milk can the Kathl one pound of
Quark machen.
cottage cheese make.
- (26) (b) Es kann sein, daß die Kathl aus dieser Kanne Milch
It may be that the Kathl from this can of milk
ein Pfund Quark macht.
one pound cottage cheese makes.
- (27) (a) In dieser Gegend können Zwetschgenbäume wachsen.
In this area can plum trees grow.
- (27) (b) Es kann sein, daß in dieser Gegend Zwetschgenbäume wachsen.
It may be that in this area plum trees grow.

Sentences (26) (a) and (27) (a) have a circumstantial reading besides an epistemic one. For sentences (26) (b) and (27) (b), the epistemic reading is prominent. Given a circumstantial reading for the (a)-sentences and an epistemic reading for the (b)-sentences, we can imagine situations where I say something true in uttering an (a)-sentence, but something false in uttering the corresponding (b)-sentence. Take the first two sentences: In view of quite general conditions concerning the production of cottage cheese, it is possible that Kathl is going to produce a pound of cottage cheese from the milk in the can. We know, however, that Kathl never uses the whole can of milk for the production of cheese. She uses a bit for her coffee, a bit for her porridge, a bit for the cat and the rest for her cheese. This means, that in view of everything we know, it is not possible that Kathl is going to produce a pound of cottage cheese from the milk in the can.

In using a circumstantial modal, we neglect certain kinds of facts. In our case, it is facts about what Kathl always actually does.

The situation is similar with the sentences (27) (a) and (27) (b). Suppose I am travelling in an exotic country and discover that soil and climate are very much like that in my own country, where plum trees prosper everywhere. In such a situation, an utterance of (27) (a) in its circumstantial sense would probably be true. But (27) (b) could very well be false, given that this country had no contacts whatsoever with western civilization and the vegetation is altogether different from ours. Since we know this, it is impossible in view of what we know that plum trees grow in this area.

Again, we have to neglect certain facts for (27) (a), although we might be aware of them.

The kind of facts we take into account for circumstantial modality are a rather slippery matter. This may give rise to misunderstandings and jokes.⁸ I once heard a philosopher say that one of the defining properties of a cup is, that you can pour things like coffee in it. A student objected to this in pointing out that – if this were true – a cup which has coffee in it already, would not be a cup anymore.

When we talk to each other, we hardly ever make explicit in view of which circumstances something should be necessary or possible. We may give hints. Usually people understand. And they all understand in pretty much the same way.

Consider the following sentence:

- (28) Ich kann nicht Posaune spielen.
I can not trombone play.

Depending on the situation in which I utter this sentence, I may say quite different things. I may mean that I don't know how to play the trombone. I am sure that there is something in a person's mind which becomes different when he or she starts learning how to play the trombone. A programme is filled in. And it is in view of this programme that it may be possible that I play the trombone.

Or suppose that I suffer from asthma. I can hardly breathe. In view of my physical condition I am not able to play the trombone, although I know how to do it. I may express this by uttering (28). Or else imagine that I am travelling by sea. The ship sinks and so does my trombone. I manage to get to a lonely island and sadly mumble (28). I could play the trombone in view of my head and my lungs, but the trombone is out of reach.

There are more conceivable interpretations for an utterance of (28), but most of them involve other conditions in addition to the facts. That is, most of them involve a non-empty ordering source.

A distinction between circumstances concerning mainly the outside world, the body or the mind of a person, plays a role in the semantic development of *können*. According to Gustav Deggau, a student of Otto Behaghel's, the Old High German equivalent of this modal was first used for intellectual capacities. Then, it could express possibilities in view of the outside situation. Only considerably later was it used for talking about physical abilities.⁹

Ferenc Kiefer (1980) has shown that similar distinctions are made in Hungarian. In Hungarian, the verbal suffix *-hat/-het* expresses possibility. In its circumstantial reading, it can only be used for possibilities in view of the outside situation. In Kiefer's own terms: "Modal sentences with *-hat/-het* can only express outer dispositions".

Taking up some of Kiefer's further observations, I would like to present some analogous facts about modern German.

Consider a phrase like *imstande sein* (to be able).

I could say

- (29) Ich bin nicht imstande, Posaune zu spielen.
I am not able trombone to play.

if I have asthma or weak nerves or if I am just too stupid. I doubt whether I would say it in a situation where I haven't learnt how to play the trombone. And I could never say it on the island with my trombone lost at sea. The prominent circumstances for *imstande sein* are concerned with the strength of our body, character or intellect.

For *kann*, there is a further type of restrictions.

Consider:

- (30) Dieses Messer kann nicht schneiden.
This knife can not cut.
- (31) Dieser Hut kann den Kopf warmhalten.
This hat can the head keep warm.
- (32) Dieser Ofen kann nicht richtig heizen.
This stove can not properly heat.

These sentences sound funny. They suggest that the knife, the hat or the stove are agents which take an active part in the cutting, the warming of the head or the heating. To avoid this effect, we would have to say:

- (33) Dieses Messer schneidet nicht.
This knife cuts not.
- (34) Dieser Hut hält den Kopf warm.
This hat keeps the head warm.
- (35) Dieser Ofen heizt nicht richtig.
This stove heats not properly.

