Accurate Updating for the Risk-Sensitive

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Abstract

Philosophers have recently attempted to justify particular belief revision procedures by arguing that they are the optimal means towards the epistemic end of accurate credences. These attempts, however, presuppose that means should be evaluated according to classical expected utility theory; and there is a long tradition maintaining that expected utility theory is too restrictive as a theory of means-end rationality, ruling out too many natural ways of taking risk into account. In this paper, we investigate what belief-revision procedures are supported by accuracy-theoretic considerations once we depart from expected utility theory to allow agents to be risk-sensitive. We argue that, if accuracy-theoretic considerations tell risk-sensitive agents anything about belief-revision, they tell them the same thing they tell risk-neutral agents: they should conditionalize.

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1 Introduction

Philosophers have recently attempted to justify particular belief revision procedures by arguing that they are the optimal means towards the epistemic end of accurate credences.[1] These attempts, however, presuppose that means should be evaluated according to classical expected utility theory;[2] and there is a long tradition maintaining that expected utility theory is too restrictive as a theory of means-end rationality, ruling out too many natural ways of taking risk into account.[3] In this paper, we investigate what belief revision procedures are supported by accuracy-theoretic considerations once we depart from expected utility theory to allow agents to be risk-sensitive. We argue that, if accuracy-theoretic considerations tell risk-sensitive agents anything about belief-revision, they tell them the same thing they tell risk-neutral agents: they should conditionalize, that is, they should respond to evidence E (in which their prior credence wasn't 0) by adopting as their new credence in any proposition X their prior credence in X conditional on E, $c(X|E) = \frac{c(X \wedge E)}{c(E)}$.

We specifically focus on Lara Buchak's ([2013]) Risk-Weighted Expected Utility Theory. Section 2 describes this theory and discusses how to think of accuracy when working with it. We then distinguish two broad ways of using accuracy considerations to evaluate update procedures. On the 'local' approach, most explicit in (Leitgeb and Pettigrew [2010]), we determine the optimal response to some particular evidence E by comparing different updated credences for accuracy across the worlds consistent with E. Section 3 argues that this approach recommends that risk-sensitive agents conditionalize. On a more 'global' approach, employed in (Greaves and Wallace [2006]), we compare the accuracy of complete update strategies that specify updated credences for each of the various pieces of evidence the agent might receive. We will see that risk-sensitive agents should often regard as optimal a global update strategy that diverges from conditionalization (section 4.1). However, we argue that, in the risk-sensitive setting, this is no reason to think that they should actually update in this non-standard way (section 4.2). We conclude that accuracy considerations either tell risk-sensitive agents nothing about how to update their beliefs, or else tell them to conditionalize.

Throughout the paper, we will make some controversial assumptions. For example, we will assume that Buchak's theory is an attractive theory of risk-sensitivity, and that considerations of (risk-weighted) expected accuracy are probative when it comes to determining the norms of belief revision.[4] Some of

[1] See (Greaves and Wallace [2006]; Leitgeb and Pettigrew [2010]; Easwaran [2013]; Pettigrew [2016], Part IV; Briggs and Pettigrew [forthcoming]; Gallow [2019]).

[2] The reliance on expected utility theory is clear in most cases. An exception is the argument in (Briggs and Pettigrew [forthcoming]) which initially seems to use only dominance reasoning; however, for reasons explained in footnote 9, even this argument implicitly relies on expected utility theory.

[3] See (Allais [1953]; Machina [1982]) for classic and (Buchak [2013]; Bradley and Stefansson [2019]) for recent discussion.

[4] For worries about accuracy-theoretic approaches to belief revision, see (Greaves [2013]; Berker [2013]; Carr [2017]; Pettigrew [2016], ch. 15). For criticism of Buchak's theory, see

the consequences we draw out from these assumptions may ultimately reduce their plausibility. However, we will not call them into question on this basis – our project, which is interesting and complex enough as it stands, is to see where they lead.

2 Preliminary material

We begin with a brief introduction to our two ingredients: risk weighted expected utility theory and accuracy. In particular, we discuss how accuracy considerations apply to the risk-sensitive; as we'll see, new accuracy measures are needed.

2.1 Risk Weighted Expected Utility Theory

In standard expected utility theory, agents face a choice between acts, which are functions from states of the world to outcomes. They assign credences c to the states, representing how likely they think each one is to obtain; and they assign utilities U to the outcomes, representing how good they take each one to be. We can then, somewhat non-standardly, define the expected utility of an act as follows:[5]

> Expected utility: Suppose an act, A, leads to outcomes $o_1, \ldots, o_n$ in states $s_1, \ldots, s_n$ with $U(o_1) \leqslant \ldots \leqslant U(o_n)$. For notational purposes, we introduce a further 'outcome' o_0 with $U(o_0) = 0$. Then the expected utility of A is:

$$\begin{aligned}\mathrm{Exp}_c\, U(A) &= \sum_i \Big(\big(\sum_{j \geqslant i} c(s_j) \big) \times \big(U(o_i) - U(o_{i-1}) \big) \Big) \\ &= U(o_1) \\ &\quad + (c(s_2) + \ldots + c(s_n)) \cdot (U(o_2) - U(o_1)) \\ &\quad \ldots \\ &\quad + \qquad\qquad c(s_n) \cdot (U(o_n) - U(o_{n-1}))\end{aligned}$$

Intuitively, the expected utility is here calculated by first taking the utility of the worst-case scenario; adding the improvement over the worst-case scenario secured in the second-worst-case scenario, weighted by the probability of securing at least that improvement; adding the improvement over the second-worst-case scenario secured in the third-worst-case scenario, weighted by the probability of securing at least that improvement; and continuing like this until all the possible improvements have been taken into account.

Risk weighted expected utility introduces an additional component: the agent's risk profile r, representing her tendency towards worst-case-scenario style reasoning. Formally, r is a continuous, strictly increasing function from $[0, 1]$

(Pettigrew [2015]; Briggs [2015]; Thoma and Weisberg [2017]; Joyce [2017]; Bradley and Stefansson [2019]).

[5] The standard definition is $\mathrm{Exp}_c\, U(A) = \sum_i \Big(c(s_i) \times U(o_i) \Big)$. This is equivalent, but the more complicated definition clarifies the relation to risk-weighted expected utilities.

to $[0,1]$ with $r(0) = 0$ and $r(1) = 1$. Risk-weighted expected utilities are then defined as follows:

> Risk-weighted expected utility: Suppose an act, A, leads to outcomes $o_1, \ldots, o_n$ in states $s_1, \ldots, s_n$ with $U(o_1) \leqslant \ldots \leqslant U(o_n)$. Let $U(o_0) = 0$, by stipulation. Then the risk-weighted expected utility of A is:

$$\begin{aligned} \mathrm{RExp}_c^r\, U(A) &= \sum_i \Big(r\big(\sum_{j \geqslant i} c(s_j)\big) \times \big(U(o_i) - U(o_{i-1})\big)\Big) \\ &= U(o_1) \\ &\quad + r(c(s_2) + c(s_3) + \ldots + c(s_n)) \times (U(o_2) - U(o_1)) \\ &\quad + \qquad r(c(s_3) + \ldots + c(s_n)) \times (U(o_3) - U(o_2)) \\ &\qquad \ldots \\ &\quad + \qquad\qquad r(c(s_n)) \times (U(o_n) - U(o_{n-1})) \end{aligned}$$

Intuitively, r has an effect by modifying the weight given to possible improvements depending on how likely they are to come about. For someone with the risk-neutral profile $r(x) = x$, the theory collapses into expected utility theory. But if, for example, $r(x) < x$, the risk-weighted expected utility of an action that is guaranteed to secure 5 utiles, and thus has risk-weighted utility of 5, exceeds that of taking a gamble with a $1/2$ chance of securing 10 utiles and a $1/2$ chance of securing none (which has risk-weighted expected utility $0 + r(1/2) \cdot (10 - 0) < 5$). Including r thus allows for a novel form of risk-avoidance.

Buchak ([2013]) argues at length that at least some risk-sensitivity, as spelled out by risk-weighted expected utility theory with a non-neutral r, is rationally permissible. Here, we simply assume she is right, and investigate the consequences for accuracy-theoretic approaches to belief revision.

2.2 Accuracy for the risk-sensitive

Accuracy-theoretic approaches to belief revision attempt to justify update procedures by the anticipated 'accuracy', or closeness to the truth, of the beliefs they result in. An accuracy measure associates with each $y \in [0,1]$ and $v \in \{0,1\}$ a real number $\mathcal{A}(y, v)$, representing how accurate a credence of y in a proposition X is when X has truth value v.[6] One prominent such measure is the Brier score:[7]

$$\mathcal{A}(y, v) = -(y - v)^2$$

We can think of the accuracy of a credence in a world as the 'epistemic utility' achieved by adopting it there. Assuming expected utility theory, accuracy-theoretic approaches then defend a particular update procedure (such as conditionalization) by showing that adopting the beliefs it recommends has higher expected accuracy than any alternative.

[6] We associate 0 with false, and 1 with true.

[7] To simplify the interaction with Buchak's theory, we consider accuracy rather than the more standard inaccuracy.

When allowing for risk-sensitivity, we should replace expected with risk-weighted expected accuracy – at least if, following Buchak, we think of risk-sensitivity not as a matter of how valuable the outcomes are, but as a matter of how their value determines the choiceworthiness of the means.[8] But this means that accuracy measures like the Brier Score are no longer suitable. The problem is that these measures satisfy

> Propriety: $\mathcal{A}$ is (strictly) proper if for all probabilistic c, $\text{Exp}_c\, \mathcal{A}(c(X)) > \text{Exp}_c\, \mathcal{A}(y)$ when $y \neq c(X)$.