I think that sentences (30) to (32) have some features in common whose co-occurrence might be responsible for the fact that they sound bizarre.

One of these properties is concerned with agency: The knife is not an agent, but an instrument for cutting something. The hat is not an agent, but an instrument for warming the head. And the stove is not an agent, but an instrument for heating a room. After all, it's *you* who cuts the bread, keeps the head warm and heats the house. Some machines, like music boxes, can do things all by themselves, thus functioning as true agents. I can't find anything peculiar about (36):

- (36) Diese Spieluhr kann "La Paloma" spielen.
This music box can "La Paloma" play.

Here, the music box is an agent and the use of *kann* is appropriate.

Another feature is concerned with the kinds of actions which are said to be possible or impossible for a knife, hat or stove to be involved in. That a knife cuts, a hat keeps the head warm or a stove heats a room, is fairly well compatible with our stereotypical notions about knives, hats or stoves. Consider in contrast:

- (37) Dieses Messer kann einen Felsen zerschneiden.
This knife can a rock cut into pieces.
- (38) Dieser Hut kann epileptische Anfälle verhindern.
This hat can epileptic attacks prevent.
- (39) Dieser Ofen kann wahlweise mit Kohle oder Öl heizen.
This stove can at choice with coal or oil heat.

Knives which cut rocks into pieces, hats which prevent epileptic attacks and stoves which work with coal or oil at choice come as a surprise. I think this is the reason why sentences (37) to (39) sound all right although the knife, the hat and the stove remain

instruments for the actions under consideration.¹⁰ Further research has to be done in this area.

What these examples (as well as Kiefer's examples) show, however, is that it is still a simplification to describe the meaning of modal expressions by specifying nothing more but a modal relation and some restrictions for possible modal bases or ordering sources. Some constraints seem to involve agency or stereotypes associated with natural kind terms.¹¹ I shall nevertheless stick to this simplification. I think it is still rewarding to examine the modal system of a language with respect to these three parameters, even if this is not the whole story.

In this chapter, I have examined the two major kinds of modal bases which are relevant for German (and all other languages I know): Circumstantial and epistemic modality are both based on realistic conversational backgrounds, but involve a different categorization of the facts.

The distinction is clearly marked in the vocabulary. Verbs with inherent modality, modal adjectives on *-lich* and *-bar* and phrases like *imstande sein* or *in der Lage sein* never express epistemic modality.

Sentence adverbs like *wahrscheinlich* or *möglicherweise* and auxiliaries like *wird* always express epistemic modality – if they express modality at all. Some of these expressions involve a grading. In the examples discussed in this chapter, I avoided grading as far as possible.

In the following sections, I will show how different modal bases interact with different kinds of ordering sources to yield the variety of the German modal system.

5. The Quest for Certainty

In section three, I gave an example of the grading of an epistemic modal base. As a result, we obtained some new modal relations which were linked to expressions like *there is a good possibility that* or *it is probable that*. In this section, I want to discuss some further issues concerning the grading of epistemic modal bases.

It has often been observed that I make stronger claim in uttering (40) than in uttering (41):¹²

- (40) Das ist die Bürgermeister-Weiß-Straße.
This is the Bürgermeister Weiß Street.
- (41) Das muß die Bürgermeister-Weiß-Straße sein.
This must the Bürgermeister Weiß Street be.

These utterances present a problem if we assume that *muß* gets a “pure” epistemic interpretation. In this case, the proposition expressed by the utterance of (40) would follow from the proposition expressed by the utterance of (41) but not vice versa. Thus, uttering (41) should lead to a stronger claim than uttering (40). Since this is not the way things are, we have good reasons to assume that the utterance of *muß* in (41) does not express “pure” epistemic necessity. In our framework, this means that the ordering source is not empty.

In uttering (41) instead of (40), I signalize that I don't reason from established facts alone. I use other sources of information which may be more or less reliable. Take for example the route description of a friend, a tourist guide or my own vague memories from years ago. These other sources of information may form ordering sources for epistemic modal bases.

A set of facts is always consistent. Other sources of information may themselves be inconsistent or else be inconsistent with the established facts. If these other sources function as ordering sources and are not part of the modal base, it can be explained why they can still be useful, even if there are inconsistencies. And why they never override the facts: In the case of a conflict, established facts have priority over route descriptions, tourist guides and memories. I shall give an illustration of the treatment of inconsistencies in section seven. So I needn't go into details here.

The next point I want to discuss, was brought up by John Lyons (1977):

In principle, two kinds of epistemic modality can be distinguished, objective and subjective. This is not a distinction that can be drawn sharply in the everyday use of language; and its epistemological justification is, to say the least, uncertain. . . . It is nonetheless of some theoretical interest to draw the distinction between objective and subjective epistemic modality.

The distinction is manifest in the vocabulary of German. Imagine that Lenz, who often has bad luck, is going to leave the Old World by boat, today, on Friday thirteenth. On hearing about this, someone might utter one of the following sentences:¹³

- (42) Wahrscheinlich sinkt das Schiff.
Probably sinks the boat.
Probably, the boat will sink.
- (43) Es ist wahrscheinlich, daß das Schiff sinkt.
It is probable that the boat sinks.
It is probable that the boat will sink.
- (44) Das Schiff wird (bestimmt) sinken.
The boat will (certainly) sink.
- (45) Das Schiff dürfte sinken.
The boat sink.
It is probable that the boat will sink.

In German, the auxiliary *wird* has a temporal and a modal use.¹⁴ I intended the modal reading for (44). I couldn't find an appropriate glosse for *dürfte*, so I left a gap. In uttering (42) or (44), I make a more subjective claim than in uttering (43) or (45). I may be rather superstitious. I couldn't defend my claim on objective grounds. But I would have to do so if I uttered (43) or (45). There are established facts about the boat, the technical equipment nowadays or the weather. And there are commonly held conceptions about the normal course of events. In a world reigned by science and technology, these conceptions don't include superstitions. *Es ist wahrscheinlich daß* and *dürfte* seem to require an "objective" stereotypical background as their

ordering source. *Wahrscheinlich* and *wird* prefer “subjective” stereotypical backgrounds.

John Lyons believes that in its subjective reading, an epistemic modal doesn't contribute to the propositional content of an utterance at all. This is a very debated issue on the border of semantics and pragmatics. I don't want to go into it, as I won't be able to examine the different positions here.

In the following section, I want to discuss ways of grading circumstantial modal bases.

6 Approaching Ideals

In this section, I am going to examine how different ordering sources interact with a circumstantial modal base and how this is reflected in German.

Circumstantial conversational backgrounds are special kinds of realistic ones. They involve the sort of categorization of facts which we have discussed in section four. We can include the empty conversational background as a special case of a circumstantial one.

Circumstances create possibilities, the set of possible worlds which are compatible with them. These worlds, which are accessible in the circumstances under consideration, may be closer or further away from

The Law,
 What my father provided in his last will,
 What is good,
 What you think is good,
 Our plans,
 Our aims,
 Our hopes,
 Our wishes,
 Our conception of a good life,
 What Ferdl recommends to his wife,
 What is rational
 :
 :

To all of these ideals correspond conversational backgrounds. In the terms of possible worlds semantics, these would be functions g from possible worlds into sets of propositions, such that for every world w , $g(w)$ is the set of all those propositions p such that

The Law provides that p in w ,
 My father provided that p in his last will,
 p is good in w
 In w , you think that p is good,
 Our plans in w provide that p ,
 It is our aim in w that p ,

We hope in w that p ,
 We wish in w that p ,
 It is in w our conception of a good life that p ,
 Ferdl recommends p to his wife in w ,
 In w , it is rational that p ,
 :
 :

All of these “normative” conversational backgrounds could be proper ordering sources for a circumstantial modal base. Just as in section two, they would induce an ordering on the set of accessible worlds. From this, we get corresponding notions of human necessity, human possibility, slight possibility and comparative possibility.

Some modal expressions of German tolerate a wide range of ordering sources. Others have to obey more restrictions. Let us look at some examples:

Können and dürfen

- (46) Du kannst doch nicht nur Häuser bauen oder Semmeln backen
 You can not only houses build or rolls bake
 und wenn du dann gestorben bist, ist alles aus,
 and when you then dead are is everything finished,
 alles weggewischt.
 everything wiped out.

Shortly before his death, the old Graf realizes that in view of some conception of an ideal life, you should do more than just care for your property or do your daily work.

- (47) Sagen kannst gewiß nicht, daß ich dir einmal schlecht
 Say can you certainly not that I you once bad
 geraten hab'.
 advice given have.

Jani Hans always advised the Heimrath widow well. Given this fact, it is impossible in view of an ideal of truthfulness and trust, that she says anything to the contrary.

- (48) Dieses Brot kann man ja direkt seiner
 This bread can one indeed straight away to his
 Majestät empfehlen.
 Majesty recommend.

This bread is good. If you recommend him something good, his Majesty will be pleased. If you recommend him something bad, however, his Majesty will hate you. Given these facts, it is possible in view of an ideal where his Majesty loves you, that you recommend this bread to his Majesty.

- (49) Kann ich jetzt gehen?
Can I now leave?

Imagine a pupil who says (49) to his teacher. The teacher is the source of law and order for him. What she wants is commanded and nothing is commanded unless she wants it. The boy wants to know whether it is possible in view of what is commanded that he leaves. In this case, the *kann* in (49) is deontic.

Welke (1965) and Buscha et al. (1977) think that this purely deontic use of *kann* is colloquial. Klaus Welke quotes from “Muttersprache” (“Mother Tongue”), where teachers of German are advised to correct pupils who use *kann* for expressing permission. They should say *darf* (*may*). For me, *kann* may express permission and I don’t feel that there is anything colloquial about it.

For *darf*, a deontic ordering source is common but not obligatory. Suppose two burglars are trying to enter a farm house and whisper to each other:

- (50) Jetzt dürfen wir keinen Lärm machen.
Now may we no noise make.

It is not that they are not allowed to make a noise. They can’t make a noise in view of their aim to burgle the farmers without getting caught.