And if we use proper accuracy measures, probabilistic agents with credences that are non-extremal and not $1/2$ will always risk-weightedly expect some other credences to be more accurate than their own.[9] This violates a plausible and widely accepted constraint often used to motivate propriety, namely that probabilistic agents should be immodest, preferring (as far as accuracy is concerned) their current beliefs to any specific alternative.[10] Consequently, it makes trouble for the accuracy-theoretic approach to belief revision; for if an agent risk-weightedly expects that other credences are more accurate than her own, this approach will tell her to revise their beliefs to those new ones even when no new evidence is encountered. This process, moreover, will either repeat indefinitely or will end with the agent reaching $1/2$.

We thus need accuracy measures that allow probabilistically coherent but risk-sensitive agents to be immodest: to assign a higher risk-weighted expected accuracy to their own credences than to any particular alternative. More precisely, we want measures that satisfy

> r-propriety: $\mathcal{A}$ is (strictly) r-proper if for all probabilistic c, $\text{RExp}_c^r\, \mathcal{A}(c(X)) > \text{RExp}_c^r\, \mathcal{A}(y)$ when $y \neq c(X)$.

where

$$\text{RExp}_c^r\, \mathcal{A}(y) = \begin{cases} \mathcal{A}(y,0) + r(c(X)) \times (\mathcal{A}(y,1) - \mathcal{A}(y,0)) & \text{if } \mathcal{A}(y,0) \leqslant \mathcal{A}(y,1) \\ \mathcal{A}(y,1) + r(c(\neg X)) \times (\mathcal{A}(y,0) - \mathcal{A}(y,1)) & \text{if } \mathcal{A}(y,1) \leqslant \mathcal{A}(y,0) \end{cases}$$

The results in section 3 require only that accuracy is measured by some measure with this feature.

In section 4.1, we will need to assume a little more. Two relatively uncontroversial assumptions are:[11]

[8] See especially (Buchak [2013], p.34-36). See also (Campbell-Moore and Salow [forthcoming]) for discussion (and defence) of the move from expected to risk-weighted expected accuracy.

[9] This follows from a slightly fixed up version of the argument from (Pettigrew [2016], Section 16.4) which shows that what maximizes $\text{RExp}_c^r \mathcal{A}(y)$ is either $r(c(X))$ if this is $> 1/2$; $1 - r(1 - c(X))$ if this is $< 1/2$; and otherwise $1/2$ itself.
This is also why the argument in (Briggs and Pettigrew [forthcoming]) presupposes risk-neutrality despite using only dominance as its decision rule: the argument requires strictly proper accuracy measures, which are plausible for risk-neutral, but not risk-sensitive, agents.

[10] See (Oddie [1997]; Greaves and Wallace [2006]; Joyce [2009]).

[11] See, for example, (Joyce [2009]).

Truth-Directedness: $\mathcal{A}$ is truth directed if $\mathcal{A}(y', v) > \mathcal{A}(y, v)$ whenever either $v \leqslant y' < y$ or $y < y' \leqslant v$.

0/1 Symmetry: $\mathcal{A}$ is 0/1 symmetric if $\mathcal{A}(y, 1) = \mathcal{A}(1 - y, 0)$.

These assumptions are primarily needed to tell us about how possible outcomes are ordered, information we need to calculate risk-weighted expectations. In addition, we will use the fact that r-propriety can be strengthened to:

Monotone r-propriety: $\mathcal{A}$ is monotone (strictly) r-proper if for all probabilistic c, $\mathrm{RExp}_c^r\ \mathcal{A}(y') > \mathrm{RExp}_c^r\ \mathcal{A}(y)$ whenever either $c(X) \leqslant y' < y$ or $y < y' \leqslant c(X)$.

This is an extremely natural generalisation of r-propriety. Furthermore, it follows from r-propriety (in fact, only a weak form of r-propriety is required), truth-directedness and 0/1-symmetry, by an extension of the argument from (Campbell-Moore and Levinstein [forthcoming]).[12]

It's worth noting explicitly that there are, in fact, accuracy measures that have all these features.[13] For example, for the risk-profile $r(x) = x^2$, the following modification of the Brier Score is truth-directed, 0/1 symmetric, and monotone r-proper:[14]

$$\mathsf{AltBS}(y, v) := \frac{-(v - y)^2}{\max\{y, 1 - y\}}$$

We should emphasize that the measures we use only evaluate the accuracy of a credence in a particular proposition, X, not of an entire credence distribution. In fact, it is unclear that there are attractive measures of the accuracy of entire distributions which obey constraints like r-propriety.[15] Fortunately, the most prominent accuracy-theoretic approaches to belief-revision require only such 'pointwise' measures; adapting other approaches will have to wait until risk-sensitive accuracy measures are better understood.[16]

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[12]To extend the argument, use the formulation of risk-weighed expectations exploited in section 4.1 to apply the argument in the case where y and y' generate the same ordering of states. To show it holds when they generate different orderings, compare both to $1/2$, which generates both orderings.

[13](Campbell-Moore and Levinstein [unpublished]) shows that these exist for any risk profile.

[14]To see that it is (monotone) r-proper, differentiate $\mathrm{RExp}_c^r\ \mathsf{AltBS}(y)$ and note this is > 0 for $y < c(X)$ and < 0 for $y > c(X)$.

[15]See (Campbell-Moore and Levinstein [unpublished]).

[16](Leitgeb and Pettigrew [2010]) and (Leinvstein [2012]) explore an approach where the agent selects the expected accuracy maximizing probability distribution amongst those meeting some additional evidential constraint like $p(E) = 1$. If we only consider a target proposition, X, then typically her prior credence in X will be in the range of options, so (r-)propriety will mean she evaluates her prior credence as optimal.

(Briggs and Pettigrew [forthcoming]) argues that pairs of priors and update strategies are accuracy-dominated if they are not conditionalization pairs. If we only look at the accuracy of credences in X, we have no hope of motivating a particular update rule because the information – for example, whether it is the result of conditionalization – isn't available to the dominance argument.

3 The Local Approach

The first accuracy-theoretic approach to belief revision maintains that agents should adopt as their new credence in X the value that maximizes (risk-weighted) expected accuracy, as calculated by their prior credences over the possibilities consistent with the evidence that has been gathered. (Leitgeb and Pettigrew [2010]) shows that, in the risk-neutral case, this means that agents should conditionalize; we will argue that this also holds for risk-sensitive agent.

There are two pictures we can use to motivate this approach. The first is a picture of belief revision on which the immediate, arational effect of learning E is that all $\neg E$ possibilities drop out from the agent's credal state. It is only once this has happened that the rational aspect of belief-revision can take place. The agent is left with her previous credences in the various possibilities compatible with E, which (remaining – as yet – unmodified) fail to add up to 1 and hence fail to constitute a probability distribution. She then uses these (non-probabilistic) credences to decide what credences to adopt, by maximizing (risk-weighted) expected accuracy. This means that she will adopt the credences that maximize (risk-weighted) expected accuracy, as calculated using her prior credences over the worlds consistent with E.

This is not, of course, an accurate representation of how agents actually revise their beliefs. For example, there probably isn't really a time when agents are in this intermediate state of having learned the evidence, but not yet having updated any of their credences. But perhaps it is a reasonable rational reconstruction of how updating, if it were a temporally extended and deliberate process, might go; and that would be enough.

On the second picture, by contrast, we do not consider an agent who has already learned E; instead, we consider an agent who is thinking about the possibility that she might learn E, and deciding how to respond if she does. The options that such an agent is evaluating ('if you learn E, adopt credence y') are partial or conditional, like the plan to take an umbrella if the weather forecast predicts rain, or to take a bet at such-and-such odds if you are offered it. Unlike ordinary actions, such partial plans correspond only to partial functions from states to outcomes; their unrestricted expected utility is thus undefined. But, intuitively, we can still compare the advisability of various partial plans (defined over the same states), by comparing their expected utility as restricted to the states in which they apply. In our case, this means calculating the (risk-weighted) expected accuracy of different new credences in the possibilities in which the agent learns E; hence, in the possibilities in which E is true.[17]

The two pictures, while quite different, motivate the same calculation: we need to determine the risk-weighted expected accuracy of different credences

[17] This move between 'possibilities in which the agent learns E' and 'possibilities in which E is true' is justified when the agent is guaranteed to receive as her evidence the true member of some fixed partition. (Bronfman [2014]; Schoenfield [2017]; Das [2019]) all argue that this doesn't always hold, and that conditionalization isn't supported by accuracy arguments in cases where it doesn't, even in the risk-neutral setting. Analogous worries clearly arise here; for simplicity, we set them aside.

in X over the set E, which doesn't include all the possibilities. To do this, we need to decide how to understand risk-weighted expectations when an agent's credences over the relevant states add up only to $c(E) < 1$. A natural first pass, analogous to the one in (Leitgeb and Pettigrew [2010]), is as follows. Suppose that E consists of exactly two states, s_{bad} and s_{good}; and suppose that A leads to outcomes o_{bad} and o_{good} in those states respectively, with $U(o_{\text{bad}}) \leqslant U(o_{\text{good}})$. Since $c(E) = c(s_{\text{bad}}) + c(s_{\text{good}})$, simply applying the original formula gets us:

$$\text{NaiveRExp}_c^r\, U(A) \restriction_E = r(c(E)) \times U(o_{\text{bad}}) + r(c(s_{\text{good}})) \times (U(o_{\text{good}}) - U(o_{\text{bad}})).$$

We think that, if r has certain further properties, this is exactly right. Let us say that r is multiplicative if it satisfies $r(x \times y) = r(x) \times r(y)$ – as does $r(x) = x^2$ and all other risk-profiles of the form $r(x) = x^k$, which provide virtually all the examples in the literature.[18] We think that NaiveRExp makes exactly the right predictions for agents with multiplicative risk-profiles. And, as we'll see later, agents of this kind need to conditionalize to maximize NaiveRExp.