Kann und *darf* have similar meanings. Both express human possibility. But there are differences.¹⁵

Darf requires an ideal in view of which possibilities are assessed. *Kann* is more neutral in this respect. Here, possibilities may depend on brute facts only, that is, the ordering source may be empty. On the other hand, *darf* doesn’t admit *any* “normative” conversational background as ordering source.

Suppose I have a horrible headache and say with a deep sigh:

- (51) Ich kann das nicht aushalten.
I can this not bear.

This use of *kann* involves standards concerning normal tolerance thresholds for pain. I couldn’t express the same thing in uttering.

- (52) Ich darf das nicht aushalten.
I may this not bear.

Darf does not tolerate a “normal standards” ordering source. On the other hand, *kann* may have difficulties with deontic ordering sources: Tomorrow is the coronation of the King and I utter

- (53) Morgen darf es nicht regnen.
Tomorrow may it not rain.

What I say here is roughly, that in view of what we all want, it shouldn’t rain tomorrow. I couldn’t get this interpretation in uttering:

- (54) Morgen kann es nicht regnen.
 Tomorrow can it not rain.

We can conclude that there are certain restrictions for *kann* and *darf* which concern the admissible ordering sources. Again, more detailed investigations have to reveal the exact nature of these restrictions.

That an expression requires a complement of a certain kind to be provided by the context of use, has important consequences for the way we understand these expressions. These rules of use can influence certain features of the utterance context itself by means of what David Lewis has called “rules of accommodation”.¹⁶ In our case, a rule of accommodation in the style of David Lewis would look as follows:

Rule of Accommodation:

If the utterance of an expression requires a complement of a certain kind to be correct, and the context just before the utterance does not provide it, then *ceteris paribus* and within certain limits, a complement of the required kind comes into existence.

This is black magic, but it works in many cases. Suppose, I have a broken leg and say:

- (55) Ich darf nicht laufen.
 I may not walk.

So far, I have been talking about how I fell down the ladder, how they plastered my leg ... just facts and nothing else. With the utterance of (55), suddenly ideals start entering the picture: ideals where people don't have crooked legs, where they don't feel pain or where they just listen to their physician. As David Lewis shows, rules of accommodation play an important role in our conversations. So this is an example of how the way we understand a particular occurrence of a modal can be at least partly explained by an interaction of independently motivated semantic and pragmatic principles.

Müssen and Sollen

- (56) Wegen der Lola Montez hat er dem Thron entsagen müssen.
 Because of Lola Montez has be the (dat.) throne abdicate must (inf.).

Ludwig I of Bavaria loved Lola Montez. People became angry. Revolution broke out. In view of the public interest, it was necessary in this situation that he resigned. (Note the use of the infinitive *müssen* here. You would expect the participle perfect passive *gemußt*. This peculiarity of German is discussed in Edmondson (1979).)

- (57) Es muß mir gehören, es muß.
 It must to me belong, it must.

Kastenjakl is desperate to buy a piece of land from the Heimrath's. In view of what he wants, it must belong to him.

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- (58) Lump muß man sein, nur als Lumpzwingt man die lumpige Welt.
 Crook must one be, only as crook conquers one the crooky world.

Lenz presents his aim in the second part of the sentence. Given our world as it is, it is necessary in view of the aim to conquer the world, to be a crook.

- (59) Arbeiten haben wir bis jetzt müssen, arbeiten werden
 Work have we up to now must (inf.), work will
 wir auch weiter müssen.
 we also in future must (inf.).

The Heimrath's are peasants. Given their social status, they have to work in view of an ideal of a decent and honest life. They don't want to be beggars or burglars.

Like *kann*, *muß* accepts a wide range of ordering sources. The ordering source may be empty too. This is suggested by sentences like:

- (60) Er mußte husten.
 He must (past) cough.

Like *darf*, *soll* requires a non-empty ordering source. Let us consider some examples:

- (61) Ein Richard Wagner Festspielhaus sollte nach den
 A Richard Wagner festival hall shall (past) after the
 Entwürfen des Architekten Semper gebaut werden.
 designs of the architect Semper built be.

In view of the plans of King Ludwig II of Bavaria, a Richard Wagner festival hall was to be built after the designs of the architect Semper.

- (62) Ich bitt' euch gar schön, der bochwürdige Herr Pfarrer soll kommen.
 I ask you very nicely, the reverend Sir curate shall come.

Gauzner Michl is dying. In view of what he wants, a priest must come.

In Luther's translation, God uses *sollen* a lot when he talks to Moses.

- (63) Sechs Tage soltu erbeiten und alle deine Werck thun.
 Six days shalt thou labour and all thy work do.

In view of what God wants, it is necessary that you work six days a week. In some societies, what God wants is commanded. In other societies, what God wants is good and recommended, but not commanded. If I lived in a society of the first kind, I would most naturally say:

- (64) Ich muß sechs Tage arbeiten und alle meine Werke tun.
 I must six days work and all my work do.

If I lived in a society of the second kind, however, I would prefer to say:

- (65) Ich soll sechs Tage arbeiten und alle meine Werke tun.
 I shall six days work and all my work do.
 I am supposed to work for six days and to do all my work.

Sollen expresses necessity. It requires an ordering source which is created by what is good, planned or recommended, or by what a particular someone wants, plans or recommends. Actually, it is not just what *anyone* wants, plans or recommends. The one who does so cannot be identical with the individual referred to by the subject of the sentence in which *sollen* occurs. I can't say

- (66) Ich soll ein Bäcker werden.
 I shall a baker become.
 I am supposed to become a baker.

if it is mine but no-one else's wish that I become a baker. Compare this with Gunnar Bech's (1949) characterization "*sollen* ... bezeichnet einen nicht dem Subjekt innewohnenden Willen" ("*sollen* refers to a will which is not inherent in the subject"). If we assume that in a passive sentence like (67), *er* is not the logical subject, (67) is not a counterexample to this principle:

- (67) Er soll in Ruhe gelassen werden.
 He shall in peace left be.

I think that I could use (67) for expressing that it is in view of what he wants himself that he shouldn't be bothered.

Muß is neutral with respect to who wants me to become a baker.

- (68) Ich muß ein Bäcker werden.
 I must a baker become.

may be used if I want to say that it is in view of my own wishes that I have to become a baker.

The suffixes *-bar* and *-lich* allow all kinds of ordering sources, depending on the adjective they are attached to.

-bar and -lich

Consider:

- (69) Dieses Eintrittsbillet ist nicht übertragbar.
 This admission ticket is not transferable.

In view of the regulations, it is not possible to give this ticket to someone else.

- (70) Diese Tasse ist zerbrechlich.
This cup is fragile.

I think that this is a case of “pure” circumstantial modality. It is in view of certain properties inherent in the cup, that it is possible that it breaks. The ordering source seems to be empty.

- (71) Dieser Vorschlag ist annehmbar.
This proposal is acceptable.

In view of our common aims, it is possible to accept this proposal.

- (72) Diese Lage ist unerträglich.
This situation is intolerable.

Every night, Marie-Louise’s living room becomes the meeting place for all the cats in the neighbourhood. This is intolerable in view of quite normal standards concerning property, noise and smell. We may add a phrase like *for Marie-Louise* to indicate that the standards involved are more subjective.

- (73) Für Marie-Louise ist diese Lage unerträglich.
For Marie-Louise is this situation intolerable.

Ordering sources permit the grading of possibilities:

- (74) Ich kann eher Bäcker als Stellmacher werden.
I can rather baker than cartwright become.
I’d rather become a baker than a cartwright.

Maxl was wounded during the war against the Prussians. Given this, he comes closer to an ideal where everyone is good in whatever his craft may be, if he becomes a baker and not a cartwright.

Kann eher ... als expresses comparative possibility. In section two, the main motivation for introducing a clear-cut distinction between conversational backgrounds functioning as modal bases or as ordering sources, was the necessity to obtain notions of graded possibility.

In the following section, I want to discuss further arguments in favour of this bipartition.

7 Practical Inference

There is an obvious connection between my way of analyzing modals and what has been called “practical inferences”.¹⁷ A practical inference may have the following form:

I want to become mayor.
I will become mayor only if I go to the pub regularly.

Therefore:
I must go to the pub regularly.

Let us adapt this inference to the present framework. If w is any possible world, we would have:

In w , all I want is to become mayor.
In w , the relevant circumstances are such that I will become mayor only if I go to the pub regularly.

Therefore:
Considering the relevant circumstances and what I want, it is necessary in w that I go to the pub regularly.

The reader can easily check that this inference should be valid. To do this, we have to interpret some expressions in a certain way, namely: *Necessary* expresses human necessity. The phrase *the relevant circumstances* contributes a modal base f . f is that function from possible worlds into sets of propositions which assigns to any world the set of propositions which constitute the relevant circumstances in it. The phrase *what I want* contributes the ordering source g . g is that function from possible worlds into sets of propositions which assigns to any possible world the set of those propositions which constitute what I want in it. For the particular world w mentioned in the inference, $f(w)$ contains just one proposition, namely that I will become mayor only if I go to the pub regularly. And $g(w)$ contains nothing but the proposition that I will become mayor. The union of $f(w)$ and $g(w)$ is a consistent set of propositions. It can be proved that if this is so, then it is a human necessity in w with respect to f and g that I go to the pub regularly if, and only if, it follows from the union of $f(w)$ and $g(w)$ that I do so. It does indeed follow. Thus the inference is valid according to our definitions.

I should like to look at a more intricate example:

In w , all I want is two things, namely to become mayor
and
not to go to the pub regularly.
In w the relevant circumstances are such that I will become mayor only if I go to the pub regularly.

Therefore:	Considering the relevant circumstances and what I want,
Conclusion one:	it is necessary in w that I go to the pub regularly.
Conclusion two:	it is necessary in w that I don't go to the pub regularly.
Conclusion three:	it is possible in w that I don't go to the pub regularly and still will become mayor.
Conclusion four:	it is possible in w that I go to the pub regularly.
Conclusion five:	it is possible in w that I don't go to the pub regularly.