For agents with non-multiplicative risk-profiles, however, maximizing NaiveRExp will not lead to conditionalization. But we think that NaiveRExp has independently implausible results for such agents, because it misinterprets how r represents their attitude to risk. Moreover, we think that, once the formula is modified to avoid this problem, we can get the stronger result that all agents, regardless of risk-profile, should conditionalize.

Consider an agent with the non-multiplicative risk-profile represented in figure 1.[19] Such an agent is neither straight-out risk-avoidant, nor straight-out

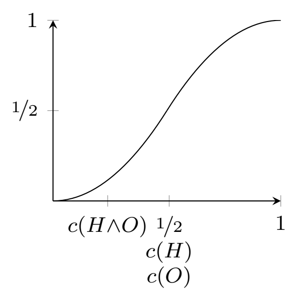

Figure 1: A risk function that is non-multiplicative and leads to problems with NaiveRExp.

risk-seeking. Rather, she seeks out small risks (she gives additional weight to

[18] (Buchak [2013]) mostly discusses $r(x) = x^2$ and $r(x) = \sqrt{x} = x^{1/2}$, while (Thoma and Weisberg [2017]) discusses a range of profiles of this form. By contrast, the literature on the closely related Cumulative Prospect Theory (Tversky and Wakker [1995]; Wakker [2010]), which usually aims to model actual rather than rational preferences, often uses non-multiplicative risk-profiles.

[19] Explicitly, r is given by: $r(x) = \begin{cases} 2x^2 & x \leqslant 1/2 \\ 1 - 2(1-x)^2 & x \geqslant 1/2 \end{cases}$. There are also non-multiplicative risk-profiles that look more uniform, such as $r(x) = \frac{x^2+x}{2}$; combining these with NaiveRExp gives rise to the same problem, but they don't illustrate it quite as intuitively.

likely improvements) but avoids large ones (she tempers the weight given to unlikely improvements). She has no strong feelings either way about medium-sized risks (the weight she gives to an improvement that has even odds of coming about matches its probability). We'll argue that applying NaiveRExp to this agent has implausible results.

To bring out the problem in an independent way, we will focus on practical rationality, considering how NaiveRExp has our agent evaluate the partial plans of taking or declining a bet, if it is offered. Suppose that our agent knows that a fair coin will be tossed tomorrow; let H be the proposition that it will land heads. Our agent also thinks that her erratic friend might, before the coin is tossed, offer her a bet that pays 2 utils if H, and loses 1 util if $\neg H$. Let O be the proposition that her friend offers the bet. Our agent thus finds herself comparing two plans for what to do if O: the plan to take the bet, and the plan to decline it. We will assume that $c(H) = c(O) = 1/2$; we also assume that, since the agent knows that the coin is fair and that her friend has no supernatural powers for predicting the outcomes of fair coins, she does not consider O to be evidentially relevant to H. Then, if we use NaiveRExp to calculate the expected utility, restricted to O, of taking the bet, we get:

$$\begin{aligned}\text{NaiveRExp}_c^r U(\text{Take})\restriction_O &= r(c(O)) \times (-1) + r(c(H \wedge O)) \times (2 - -1) \\ &= r(1/2) \times (-1) + r(1/2 \times 1/2) \times 3 \approx -0.125\end{aligned}$$

So this formula has her evaluating the plan of taking the bet, if she is offered it, as worse than that of declining (which guarantees a utility of 0).

This is, already, counterintuitive. Our agent considers a bet on H to have the same chance of winning as it has of losing, regardless of whether O. Her risk-profile suggests that she is indifferent to the kind of risk involved in such even bets, neither seeking nor avoiding it. But then it's hard to see why she should think negatively of this bet which, after all, pays more when she wins than it costs when she loses.

Things, however, get weirder still. For suppose we change the case slightly, merely by making our agent more confident that her friend will offer the bet, so that $c(O) = 0.9$. Then

$$\text{NaiveRExp}_c^r U(\text{Take})\restriction_O = r(0.9) \times (-1) + r(1/2 \times 0.9) \times 3 \approx 1/2.$$

By changing the likelihood of being offered the bet, and without altering the fact that our agent thinks that heads and tails are equally likely and that the outcome is independent of whether she is offered the bet, we have reversed her preference between the plan of taking the bet, if she is offered it, and the plan of declining it. This is a bad result. How likely a plan is to be called upon might affect whether it is worth making; but, if one is already in the business of making a plan for that possibility, it should not affect which plan one makes.

The problems have a common source. For note that, unless $c(O)$ is very high, $c(H \wedge O)$ is significantly below $1/2$. This means that r, the agent's risk-profile, treats it as a low probability outcome, and thus tempers the weight given to it in $\text{NaiveRExp}_c^r U(\text{Take})$. But, intuitively, that is a mistake. For while the absolute probability of winning the bet in the O states is, indeed, relatively low, winning is just as likely as losing, which is the only other relevant outcome. So the relative probability of winning is moderate; and so the agent's risk-profile (which neither boosts nor tempers moderate risks) should neither boost nor temper the weight it receives.

Analogous problems cannot arise with multiplicative risk-profiles. This is because, when H and O are independent and r is multiplicative, we have

$$\begin{aligned}&\text{NaiveRExp}^r_c\, U(\text{Take})\restriction_O \\ &\quad = r(c(O)) \times U(\text{lose}) + r(c(H \wedge O)) \times (U(\text{win}) - U(\text{lose})) \\ &\quad = r(c(O)) \times \Big(U(\text{lose}) + r(c(H)) \times (U(\text{win}) - U(\text{lose}))\Big)\end{aligned}$$

More generally, multiplicative risk profiles are only sensitive to the relative probabilities of the various outcomes. So multiplying the absolute probability of those outcomes without altering their relative probabilities, such as modifying the probability of the independent proposition that the bet is offered, cannot affect whether the bet is desirable or not.[20]

We can fix this problem NaiveRExp has with non-multiplicative profiles by modifying the credences before applying r, aligning absolute and relative probabilities. The standard implementation is renormalization, dividing all credences by $c(E)$. Since $r(c(E)/c(E)) = r(1) = 1$, doing this with NaiveRExp yields

$$\text{NormRExp}^r_c\, U(A)\restriction_E = U(o_{\text{bad}}) + r\left(\frac{c(s_{\text{good}})}{c(E)}\right) \times (U(o_{\text{good}}) - U(o_{\text{bad}}))$$

Unlike NaiveRExp, this formula has the agent above approve of the plan of taking the bet if offered it even if she is not particularly likely to be offered it; more generally, it makes her evaluations of the partial plan to take a bet on a propositions like H independent of the probability of being offered that bet. But it's also easy to see that, when r is multiplicative, $\text{NaiveRExp}^r_c\, U(A)\restriction_E = r(c(E)) \times \text{NormRExp}^r_c\, U(A)\restriction_E$, meaning that the two agree on the relative merits of all plans restricted to E; so NormRExp doesn't depart from NaiveRExp more than it must to solve our problem.

What predictions does this make in the epistemic case? We can think of the options as simply the various $y \in [0, 1]$ the agent could adopt as her credence in the relevant proposition X. The two relevant states, since they need to satisfy E,

[20] Multiplicativity is not only sufficient for this, but also necessary. For if r isn't multiplicative, we can find values for $c(H)$ and $c(O)$ such that $r\big(c(H) \times c(O)\big) \neq r(c(H)) \times r(c(O))$. So we can choose values for $U(\text{win})$ and $U(\text{lose})$ such that

$$\begin{aligned}r(c(O)) \times U(\text{lose}) + r\big(c(H) \times c(O)\big) \times (U(\text{win}) - U(\text{lose})) > 0 \\ \text{and } r(c(O)) \times U(\text{lose}) + r(c(H)) \times r(c(O)) \times (U(\text{win}) - U(\text{lose})) < 0\end{aligned}$$

or visa versa. But then $\text{NaiveRExp}^r_U\, c(\text{Take})\restriction_O > 0$ at this $c(O)$; but if we increase $c(O)$ to 1 (or close to 1), we would reduce this to < 0 (or visa versa). So whether taking is preferable to declining will depend on $c(O)$.

are simply $E \wedge X$ and $E \wedge \neg X$. The risk weighted expected accuracy, as restricted to E, is thus

$$\begin{aligned}&\text{NormRExp}^r_c\,\mathcal{A}(y)\restriction_E\\&\quad=\begin{cases}\mathcal{A}(y,0)+r(\frac{c(E\wedge X)}{c(E)})\times(\mathcal{A}(y,1)-\mathcal{A}(y,0)) & \text{if } \mathcal{A}(y,0)\leqslant\mathcal{A}(y,1)\\ \mathcal{A}(y,1)+r(\frac{c(E\wedge\neg X)}{c(E)})\times(\mathcal{A}(y,0)-\mathcal{A}(y,1)) & \text{if } \mathcal{A}(y,1)\leqslant\mathcal{A}(y,0)\end{cases}\end{aligned}$$

It's easy to see that $\text{NormRExp}^r_c\,\mathcal{A}(y)\restriction_E = \text{RExp}^r_{c(\cdot|E)}\,\mathcal{A}(y)$, so $\mathcal{A}$'s r-propriety guarantees that it is uniquely maximized at $y = c(X|E)$. So if this is how we do the calculations, we get that the credence in X which the agent should adopt in response to E is exactly what was previously her credence in X conditional on E. In other words, the agent should conditionalize. (Since NaiveRExp and NormRExp agree when r is multiplicative, this also establishes that, as we claimed earlier, NaiveRExp also requires agents with multiplicative risk-profiles to conditionalize.)