This is the horrible story of someone who wants something but rejects the necessary means leading to the fulfillment of her desires. Which conclusion can we draw in such a case? I think that the first three conclusions are faulty, but the last two are correct. The above analysis predicts this. Let us see why. The expressions *necessary*, *the circumstances* and *what I want* are interpreted as above. *Possible* expresses human possibility. This time, $g(w)$ contains exactly two propositions: That I will become mayor and that I don't go to the pub regularly. We may now reason as follows:

$\cap f(w)$ is the set of worlds which are accessible from w .

(a) For all worlds $v \in \cap f(w)$, we have:

If I don't go to the pub regularly in v , I won't become mayor in v .

Given the definition of human possibility, it follows immediately that conclusion three is false. Let us now consider the set $g(w)$. It induces a tripartition of the set $\cap f(w)$ of accessible worlds as follows:

A is the set of all those possible worlds v of $\cap f(w)$ such that I will become mayor in v .

B is the set of all those possible worlds v of $\cap f(w)$ such that I don't go to the pub regularly in v .

C is the set of all those possible worlds v of $\cap f(w)$ such that I won't become mayor but yet do go to the pub regularly in v .

The reader may verify that

(b) A, B and C are not empty, they are pairwise disjoint and
 $A \cup B \cup C = \cap f(w)$.

It is easy to check now that all of the following statements concerning the ordering relation $\leq_{g(w)}$ are true:

(c) For all v and $z \in \cap f(w)$:

If $v \in A$ and $z \in B$, then neither $v \leq_{g(w)} z$ nor $z \leq_{g(w)} v$.

(d) For all v and $z \in A$: $v \leq_{g(w)} z$.

(e) For all v and $z \in B$: $v \leq_{g(w)} z$.

(f) For all v and $z \in \cap f(w)$: If $z \in C$ and $v \in A \cup B$, then not $z \leq_{g(w)} v$.

It follows from (b), (c), (d) and (f), that there is a world v in $\cap f(w)$ such that for any world z in $\cap f(w)$ such that $z \leq_{g(w)} v$, I will become mayor in z .

Given (a), it follows that there is a world v in $\cap f(w)$ such that for any world z of $\cap f(w)$ such that $z \leq_{g(w)} v$, I go to the pub regularly in z .

This means that it is a human possibility in w with respect to f and g that I go to the pub regularly.

Thus, conclusion two is false and conclusion four is correct. An analogous argument would show that conclusion one is false and conclusion five is correct.

In a practical inference, facts have priorities over ideals. You can't give up facts in favour of an ideal. That's why conclusion three is false.

The analysis I proposed (1977, 1978) cannot cope with these more complicated examples in a straightforward way. I did not distinguish facts and ideals.¹⁸ For the second example, there would be false predictions since we would proceed as follows: We would not have two conversational backgrounds f and g , but just one, h . For any world w , $h(w) = f(w) \cup g(w)$. $h(w)$ is an inconsistent set of propositions. We would try to make the best out of this inconsistent set by looking at all its maximal consistent subsets. If a proposition follows from all of them, it would be necessary in w with respect to h . If it is compatible with one of them, it would be possible in w with respect to h . Unfortunately, there is a maximal consistent subset of $h(w)$ which contains all I want in w , namely that I will become mayor and that I don't go to the pub regularly. Thus, conclusion three should be correct under this interpretation of possibility. As it isn't, we have good reasons to prefer my new analysis to the old one. There is another reason. The new analysis offers a very natural way for treating certain kinds of conditionals. In Kratzer (1978, 1979) I was not able to say what happens, if an *if*-clause modifies an arbitrary modal. I had to give meaning rules for each modal separately. Doing this, I missed an obvious generalization.

In the following section, I will sketch how conditional modalities fit into the present framework.

8 Conditionals

I argued in Kratzer (1978, 1979) that many conditionals seem to involve modals in an explicit or implicit way. I want to talk about these conditionals in this section. They may have the following form:

(If.....), (then necessarily.....)
 (If.....), (then possibly.....)
 (If.....), (then probably.....)
 etc.

The second part of these constructions is a normal modalized sentence of the kind we have discussed so far. (Let us forget about the *then* in what follows). The first part is an *if*-clause. Its job is very easy: It makes sure that a hypothesis is added to the modal base required by the modal expression to follow. I would like to make this more precise:

Conditional modality

Consider an utterance of a sentence of the following form:

(if α), (then modal. . . .)

This utterance has two parts: the first part consists of the utterance of the *if*-clause, and the second part consists of the utterance of the *then*-clause. Suppose that the proposition p is expressed by the utterance of α .

The rule is now:

- (i) The first part of the utterance requires one, and only one, modal base and one, and only one, ordering source to be correct.¹⁹
- (ii) If f is the modal base and g the ordering source for the first part of the utterance, then f^+ is the modal base and g the ordering source for the second part of the utterance. f^+ is that function from possible worlds to sets of propositions, such that for any world w , $f^+(w) = f(w) \cup \{p\}$.

We obtain different kinds of conditionals by fixing the parameters f and g in different ways. I want to demonstrate this with a few examples.