However, one might object to the use of renormalized probabilities in the definition of NormRExp. To require that agents conditionalize just is to require that they renormalize their credences after the possibilities inconsistent with their evidence have been eliminated. If we build this renormalizing process into the calculation, it is not surprising that conditionalization will come out.

It's thus worth noting another way to solve the problem we identified for NaiveRExp: instead of modifying the probabilities, we can rescale the risk-function to make it sensitive to relative probabilities instead of absolute ones. More precisely, given a risk-function r, we can define $r_z(x) := r(z) \times r(x/z)$ as the risk-function relevant to calculating restricted expected utility, where the restriction itself has probability z. As should be clear from comparing figures 1 and 2, r_z scales r from the interval $[0, 1]$ to the smaller interval $[0, z]$. We can

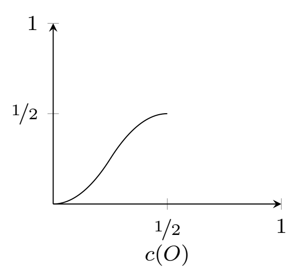

Figure 2: The rescaled risk function $r_{1/2}(x)$

then replace r with $r_{c(E)}$ in NaiveRExp; this yields the following formula for determining the risk weighted expected utility of doing A over the E states:

$$\begin{aligned}\text{ScaledRExp}^r_c\,U(A)\restriction_E &= r_{c(E)}(c(E))\times U(o_{\text{bad}})\\&\quad+r_{c(E)}(c(s_{\text{good}}))\times(U(o_{\text{good}})-U(o_{\text{bad}})).\end{aligned}$$

It's easy to see that $\text{ScaledRExp}^r_c\,U(A)\restriction_E = r(c(E)) \times \text{NormRExp}^r_c\,U(A)\restriction_E$. It follows that ScaledRExp is just as effective at solving NaiveRExp's problem with non-multiplicative profiles; and also that ScaledRExp, like NormRExp, requires agents with any risk profile to conditionalize. Yet ScaledRExp doesn't modify the probabilities used in NaiveRExp, and is thus arguably more neutral between different update rules.

But does ScaledRExp really avoid the problem we raised for NormRExp, of building in some form of renormalization and hence of conditionalization?[21] We're not sure. But it's worth being clear about the dialectic here. NaiveRExp is the most obvious suggestion for how to calculate restricted risk-weighted expectations. And we have seen no reason to think it makes bad predictions for agents with multiplicative risk-profiles, for whom it supports conditionalization. However, it does run into problems when applied to agents with non-multiplicative profiles. Two natural modifications of NaiveRExp that avoid these problems, while agreeing with NaiveRExp in the cases where we've seen no reason to doubt it, are ScaledRExp and NormRExp; both of these support conditionalization regardless of the risk profile. Perhaps there are other natural modifications to NaiveRExp that also solve the problem we identified, and which support alternative update rules. But it's not obvious that there are or what they look like; and so the onus is on the opponent of conditionalization to propose one.[22] We thus conclude that this first, local way of using accuracy-considerations to determine how one should update, gives us strong *prima facie* reason to believe that even risk-sensitive agents are required to conditionalize.[23]

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[21]Thanks to an anonymous referee for pushing us to address this worry.

[22]One option would be to mimic NormRExp and propose AltRExp$^r_c U(A)\upharpoonright_E = r(c^*(E)) \times U(o_{\text{bad}}) + r(c^*(s_{\text{good}})) \times (U(o_{\text{good}}) - U(o_{\text{bad}}))$ where c^* is the credence distribution recommended by some alternative update procedure for agents with prior c and risk-profile r who receive evidence E. Provided that c^* is probabilisitic with $c^*(E) = 1$, evaluating new credences with this formula will support the alternative update rule for the same reason that evaluating them with NormRExp supported conditionalization. However, for a wide range of alternative update procedures, AltRExp will be either overkill, departing from NaiveRExp even for the non-multiplicative risk-profiles, or else fail to solve the problem faced by NaiveRExp, by failing to makes someone's evaluations of the partial plan to take a bet on a propositions like H independent of the probability of being offered that bet.

[23](Gallow [2019]) offers a third picture to motivate a calculation which, in the risk-neutral setting, is very similar to that in (Leitgeb and Pettigrew [2010]). The idea is that agents calculate expected accuracy across all worlds, including those in which E is false; however, they use a revised accuracy measure which simply doesn't care about accuracy in worlds in which E is false. Formally, this is implemented by letting $\mathcal{A}$ be constant in these worlds.
When attempting to extend this approach to risk-weighted expectations, it matters how the relevant constant compares to the accuracy in the various scenarios in which E is true. If the constant is low enough for $\neg E$ to always be the worst case scenario, this formula has the same maximum as NaiveRExp. If, however, the constant is higher, we can get different results.
To our mind, the very fact that it matters which constant we choose suggests that this is not a good implementation of the intuitive idea that the agent doesn't care about accuracy in $\neg E$ possibilities when working with risk-sensitive agents. Absent an alternative implementation, we thus can't tell which belief-revision procedure is favoured by this picture.

4 The Global Approach

In the previous section, we evaluated potential updated credences over the states consistent with some particular piece of evidence E. There is, however, also an alternative approach which follows (Greaves and Wallace [2006]) and does not require us to use restricted (risk-weighted) expected utilities. On this approach, we compare global belief revision strategies, which specify new credences for each of the various things the agent might learn; since these specify outcomes for every state, we can use standard (risk-weighted) expectations to compare them.

It's worth noting that this approach, unlike the local one, really only goes with one picture: that of an agent evaluating, before knowing what she will learn, the different strategies for revising her credences in response to E. What these accuracy considerations determine is thus not, in the first instance, how the agent should update, but instead something like which updating strategy she should consider optimal. Before zooming in on that gap, and arguing that it can't be bridged in the case of risk-sensitive agents, let's look at which belief revision strategies this approach would support. This discussion will be slightly more technical than the remainder of the paper, so readers should feel free to skip to section 4.2; the important take away message is that risk-sensitive agents should almost never take conditionalization to be optimal.

4.1 Which global strategies are optimal?

We will restrict our attention to particularly simple cases, in which the agent is guaranteed to receive as her evidence either E or $\neg E$.[24] For a fixed prior distribution c, we can thus represent a (global) belief revision strategy by a pair of real numbers $\langle y_E, y_{\neg E}\rangle$, specifying the credence in X that the strategy tells agents with prior c to adopt if they learn E and $\neg E$, respectively.

The accuracy of an update strategy in a state is given by the following table:

State s	$E \wedge \neg X$	$\neg E \wedge \neg X$	$\neg E \wedge X$	$E \wedge X$
$\mathcal{A}(\langle y_E, y_{\neg E}\rangle, s)$	$\mathcal{A}(y_E, 0)$	$\mathcal{A}(y_{\neg E}, 0)$	$\mathcal{A}(y_{\neg E}, 1)$	$\mathcal{A}(y_E, 1)$

Given the probabilities c assigns to these states, we can then calculate each strategy's expected or risk-weighted expected utility.

Focusing on the risk-neutral case, (Greaves and Wallace [2006]) shows:

Proposition 1. *For all c probabilistic with $c(E) \in (0,1)$ and all strictly proper accuracy measures, $\mathcal{A}$,*

$$\text{Exp}_c\, \mathcal{A}\langle c(X|E), c(X|\neg E)\rangle > \text{Exp}_c\, \mathcal{A}\langle y_E, y_{\neg E}\rangle$$

whenever $y_E \neq c(X|E)$ or $y_{\neg E} \neq c(X|\neg E)$.

[24] The more general case of learning the true member of any fixed partition will involve natural extensions of our results. We also think that the general setting won't alter the considerations in section 4.2.

So conditionalization is the unique (epistemically) optimal update strategy for risk-neutral agents.

In the risk-sensitive setting, an optimal update strategy is one that maximizes risk-weighted expected accuracy, as evaluated by our prior c. We'll describe what such strategies look like, showing that they differ from conditionalization.[25]

To adapt the proof strategy in (Greaves and Wallace [2006]), we need to consider a reformulation of risk-weighted expected utility theory. For A leading to outcomes $o_1, \ldots, o_n$ in states $s_1, \ldots, s_n$ with $U(o_1) \leqslant \ldots \leqslant U(o_n)$, the risk-weighted expected utility formula can be rearranged as:[26]

$$\mathrm{RExp}^r_c\, U(A) = \sum_i \Bigg(\Big(r\big(\sum_{j \geqslant i} c(s_j)\big) - r\big(\sum_{j > i} c(s_j)\big) \Big) \times U(o_i) \Bigg)$$

In this formulation, $U(o_i)$ is multiplied by a term representing how much weight the state s_i receives in the calculation. This is relative to a strict ordering $\prec$ of the states (which was specified by the subscripts).[27] Making this explicit, we introduce a label for these decision weights:

$$\mathtt{d}^{\prec}(s) := r\big(\sum_{s' \succeq s} c(s')\big) - r\big(\sum_{s' \succ s} c(s')\big).$$

In our particular case, a strategy $\langle y_E, y_{\neg E}\rangle$ generates an ordering of the states, $\prec$, according to how $\mathcal{A}(y_E, 0)$, $\mathcal{A}(y_{\neg E}, 0)$, $\mathcal{A}(y_{\neg E}, 1)$ and $\mathcal{A}(y_E, 1)$ are ordered. (We will discuss this in more detail shortly.) And if $\langle y_E, y_{\neg E}\rangle$ generates $\prec$ we have

$$\mathrm{RExp}^r_c \mathcal{A}\langle y_E, y_{\neg E}\rangle = \begin{matrix} \mathtt{d}^{\prec}(E \wedge X) \times \mathcal{A}(y_E, 1) + \mathtt{d}^{\prec}(E \wedge \neg X) \times \mathcal{A}(y_E, 0) \\ + \mathtt{d}^{\prec}(\neg E \wedge X) \times \mathcal{A}(y_{\neg E}, 1) + \mathtt{d}^{\prec}(\neg E \wedge \neg X) \times \mathcal{A}(y_{\neg E}, 0) \end{matrix} \quad (1)$$

These $\mathtt{d}^{\prec}(s)$ play a special role in describing the optimal update via the following definitions:

$$\mathtt{d}^{\prec}_E(X) := \frac{\mathtt{d}^{\prec}(E \wedge X)}{\mathtt{d}^{\prec}(E \wedge X) + \mathtt{d}^{\prec}(E \wedge \neg X)}$$

$$\mathtt{o}^{\prec}_E(X) := \begin{cases} r^{-1}(\mathtt{d}^{\prec}_E(X)) & \text{if } E \wedge \neg X \prec E \wedge X \\ 1 - r^{-1}(1 - \mathtt{d}^{\prec}_E(X)) & \text{if } E \wedge X \prec E \wedge \neg X \end{cases}$$

and the corresponding definitions of $\mathtt{d}^{\prec}_{\neg E}(X)$ and $\mathtt{o}^{\prec}_{\neg E}(X)$. Note that if r is risk-neutral, $\mathtt{o}^{\prec}_E(X) = c(X|E)$ and $\mathtt{o}^{\prec}_{\neg E}(X) = c(X|\neg E)$.