For the following, consider utterances of sentences which have the following form:

(if α), (then necessarily β)

Suppose that p and q are the propositions expressed by α and β respectively, and that *necessarily* expresses human necessity. As our first example, let us look at material implication:

Material Implication:

A material implication is characterized by a *totally realistic modal base* f and an *empty ordering source* g . We have to prove that these requirements for f and g indeed give us material implication.

Sketch of a Proof:

Let w be any possible world.

We must show that q is a human necessity in w with respect to f^+ and g if, and only if, q is true or p is false in w .

Case one: Suppose that p is true in w .

Then $f(w) \cup \{p\} = f^+(w)$ is a consistent set of propositions.

Since $\cap f(w) = \{w\}$ and $f^+(w)$ is a consistent superset of $f(w)$, $\cap f^+(w) = \{w\}$ as well. It follows immediately, that in this case, q is a human necessity in w with respect to f^+ and g if, and only if, q is true in w .

Case two: Suppose that p is false in w . Then $f(w) \cup \{p\} = f^+(w)$ is an inconsistent set of propositions and $\cap f^+(w)$ is the empty set. Then it is vacuously true that q is a human necessity in w with respect to f^+ and g .

Our next example is strict implication:

Strict implication:

A strict implication is characterized by an *empty modal base* f and an *empty ordering source* g . Again, we have to prove that these requirements for f and g yield strict implication.

Sketch of a Proof:

Let w be any possible world.

We must show that q is a human necessity in w with respect to f^+ and g if, and only if, q is true in all worlds in which p is true. Since $g(w)$ is the empty set, we have:

For all worlds u and $v \in \cap f^+(w)$: $u \leq_{g(w)} v$.

Since $f^+(w) = f(w) \cup \{p\} = \{p\}$, this means that q is a human necessity in w with respect to f^+ and g if, and only if, q is true in all worlds of $\cap \{p\} = p$.

The most interesting kinds of conditionals are counterfactuals. They are the exact mirror images of material implications.

Counterfactuals

A counterfactual is characterized by an empty modal base f and a totally realistic ordering source g .

It follows from David Lewis' work mentioned above, that this analysis of counterfactuals is equivalent to the one I give in Kratzer (1981). I don't want to discuss counterfactuals in detail here. I do this in Kratzer (1981). The idea is this: All possible worlds in which the antecedent p is true, are ordered with respect to their being more or less near to what is actually the case in the world under consideration. "What is actually the case" is a vague concept. There are many ways of uniquely characterizing a world.

In formal terms: There are many functions g from W which assign to any world w of W a subset of the power set of W such that $\cap g(w) = \{w\}$.

Let us consider an example:

Two totally realistic conversational backgrounds g_1 and g_2 may differ in the following way: for some world w

$$\begin{aligned} g_1(w) &= \{r, s\} \\ g_2(w) &= \{r \cap s\} \end{aligned}$$

g_1 assigns to w a set which contains two propositions, the propositions r and s . g_2 assigns to w a set which contains one proposition, the conjunction (that is the intersection) of r and s .

If g_1 and g_2 function as ordering sources, such a difference may become important. $g_1(w)$ and $g_2(w)$ induce different orderings on the set of all possible worlds. Consider two worlds u and v such that r is true and s is false in u , and r and s are both false in v .

We have now: $v \leq_{g_2(w)} u$, but not $v \leq_{g_1(w)} u$.

I think that this vagueness about "what is the case" is responsible for the vagueness of counterfactuals. It is worth noticing that no such vagueness can arise for material implications where totally realistic conversational backgrounds function as modal bases.

As a last example, I would like to discuss a kind of conditional which has led to paradoxes in the past.²⁰

Deontic paradox

Consider utterances of the following sentences:

- (75) Jedem Menschen muß Gerechtigkeit widerfahren.
To every person must justice be given.

- (76) Wenn jemand ungerecht behandelt wurde, muß das Unrecht
 If someone unjustly treated was, must the injustice
 wieder gutgemacht werden.
 amended for be.
- (77) Wenn jemand ungerecht behandelt wurde, muß er mundtot gemacht
 If someone unjustly treated was, must be reduced to silence
 werden.
 be.

In traditional modal logic, sentences like this lead to problems. I think that these problems arise because of two reasons: On the one hand, conditional sentences like (76) or (77) are analyzed as modalised material implications. They would have the following logical form:

Necessarily ($\alpha \rightarrow \beta$)

On the other hand, the interpretation of the modal is based on nothing else but a simple accessibility relation.

In our case, the traditional analysis would look as follows: The proposition I express by my utterance of (75) would be true in a world w if, and only if, it is true in all worlds which are *morally accessible* from w , that justice is given to everyone. A world is morally accessible from a world w if, and only if, the moral ideals prevailing in w are all realized in it. The proposition I express by my utterance of (76) would be true in a world w if, and only if, for all worlds w^+ which are morally accessible from w , the following holds: If someone has been treated unjustly in w^+ , the injustice is amended for in w^+ . And the proposition I express by my utterance of (77) would be true in a world w if, and only if, for all worlds w^+ which are morally accessible from w , the following is true: If someone has been treated unjustly in w^+ , he is reduced to silence in w^+ .