Using this set up, we can generalize the proof strategy in (Greaves and Wallace [2006]) to obtain (see section A.1 for the proof):

[25] That they differ from conditionalization also follows from a result in (Campbell-Moore and Salow [forthcoming]), which shows that, when r is risk-avoidant, the risk-weighted expected accuracy of sticking to one's original beliefs is sometimes higher than that of conditionalizing on the evidence. That result, however, tells us little about what optimal strategies look like.

[26] This formulation is used in rank dependent expected utility theory; see, for example, (Wakker [2010], ch.6).

[27] Note that ties can be broken either way without affecting the result.

Proposition 2. *Suppose c is probabilistic with $c(E) \in (0,1)$, and $\mathcal{A}$ is monotone r-proper. Then if $\langle y_E, y_{\neg E}\rangle$ generates the ordering $\prec$, moving y_E towards $\mathsf{o}_E^{\prec}(X)$ or $y_{\neg E}$ towards $\mathsf{o}_{\neg E}^{\prec}(X)$ or both will increase* $\mathrm{RExp}_c^r\,\mathcal{A}\langle y_E, y_{\neg E}\rangle$*, so long as the resultant strategy still generates $\prec$.*

It's tempting to infer that optimal strategies must always be $\langle \mathsf{o}_E^{\prec}(X), \mathsf{o}_{\neg E}^{\prec}(X)\rangle$ for some ordering $\prec$. But that doesn't follow, since the strategy $\langle \mathsf{o}_E^{\prec}(X), \mathsf{o}_{\neg E}^{\prec}(X)\rangle$ sometimes generates an ordering other than $\prec$. To explain this and get more information from proposition 2, we need to discuss in more depth when a strategy generates an ordering $\prec$.

Which order (or orders) are generated by $\langle y_E, y_{\neg E}\rangle$ depends on how $\mathcal{A}(y_E, 0)$, $\mathcal{A}(y_{\neg E}, 0)$, $\mathcal{A}(y_{\neg E}, 1)$ and $\mathcal{A}(y_E, 1)$ compare. Assuming that $\mathcal{A}$ is truth-directed and 0/1-symmetric, this is in turn determined by how y_E and $y_{\neg E}$ compare to each other and to $1/2$, the point at which $\mathcal{A}(y,1)$ and $\mathcal{A}(y,0)$ switch their order. For example, if we have $1/2 < y_{\neg E} < y_E$, then $\mathcal{A}(y_E, 0) < \mathcal{A}(y_{\neg E}, 0) < \mathcal{A}(y_{\neg E}, 1) < \mathcal{A}(y_E, 1)$, generating the ordering of states $E\wedge\neg X \prec \neg E\wedge\neg X \prec \neg E\wedge X \prec E\wedge X$. We represent the full range of options in figure 3.

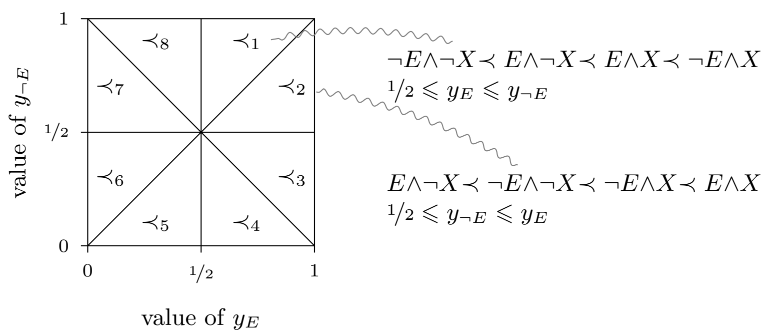

Figure 3: The possible strategies $\langle y_E, y_{\neg E}\rangle$ and the orderings they generate.

Choices of $\langle y_E, y_{\neg E}\rangle$ that fall inside the same small triangle generate the same order and choices that fall inside different triangles generate different ones. Some choices of $\langle y_E, y_{\neg E}\rangle$, such as $y_E = y_{\neg E}$ or $y_E = 1/2$, result in ties amongst the resultant accuracies; these generate all of the strict orderings on states obtained by arbitrarily breaking the ties, and equation (1) holds using any of these orderings. In the diagram, these strategies fall on the boundaries between triangles. As we can see from figure 3, there are eight possible orderings overall.

Now if we consider, for example, $r(x) = x^2$ and c given by $c(X|E) = 0.7$, $c(X|\neg E) = 0.8$, and $c(E) = 0.6$ and focus on $\prec_1$ arising from $1/2 \leqslant y_E \leqslant y_{\neg E}$, then simple (if lengthy) calculations show that $\mathsf{o}_E^{\prec_1}(X) \approx 0.77$ and $\mathsf{o}_{\neg E}^{\prec_1}(X) \approx 0.63$. But this means that $\mathsf{o}_E^{\prec_1}(X) > \mathsf{o}_{\neg E}^{\prec_1}(X)$, hence that $\langle \mathsf{o}_E^{\prec_1}(X), \mathsf{o}_{\neg E}^{\prec_1}(X)\rangle$ does not generate the initial ordering, $\prec_1$, which required $y_E \leqslant y_{\neg E}$. This is illustrated in figure 4.

In cases like this, proposition 2 still tells us something about which update strategies maximize risk-weighted expected accuracy from amongst those respecting $\prec$: they must lie along the boundary line reached when moving

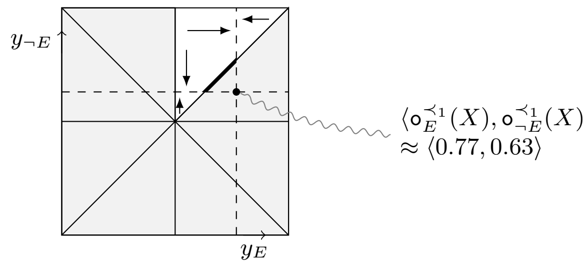

Figure 4: Arrows indicate how moving y_E and $y_{\neg E}$ within the shaded region will increase $\mathrm{RExp}_c^r \mathcal{A}\langle y_E, y_{\neg E}\rangle$.

y_E towards $\mathsf{o}_E^{\prec}(X)$ or $y_{\neg E}$ towards $\mathsf{o}_{\neg E}^{\prec}(X)$. For example, in the case just given, the best strategies from amongst those generating $\prec_1$ must lie on the line given by $y_{\neg E} = y_E$, and, more specifically, on the bold line in figure 4. Moreover, knowing that an update strategy lies on a particular boundary line tells us quite a lot about it. For example, knowing that $y_{\neg E} = y_E$ tells us that $\mathrm{RExp}_c^r \mathcal{A}\langle y_E, y_{\neg E}\rangle = \mathrm{RExp}_c^r \mathcal{A}(y_E)$, which, by r-propriety, is uniquely maximized at $y_E = c(X)$; so the only such strategy that could be optimal is $\langle c(X), c(X)\rangle$.

Combining proposition 2 with considerations of this kind then allows us to say which strategies can be epistemically optimal in the risk-sensitive setting (see section A.2 for the proof):

Theorem 3. *Suppose c is probabilistic with $c(E) \in (0,1)$, and $\mathcal{A}$ is r-proper, truth-directed and 0/1-symmetric. Then any optimal strategy is one of:*

- $\langle \mathsf{o}_E^{\prec}(X), \mathsf{o}_{\neg E}^{\prec}(X)\rangle$ *for some* $\prec$,
- $\langle \mathsf{o}_E^{\prec_2}(X), 1/2\rangle = \langle \mathsf{o}_E^{\prec_3}(X), 1/2\rangle$ *or* $\langle \mathsf{o}_E^{\prec_6}(X), 1/2\rangle = \langle \mathsf{o}_E^{\prec_7}(X), 1/2\rangle$,
- $\langle 1/2, \mathsf{o}_{\neg E}^{\prec_8}(X)\rangle = \langle 1/2, \mathsf{o}_{\neg E}^{\prec_1}(X)\rangle$ *or* $\langle 1/2, \mathsf{o}_{\neg E}^{\prec_4}(X)\rangle = \langle 1/2, \mathsf{o}_{\neg E}^{\prec_5}(X)\rangle$,
- $\langle c(X), c(X)\rangle$,
- $\langle c(X \leftrightarrow E), c(X \leftrightarrow \neg E)\rangle$.

This result does not pin down a unique optimal strategy: given c and r, it leaves us up to fourteen possible options. But there is little more to say at this level of generality. As section A.3 shows, any one of these can be uniquely optimal with the right choice of c and r. There can also be cases where two or more of them are tied, or where different ones are optimal depending on the particular choice of $\mathcal{A}$.

Despite not pinning down a unique strategy, however, theorem 3 tells us a lot. In particular, if r isn't risk-neutral, none of the fourteen options will correspond to conditionalization except in very unusual cases. It follows that, usually, conditionalization is not optimal. In fact, since there are cases (such as the one discussed above, as we show in section A.3) where $\langle c(X), c(X)\rangle$ is uniquely optimal even though $c(X) \neq c(X|E)$, conditionalization can disagree with the optimal strategies even about whether E calls for any revision at all.

4.2 Should we implement optimal global strategies?

As we noted at the beginning of the section this only shows that risk-sensitive agents should consider some non-conditionalization strategy to be globally optimal; it does not yet show that they should actually adopt it. The obvious way to bridge this gap is by appeal to a principle linking previous evaluations of strategies to rational behaviour, along the lines of:

> Implementing Optimal Strategies: Agents are rationally required to implement the strategies they previously considered optimal.

Such a principle is highly non-trivial. Several philosophers have recently argued that, from the perspective of the later agent, it's hard to find a reason for treating her previous evaluations with any more deference than the evaluations of any other equally rational and well-informed stranger.[28] Moreover, one might also think that some of these strangers will consider different strategies to be optimal, say because they are reasoning with different priors. If both of those are right, Implementing Optimal Strategies is false.[29]

Moreover, in the risk-sensitive setting there is additional reason to be worried. For we saw in section 3 that, for a risk-sensitive agent, the optimal local plan for responding to evidence E is conditionalizing on E; section 4.1 now adds that the optimal global strategy will prescribe a different response. Which of these – local plans or global strategies – is the agent required to implement when she receives E?

To resolve this question, it makes sense to consider instrumental rationality. We can show that risk-sensitive agents definitely should not always follow through on the instrumentally optimal global strategy. Furthermore, this argument does not rely on the specifics of how these agents update; they can update in the epistemically optimal way and still be rationally required not to follow through on the instrumental strategy they initially considered optimal. This provides a compelling reason to reject Implementing Optimal Strategies, at least as applied to global strategies rather than local plans.

Consider an agent with the Allais preferences, rationalization of which is a key desideratum for a risk-sensitive decision theories:

	Ticket 1	Ticket 2–11	Ticket 12–100
Good & Safe	\$1m	\$1m	\$1m
Good & Risky	\$0	\$5m	\$1m
Bad & Safe	\$1m	\$1m	\$0
Bad & Risky	\$0	\$5m	\$0

Expected utility theory tells us that, regardless of how you value money, if you prefer Safe to Risky in the Good case, you should prefer Safe to Risky in the

[28] See (Christensen [1991]; Moss [2015]; Hedden [2015]).

[29] Pettigrew ([2016], section 15.2) denies that the (Greaves and Wallace [2006]) argument establishes that conditionalization is rationally required for roughly this reason.

Bad case. But Risk-Weighted Expected Utility Theory, like other risk-sensitive decision theories, is designed to allow the intuitive preferences of Safe over Risky in the Good case but Risky over Safe in the Bad case.

Now suppose our Allais agent considers strategies for which lottery – Safe or Risky – to choose once she learns whether her ticket is one of 1–11. Since Safe and Risky are the same if the ticket is 12–100, which global strategy is optimal doesn't depend on what it prescribes in the 12–100 case. So the optimal strategy matches the pre-learning preferences: after learning 1–11 she should pick Safe over Risky in the Good case, and Risky over Safe in the Bad case. But, once she actually learns 1–11 and updates her preferences, the decision to be made in the Good case is the same as the decision to be made in the Bad case. So although the optimal strategy for the Good and Bad versions of the lottery differ, after she actually learns that it's 1–11, her preferred options cannot depend on whether it was Good or Bad. So at least one of her preferences has to switch; and, when it does, she will rationally fail to follow through on the strategy she initially preferred.

Note that this argument does not assume that the agent updates by conditionalization. All that matters is that learning 1–11 has the same effect on her credences in both the Good and Bad cases; and no one who thinks that norms of belief revision should be sensitive only to epistemic factors, such as accuracy, would want to deny this. The argument does assume that after learning 1–11, the agent is rationally required to do whatever maximizes risk-weighted expected utility with her updated credences (since otherwise the agent's preference between Safe and Risky could be different depending on whether she started in the Good or Bad case, even after learning 1–11). Advocates of 'resolute choice' would reject this assumption; but, since assessing this move in depth would take us quite far afield, we simply note that we find it implausible.[30]

We conclude, then, that Implementing Optimal Strategies should be rejected by anyone sympathetic to risk-sensitive decision theory, since it should be rejected in the case of instrumental strategies. Is there any reason to accept its restriction to belief-revision strategies in particular? We can think of only one. It's tempting to think that Implementing Optimal Strategies fails in the instrumental case only because there is an intermediate time in the 'implementation' process, at which the agent in question can re-assess her judgments about which Strategy is optimal. In the Allais case, for example, there is some time at which the agent has discovered whether her ticket is 1–11, but has not yet decided between Safe and Risky; at that time, she may (and, in one of the cases, will) re-evaluate which strategy she regards as instrumentally optimal. We could then qualify Implementing Optimal Strategies to obtain

> Implementing Stably Optimal Strategies: Agents are rationally required to implement the strategies they previously considered optimal, unless there is a stage between now and the point the strategy will have been fully implemented at which they can reconsider their choices.

[30] See (Buchak [2013], chapter 6) for discussion and further references.

This principle does not conflict with the Allais Preferences. And it can be used to bridge the gap between regarding conditionalization as optimal and being required to conditionalize if, in the epistemic case, there is no intermediate stage between the initial evaluation and the point at which the agent needs to adopt her final credences, so that there would be no point at which the agent's preferences could shift. However, it seems plausible that, at least in a rational reconstruction of the process, there is such an intermediate stage (unless the agent considers conditionalization optimal from the start). After all, the belief revision strategies in question require different things depending on what the evidence is. So a rational reconstruction of the process should feature a point at which the agent has discovered what the evidence is, but has not yet used this information to implement her favoured strategy. And, as we argued in section 3, at this point the agent should always prefer conditionalizing to every other strategy.

We thus conclude that, whilst the epistemically optimal global update strategy diverges substantially from conditionalization, this tells us little about how a risk-sensitive agent should update her beliefs. The gap between showing that a global strategy is optimal and that it should be implemented is simply too large in the risk-sensitive setting.

5 Conclusion

What do accuracy-theoretic considerations tell a risk-sensitive agent about belief revision? We have seen that this depends on relatively subtle features of how these considerations are applied: approaches that all support the same belief-revision process in the risk-neutral setting lead to different results once risk-sensitivity is on the table. In particular, 'local' approaches, which consider how to respond to some particular piece of evidence tend, when properly adapted, to support conditionalization; while 'global' approaches, which compare strategies for responding to any one of a range of propositions one might learn, will generally not. In a way, however, this divergence is unsurprising. A standard characterization of risk-sensitive decision theories is that they reject the 'Independence Axiom' or the 'Sure Thing Principle', which state, very roughly, that if a particular option is optimal in each of a number of situations which together exhaust all the possibilities, then that option must be optimal overall. What we find when looking at belief revision is just a further case in which this principle will fail: conditionalization is the accuracy-optimal response to each of $E_1, \ldots, E_n$, and we know that one of these will be the evidence obtained; but, for risk-sensitive agents, it just doesn't follow that it is also the accuracy-optimal global strategy.

However, we have also argued that there is, from a perspective sympathetic to risk-sensitivity, compelling reason to reject global approaches; for risk-sensitive agents are, quite generally, not required to stick with the (global) strategies they previously considered optimal once new information comes in. Perhaps the local approaches are also problematic for more general reasons that we did not consider here; in that case, neither kind of accuracy-theoretic argument will be compelling for risk-sensitive agents. But if accuracy-theoretic considerations can tell risk-sensitive agents anything, they must be of the 'local' type; and they will, therefore, tell the agent to conditionalize.

Appendix A Optimal Global Strategies

We say that a strategy $\langle x'_E, x'_{\neg E}\rangle$ 'improves on' a strategy $\langle x_E, x_{\neg E}\rangle$ if

$$\mathrm{RExp}^r_c\,\mathcal{A}\langle x'_E, x'_{\neg E}\rangle > \mathrm{RExp}^r_c\,\mathcal{A}\langle x_E, x_{\neg E}\rangle.$$

A strategy is thus optimal if no strategy improves on it. Note that both improvement and optimality are relative to a fixed c and r.