What is paradoxical about all this is that, supposing that the proposition I expressed in uttering (75) is true in a world, the propositions I expressed in uttering (76) and (77) would both be vacuously true in this world. If there is no injustice in any morally accessible world, anything you like is true in all those morally accessible worlds where someone has been treated unjustly.

The analysis of conditionals which I proposed above, avoids this paradox: Assume that for my utterance of (75) and the first part of (76) and (77), the modal base f was empty²¹ and the ordering source g was determined by what is morally commanded. If f^+ is the modal base for the second part of (76) and (77), then for any world w , $f^+(w)$ contains nothing but the proposition that someone has been treated unjustly. Roughly speaking, the three propositions which I expressed in uttering (75), (76) and (77) would be true under the following conditions: The first proposition would be true in a world w if, and only if, justice is given to everyone in all those possible worlds which are closest to what is morally commanded in w . The second proposition would be true in a world w if, and only if, the injustice is amended for in all those possible worlds of $\cap f^+(w)$ which are closest to what is morally commanded in w . And the third proposition would be true in a world w if, and only if, the one who has been treated unjustly is

reduced to silence in all those worlds of $\cap f^+(w)$ which are closest to what is morally commanded in w .

Under this analysis, it is not excluded, for example, that the first two propositions are true, but the third is false in a world. For us, a world where injustice is amended for, is not ideal, since there is no injustice in an ideal world. But it may still be closer to what is ideal than any world where people who suffered injustice are reduced to silence.

Whether an analysis of conditionals is appropriate is usually assessed by examining their predicted behaviour in certain kinds of inferences like “transitivity”, “strengthening the antecedent” or “contraposition”.²² The analysis I am proposing here predicts that these three inference patterns can’t be expected to be valid for all those types of conditionals which involve a non-empty ordering source.

In the literature, the failure of these inference patterns is often discussed in connection with deontic conditionals, probability conditionals and counterfactuals. If we analyse these conditionals in the way outlined above, their specific behaviour in inferences is an automatic consequence of the analysis.

Conclusion

A person who has a complete grasp of the modal system of German has certain abilities. It was the aim of this paper to say exactly what these abilities are. As a result we have

- (i) The ability of categorizing conversational backgrounds according to the requirements imposed by the vocabulary.
- (ii) The ability of drawing inferences of various strength involving two conversational backgrounds: a modal base and an ordering source.

Actually, it is a simplification to assume that there is never more than one ordering source involved in modal reasoning. Suppose I draw conclusions which involve established facts, the Encyclopedia Britannica, the local newspaper and the gossip I picked up at the corner. And suppose further that the established facts have priority over the Encyclopedia Britannica, the Encyclopedia Britannica has priority over the local newspaper and the local newspaper has priority over the gossip I picked up at the corner. How do we reason in such a case?

I think that the semantics of modals which I have presented so far can be extended in a straightforward way to handle these cases. The interpretation of a modal expression would have to depend on a modal base f and a finite sequence of ordering sources g_1, \dots, g_n . For any world w , $g_1(w)$ would induce an ordering on $\cap f(w)$ in the usual way. $g_2(w)$ would – if necessary – refine this ordering in undoing the “ties” left by its predecessor and so on for every successive member in the sequence.

Probably, we can’t assume that the different ordering sources form a natural sequence with respect to having priority over each other. There may be ordering sources which have equal priority. This all sounds as if it were the beginning of my next paper.

Notes

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- 1 Bech (1949), Calbert (1979), Debrunner (1951), Grabski (1974), Raynaud (1974), Reinwein (1977), Welke (1965).
- 2 Vater (1975).
- 3 Strictly speaking, rules like this would have to apply on a level of logical form, where all modal operators are sentential operators.
- 4 See Pinkal (1977, 1979).
- 5 The term is inspired by what Franziska Raynaud calls "source" in French.
- 6 Lewis, personal communication. See also Burgess (1979).
- 7 A proposition p is compatible with a proposition q if, and only if, p is compatible with $\{q\}$. Likewise: A proposition p follows from a proposition q if, and only if, p follows from $\{q\}$.
- 8 See Horgan (1977), Kratzer (1977) or Lewis (1979) for a further illustration of this point.
- 9 Gustav Deggau (1907).
- 10 Ewald Lang proposed an explanation along these lines (personal communication).
- 11 See Putnam (1975).
- 12 See for example Karttunen (1972) or Brünner and Redder (1979).
- 13 The inspiration for these examples came from Gerald Gazdar.
- 14 See Vater (1975).
- 15 I neglect the epistemic use of "*kann*" in what follows, which is, of course, another difference. "*Darf*" can never have an epistemic interpretation.
- 16 See Lewis (1979).
- 17 See Anscombe (1957), Brünner (1979) or von Wright (1963, 1972).
- 18 Franziska Raynaud raised an objection of this kind, personal communication.
- 19 Instead of the uniqueness condition, a Pinkal solution would be preferable here as well. There is quite a bit of vagueness around conditionals.
- 20 Hansson (1969), van Fraassen (1972), Lewis (1973) give a detailed discussion of the problem.
- 21 This assumption is not essential.
- 22 See for example Lewis (1973), Kratzer (1979).

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