A.1 Proof of Proposition 2

Recall that, if $\langle y_E, y_{\neg E}\rangle$ generates $\prec$,

$$\mathrm{RExp}^r_c\mathcal{A}\langle y_E, y_{\neg E}\rangle = \begin{array}{l}\mathtt{d}^{\prec}(E\wedge X)\times\mathcal{A}(y_E,1)+\mathtt{d}^{\prec}(E\wedge\neg X)\times\mathcal{A}(y_E,0)\\ +\,\mathtt{d}^{\prec}(\neg E\wedge X)\times\mathcal{A}(y_{\neg E},1)+\mathtt{d}^{\prec}(\neg E\wedge\neg X)\times\mathcal{A}(y_{\neg E},0)\end{array} \quad (1)$$

We defined $\mathtt{o}^{\prec}_E(X)$ and $\mathtt{o}^{\prec}_{\neg E}(X)$; we can extend them to probability functions on $\{X,\neg X\}$ by setting $\mathtt{o}^{\prec}_E(\neg X) = 1-\mathtt{o}^{\prec}_E(X)$ and $\mathtt{o}^{\prec}_{\neg E}(\neg X) = 1-\mathtt{o}^{\prec}_{\neg E}(X)$. We can then show that, if $\langle y_E, y_{\neg E}\rangle$ generates $\prec$,

$$\mathrm{RExp}^r_{\mathtt{o}^{\prec}_E}\,\mathcal{A}(y_E) = \frac{\mathtt{d}^{\prec}(E\wedge X)\times\mathcal{A}(y_E,1)+\mathtt{d}^{\prec}(E\wedge\neg X)\times\mathcal{A}(y_E,0)}{\mathtt{d}^{\prec}(E\wedge X)+\mathtt{d}^{\prec}(E\wedge\neg X)} \quad (2)$$

$$\mathrm{RExp}^r_{\mathtt{o}^{\prec}_{\neg E}}\,\mathcal{A}(y_{\neg E}) = \frac{\mathtt{d}^{\prec}(\neg E\wedge X)\times\mathcal{A}(y_{\neg E},1)+\mathtt{d}^{\prec}(\neg E\wedge\neg X)\times\mathcal{A}(y_{\neg E},0)}{\mathtt{d}^{\prec}(\neg E\wedge X)+\mathtt{d}^{\prec}(\neg E\wedge\neg X)} \quad (3)$$

The details of the derivation depend on $\prec$. For example, if $E\wedge X \prec E\wedge\neg X$, then by definition $\mathtt{o}^{\prec}_E(X) = 1-r^{-1}(1-\mathtt{d}^{\prec}_E(X))$; so $r(\mathtt{o}^{\prec}_E(\neg X)) = 1-\mathtt{d}^{\prec}_E(X)$. If $\langle y_E, y_{\neg E}\rangle$ generates $\prec$ with $E\wedge X\prec E\wedge\neg X$, we have $\mathcal{A}(y_E,1)\leqslant\mathcal{A}(y_E,0)$, so

$$\begin{aligned}\mathrm{RExp}^r_{\mathtt{o}^{\prec}_E}\,\mathcal{A}(y_E) &= \mathcal{A}(y_E,1)+r(\mathtt{o}^{\prec}_E(\neg X))(\mathcal{A}(y_E,0)-\mathcal{A}(y_E,1))\\ &= \mathtt{d}^{\prec}_E(X)\times\mathcal{A}(y_E,1)+(1-\mathtt{d}^{\prec}_E(X))\times\mathcal{A}(y_E,0)\\ &= \frac{\mathtt{d}^{\prec}(E\wedge X)\times\mathcal{A}(y_E,1)+\mathtt{d}^{\prec}(E\wedge\neg X)\times\mathcal{A}(y_E,0)}{\mathtt{d}^{\prec}(E\wedge X)+\mathtt{d}^{\prec}(E\wedge\neg X)}\end{aligned}$$

Now, suppose $\langle x'_E, x'_{\neg E}\rangle$ is the result of moving the y_E co-ordinate from $\langle x_E, x_{\neg E}\rangle$ towards $\mathtt{o}^{\prec}_E(X)$, so that $x'_{\neg E} = x_{\neg E}$ and either $x_E < x'_E \leqslant \mathtt{o}^{\prec}_E(X)$ or $\mathtt{o}^{\prec}_E(X)\leqslant x'_E < x_E$; or of moving the $y_{\neg E}$ co-ordinate towards $\mathtt{o}^{\prec}_{\neg E}(X)$; or of both. Then, by monotone r-propriety, $\mathrm{RExp}^r_{\mathtt{o}^{\prec}_E}\,\mathcal{A}(x'_E)\geqslant\mathrm{RExp}^r_{\mathtt{o}^{\prec}_E}\,\mathcal{A}(x_E)$ and $\mathrm{RExp}^r_{\mathtt{o}^{\prec}_{\neg E}}\,\mathcal{A}(x'_{\neg E})\geqslant\mathrm{RExp}^r_{\mathtt{o}^{\prec}_{\neg E}}\,\mathcal{A}(x_{\neg E})$, with at least one inequality being strict. Thus, if both generate $\prec$, then by consideration of equations (1) to (3) we see that $\langle x'_E, x'_{\neg E}\rangle$ improves on $\langle x_E, x_{\neg E}\rangle$.

A.2 Proof of Theorem 3

Proposition 2 is the main result we need for this theorem. Lemma 4 is a simple consequence of it, which, together with lemma 5, suffices for our result.

Lemma 4. *If* $\langle y_E, y_{\neg E}\rangle$ *is optimal, then either it is one of:*

- $\langle \mathsf{o}_E^{\prec}(X), \mathsf{o}_{\neg E}^{\prec}(X)\rangle$ *for some* $\prec$*,*
- $\langle \mathsf{o}_E^{\prec_2}(X), 1/2\rangle = \langle \mathsf{o}_E^{\prec_3}(X), 1/2\rangle$ *or* $\langle \mathsf{o}_E^{\prec_6}(X), 1/2\rangle = \langle \mathsf{o}_E^{\prec_7}(X), 1/2\rangle$*.*
- $\langle 1/2, \mathsf{o}_{\neg E}^{\prec_8}(X)\rangle = \langle 1/2, \mathsf{o}_{\neg E}^{\prec_1}(X)\rangle$ *or* $\langle 1/2, \mathsf{o}_{\neg E}^{\prec_4}(X)\rangle = \langle 1/2, \mathsf{o}_{\neg E}^{\prec_5}(X)\rangle$

or $y_E = y_{\neg E}$ *or* $y_E = 1 - y_{\neg E}$*.*

Proof. It is easy to check the equalities in the statement by showing that the corresponding $\mathtt{d}^{\prec}$ are equal. For example to show $\langle \mathsf{o}_E^{\prec_2}(X), 1/2\rangle = \langle \mathsf{o}_E^{\prec_3}(X), 1/2\rangle$, we note that, for both $\prec_2$ and $\prec_3$,

$$\mathtt{d}^{\prec}(E\wedge X) = r(c(E\wedge X))$$
$$\mathtt{d}^{\prec}(E\wedge\neg X) = 1 - r(1 - c(E\wedge\neg X))$$

We now argue that an optimal strategy satisfies the disjunction, by arguing that any given $\langle x_E, x_{\neg E}\rangle$ failing to satisfy the disjunction can be improved on. The choice of improving strategy depends on the orders generated by $\langle x_E, x_{\neg E}\rangle$ and $\langle \mathsf{o}_E^{\prec}(X), \mathsf{o}_{\neg E}^{\prec}(X)\rangle$, where $\prec$ is an order generated by $\langle x_E, x_{\neg E}\rangle$. We list the improving strategies assuming that $\langle \mathsf{o}_E^{\prec}(X), \mathsf{o}_{\neg E}^{\prec}(X)\rangle$ generates $\prec_1$; the other situations are analogous. As illustrated in figure 5, one can verify that in each case the improving strategy generates the same order as $\langle x_E, x_{\neg E}\rangle$, and results from moving x_E towards $\mathsf{o}_E^{\prec}(X)$, $x_{\neg E}$ towards $\mathsf{o}_{\neg E}^{\prec}(X)$, or both; it thus improves on $\langle x_E, x_{\neg E}\rangle$ by proposition 2.

If $\langle x_E, x_{\neg E}\rangle$ generates	then $\langle x_E, x_{\neg E}\rangle$ can be improved on by
$\prec_1$	$\langle \mathsf{o}_E^{\prec_1}(X), \mathsf{o}_{\neg E}^{\prec_1}(X)\rangle$
$\prec_2$	if $x_E \geqslant \mathsf{o}_E^{\prec_2}(X)$, then $\langle x_{\neg E}, x_{\neg E}\rangle$ otherwise $\langle \mathsf{o}_E^{\prec_2}(X), \mathsf{o}_E^{\prec_2}(X)\rangle$
$\prec_3$	$\langle \mathsf{o}_E^{\prec_3}(X), 1/2\rangle$
$\prec_4$	$\langle x_E, 1 - x_E\rangle$
$\prec_5$ or $\prec_6$	$\langle 1/2, 1/2\rangle$
$\prec_7$	$\langle x_{\neg E}, 1 - x_{\neg E}\rangle$
$\prec_8$	$\langle 1/2, \mathsf{o}_{\neg E}^{\prec_8}(X)\rangle$

□

Lemma 5.

- $\langle c(X), c(X)\rangle$ *improves on any other strategy with* $y_E = y_{\neg E}$*.*
- $\langle c(X\leftrightarrow E), c(X\leftrightarrow\neg E)\rangle$ *improves on any other strategy with* $y_E = 1 - y_{\neg E}$*.*

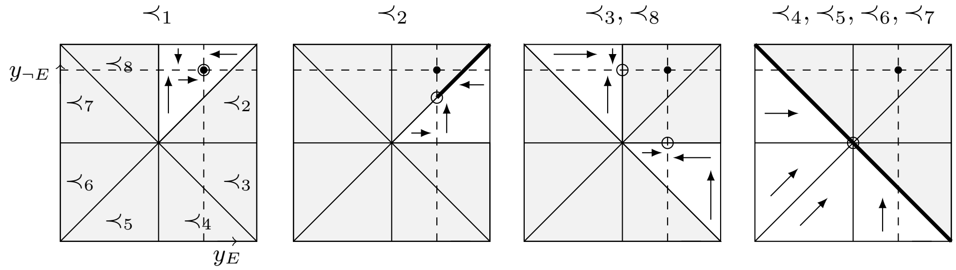

Figure 5: How to improve $\langle x_E, x_{\neg E}\rangle$ in line with proposition 2.

Proof. If $y_E = y_{\neg E}$, $\mathrm{RExp}^r_c\, \mathcal{A}\langle y_E, y_{\neg E}\rangle = \mathrm{RExp}^r_c\, \mathcal{A}(y_E)$; which, by r-propriety, is uniquely maximized at $y_E = c(X)$.

Now suppose $y_E = 1 - y_{\neg E}$. Since $\mathcal{A}$ is 0/1-symmetric, there are just two possible outcomes: $\mathcal{A}(y_E, 1) = \mathcal{A}(y_{\neg E}, 0)$ (with $c(E\wedge X) + c(\neg E\wedge\neg X) = c(X\leftrightarrow E)$), or $\mathcal{A}(y_E, 0) = \mathcal{A}(y_{\neg E}, 1)$. Consider some probability function b with $b(X) = c(X \leftrightarrow E)$; then $\mathrm{RExp}^r_c\, \mathcal{A}\langle y_E, y_{\neg E}\rangle = \mathrm{RExp}^r_b\, \mathcal{A}(y_E)$, which, by r-propriety, is uniquely maximized at $y_E = b(X) = c(X\leftrightarrow E)$; thus $y_{\neg E} = 1 - c(X\leftrightarrow E) = c(X\leftrightarrow\neg E)$. □

A.3 Examples

Theorem 3 leaves fourteen options for what an optimal update can look like. For each of these, there is a choice of r and c such that it is uniquely optimal. These choices work for any truth directed, 0/1-symmetric and monotone r-proper $\mathcal{A}$.

Example 6. For $c(X|E) = 0.7, c(X|\neg E) = 0.95, c(E) = 0.5$ and $r(x) = x^2$, $\langle \mathsf{o}^{\prec_1}_E(X), \mathsf{o}^{\prec_1}_{\neg E}(X)\rangle$ is uniquely optimal.

Proof. We start by plotting $\langle \mathsf{o}^{\prec}_E(X), \mathsf{o}^{\prec}_{\neg E}(X)\rangle$ for all eight orderings in figure 6.

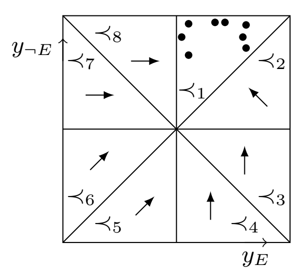

Figure 6: Plotting the $\langle \mathsf{o}^{\prec}_E(X), \mathsf{o}^{\prec}_{\neg E}(X)\rangle$ corresponding to each $\prec$ for example 6.

This graph shows that all eight points generate $\prec_1$. As we'll show, it follows that $\langle \mathsf{o}^{\prec_1}_E(X), \mathsf{o}^{\prec_1}_{\neg E}(X)\rangle$ is uniquely optimal.

The reasoning in lemma 4 shows that, if $\langle \mathsf{o}^{\prec}_E(X), \mathsf{o}^{\prec}_{\neg E}(X)\rangle$ doesn't generate $\prec$, then strategies generating $\prec$ are always improved on by strategies on the boundary of that region in the direction of $\langle \mathtt{o}_E^{\prec}(X), \mathtt{o}_{\neg E}^{\prec}(X)\rangle$ – unless they themselves lie on that boundary (a qualification we leave implicit in the following).

Consider, then, an update strategy generating $\prec_3$. This strategy is improved on by something on the $\prec_3$ / $\prec_2$ boundary. But that improved strategy will also generate $\prec_2$. So, because $\langle \mathtt{o}_E^{\prec_2}(X), \mathtt{o}_{\neg E}^{\prec_2}(X)\rangle$ generates $\prec_1$, this will be improved on by something on the $\prec_2$ / $\prec_1$ boundary. But such a strategy also generates $\prec_1$; so proposition 2 implies that $\langle \mathtt{o}_E^{\prec_1}(X), \mathtt{o}_{\neg E}^{\prec_1}(X)\rangle$ will improve on it. So $\langle \mathtt{o}_E^{\prec_1}(X), \mathtt{o}_{\neg E}^{\prec_1}(X)\rangle$ improves on any strategy generating $\prec_3$

The same argument can be run for update strategies generating any of the other orderings, following the arrows in figure 6. So $\langle \mathtt{o}_E^{\prec_1}(X), \mathtt{o}_{\neg E}^{\prec_1}(X)\rangle$ is uniquely optimal. □

Example 7. For $c(X|E) = 0.7, c(X|\neg E) = 0.8, c(E) = 0.5$ and $r(x) = x^2$, $\langle c(X), c(X)\rangle$ is uniquely optimal.

Proof. Again, we begin by plotting $\langle \mathtt{o}_E^{\prec}(X), \mathtt{o}_{\neg E}^{\prec}(X)\rangle$ for the various orderings (figure 7).

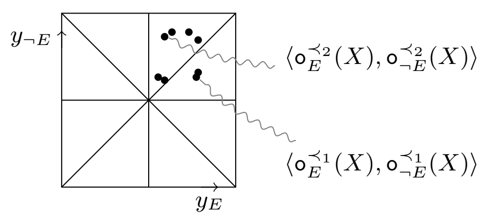

Figure 7: Plotting the $\langle \mathtt{o}_E^{\prec}(X), \mathtt{o}_{\neg E}^{\prec}(X)\rangle$ corresponding to each $\prec$ for example 7.

Note that all the $\langle \mathtt{o}_E^{\prec}(X), \mathtt{o}_{\neg E}^{\prec}(X)\rangle$ generate either $\prec_1$ or $\prec_2$. So, by the reasoning used in example 6, any optimal strategy must also generate one of these.

Note also that $\langle \mathtt{o}_E^{\prec_1}(X), \mathtt{o}_{\neg E}^{\prec_1}(X)\rangle$ generates $\prec_2$ and that $\langle \mathtt{o}_E^{\prec_2}(X), \mathtt{o}_{\neg E}^{\prec_2}(X)\rangle$ generates $\prec_1$. It follows that any optimal strategy generating either $\prec_1$ or $\prec_2$ must fall on the line between them, and hence have $y_E = y_{\neg E}$. By lemma 5, the only such strategy that can be optimal is $\langle c(X), c(X)\rangle$. □

Similar reasoning shows that each of the other options is uniquely optimal in the cases outlined in table 1.

These examples are quite robust. For example, it's possible to show that for any differentiable risk profile with $r(x) < x$, there will be cases like example 7, where the uniquely optimal strategy is not to alter one's beliefs in light of conditionally relevant evidence. This establishes what we asserted without proof in **?**, footnote 29

To show this, one argues that choosing $\mathtt{d}_{\neg E}^{\prec}(X) > 1 - r(1/2)$, $\mathtt{d}_E^{\prec}(X) = r^{-1}(\mathtt{d}_{\neg E}^{\prec}(X))$, and $c(E)$ sufficiently close to 1, yields a situation analogous to example 7: $\langle \mathtt{o}_E^{\prec}(X), \mathtt{o}_{\neg E}^{\prec}(X)\rangle$ all generate $\prec_1$ or $\prec_2$, and $\langle \mathtt{o}_E^{\prec_1}(X), \mathtt{o}_{\neg E}^{\prec_1}(X)\rangle$ generates $\prec_2$ and visa versa. Since the argument is quite involved, we omit it here. Readers wishing to reconstruct it might find it helpful to look at the proof of Theorem 5 in Campbell-Moore and Salow (forthcoming).

optimal	$r(x)$	$c(X\|E)$	$c(X\|\neg E)$	$c(E)$
$\langle \mathsf{o}_E^{\prec_1}(X), \mathsf{o}_{\neg E}^{\prec_1}(X)\rangle$	x^2	0.7	0.95	0.5
$\langle \mathsf{o}_E^{\prec_2}(X), \mathsf{o}_{\neg E}^{\prec_2}(X)\rangle$	x^2	0.95	0.7	0.5
$\langle \mathsf{o}_E^{\prec_3}(X), \mathsf{o}_{\neg E}^{\prec_3}(X)\rangle$	x^2	0.95	0.3	0.5
$\langle \mathsf{o}_E^{\prec_4}(X), \mathsf{o}_{\neg E}^{\prec_4}(X)\rangle$	x^2	0.7	0.05	0.5
$\langle \mathsf{o}_E^{\prec_5}(X), \mathsf{o}_{\neg E}^{\prec_5}(X)\rangle$	x^2	0.3	0.05	0.5
$\langle \mathsf{o}_E^{\prec_6}(X), \mathsf{o}_{\neg E}^{\prec_6}(X)\rangle$	x^2	0.05	0.3	0.5
$\langle \mathsf{o}_E^{\prec_7}(X), \mathsf{o}_{\neg E}^{\prec_7}(X)\rangle$	x^2	0.3	0.05	0.5
$\langle \mathsf{o}_E^{\prec_8}(X), \mathsf{o}_{\neg E}^{\prec_8}(X)\rangle$	x^2	0.3	0.95	0.5
$\langle c(X), c(X)\rangle$	x^2	0.7	0.8	0.5
$\langle c(X \leftrightarrow E), c(X \leftrightarrow \neg E)\rangle$	x^2	0.7	0.2	0.5
$\langle \mathsf{o}_E^{\prec_2}(X), 1/2\rangle$	$\sqrt{x}$	0.8	0.5	0.5
$\langle \mathsf{o}_E^{\prec_6}(X), 1/2\rangle$	$\sqrt{x}$	0.2	0.5	0.5
$\langle 1/2, \mathsf{o}_{\neg E}^{\prec_4}(X)\rangle$	$\sqrt{x}$	0.5	0.8	0.5
$\langle 1/2, \mathsf{o}_{\neg E}^{\prec_8}(X)\rangle$	$\sqrt{x}$	0.5	0.2	0.5

Table 1: Choices of r and c for which the various options left open by theorem 3 are uniquely optimal

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