Numerical Investigation of Resonance in the Bristol Channel

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ABSTRACT

The Bristol Channel has the second largest tidal range in the world but the complex tidal dynamics of the Bristol Channel are not yet fully understood, in particular the sensitivity of the quarter wavelength resonance to changes such as those caused by energy extraction. This was the motivation for the development of a simplified two-dimensional model to simulate the tidal flows in the Bristol Channel. The simulation was first calibrated against the observational data. Subsequently, the model was excited with different frequencies to find the fundamental mode of oscillation, and a series of sensitivity studies were then carried out on tuneable model parameters.

KEY WORDS: ADCIRC; tidal resonance; shallow water equations; numerical modelling; Bristol Channel.

INTRODUCTION

The Bristol Channel and Severn Estuary constitute one of the largest, semi-enclosed water basins in the UK. The Bristol Channel is located in the south-west coast of Great Britain (Fig. 1). The Severn Estuary is situated at the upper reaches of the Bristol Channel, which has one of the largest semi-diurnal tidal ranges worldwide. The typical mean spring tidal range is 12.2m, with the high spring tidal range approaching 14m at the Severn mouth (Liang et al., 2014). The large tidal ranges and fast currents observed in the Bristol Channel and the Severn Estuary are driven by two main mechanisms ((Robinson, 1980; Xia et al., 2012; Serhadlıoğlu, 2014). One is the funnelling effect at the upper reaches of the Bristol Channel due to its wedge-shaped geometry and shallow bathymetry. However, it has long been pointed out by Marmer (1922) that this effect is not enough to produce the observed tidal range. The other mechanism is the quarter wavelength resonance of the Channel with the incident North Atlantic tidal wave (Fong and Heaps, 1978).

Despite a number of previous model studies having been undertaken for the Bristol Channel, the complex hydrodynamic system of the Channel is not yet fully understood, particularly given its resonant nature since resonant systems are typically very sensitive to small changes and are

Fig. 1 Map of the Bristol Channel (accessed from worldatlas.com).

highly site dependent (Adcock et al., 2015). In this study we seek to improve the understanding of the resonance in the Bristol Channel. A simplified two-dimensional model has been developed from Serhadlıoğlu's (2013) model to investigate the resonances in the Bristol Channel using the discontinuous Galerkin (DG) version of ADCIRC, which is a well-developed hydrodynamic finite element model. ADCIRC is widely used for tide and surge modelling, and is in constant development, with work ongoing in the areas of adaptive grids, three-dimensional modelling, sediment transport and biological processes (Kubatko et al., 2006).

In this paper, the model equations and the model parameters used for the Bristol Channel region are firstly considered; then, the model is tested

Table 1 . Estimates of the fundamental seiche period of the Bristol Channel and the Celtic Sea derived from various theoretical models (extended table taken from Fong and Heaps (1978).

References	Comment	Areas included	Friction coefficient	Fundamental period (hours)	ω/ω_{M_2}
Fong and Hepas (1978)	1-D numerical model to investigate the quarter-wave tidal resonance in the Bristol Channel-Celtic Sea shelf area	Celtic Sea and Bristol Channel	Not included	Celtic Sea: 12.2-12.6	0.98-1.02
Heath (1981)	A linear resonant model fitted to the semi-diurnal tidal constituents	Ocean and Celtic Sea	Not included	Celtic Sea: 10.8-11.1; Bristol Channel: 7.3-9.0	Celtic Sea: 1.12-1.15; Bristol Channel: 1.38-1.7
Liang et al. (2014)	1-D computational model to predict the tidal characteristics response	Severn Estuary and Bristol Channel	Roughness height:35mm	Bristol Channel: 8-9	1.38-1.55
Serhadlıoğlu (2014)	2-D unstructured computational model to investigate the resonance of the system	European continental shelf, Irish Sea, Celtic Sea and Bristol Channel	0.0025	Bristol Channel: 10.3-11.3	1.1-1.2

by comparing its results with previous model studies and observations; finally, the resonant period and key properties that influence the resonances are investigated.

RELATED WORK

Systems that are forced by oscillations close to their natural period have large amplitude responses, this phenomenon is called resonance (Pugh, 1996). In oceanography, a tidal resonance occurs when the tide excites one of the resonant modes of the ocean. The effect is most striking when a continental shelf is about a quarter wavelength wide. The whole global ocean system seems to be near to resonance at semidiurnal tidal frequencies and the observed tides are substantially larger than the Equilibrium Tide (Baker, 1991; Pugh and Woodworth, 2014).

In some studies, the phenomenon of quarter wavelength resonance was explained by standing wave theory (Pugh, 1996; Pugh and Woodworth, 2014). Consider the simplest case of a wave travelling in a long channel being reflected without loss of amplitude at a closed end. The superposition of incident and reflected waves can resemble a standing wave which have alternate nodes, positions where the amplitude is zero, and antinodes, positions where the amplitude is a maximum, each separated by a distance $\lambda/4$ where λ is the wavelength of the original progressive wave (Pugh, 1996). A model of an open box approximates to the tidal behaviour of many shelf sea basins (Pugh, 1996). If we describe the movement of water in a box whose lengths is a quarter wavelength with one closed end and one open end, and the water is driven by oscillatory in and out currents at the open end, thus the open end is at the first node and currents at the entrance could produce large changes of level at the head. Although exact quarter-wave dimension would be very unlikely, the possibility of tidal amplification still exists.

The resonances in the Bristol Channel have practical significance and theoretical interest since cross-channel tidal barrages have been and are being considered for the Channel, and to assess how these might alter the tidal characteristics of the area it is obviously important to understand the tidal dynamics as they presently exist (Fong and Heaps, 1978).

In recent years, the Bristol Channel and Severn Estuary has been of particular interest to engineers and scientists for this reason and several studies have been undertaken over the past 30 years or so. Fong and Heaps (1978) presented a summary of the significant numerical studies of estimating the resonant period of the Bristol Channel/Celtic Sea system. Table 1 is an extended version of this. Serhadlıoğlu (2014) has worked on the resonance in Bristol Channel using a two-dimensional unstructured triangular model mesh, which was the one that the present study built on. The model domain includes the Irish Sea, the Celtic Sea, the English Channel and the Bristol Channel. In Serhadlıoğlu's study the model was excited with a single tidal component with the amplitude of the M_2 tide but with the frequency varied, and a resonant period of 10.3-11.3 hours was found.

METHOD

Shallow Water Equations

A simplification can be made by integrating the horizontal velocity over the vertical direction to obtain a representative velocity flow field, which satisfies the shallow water equations (SWEs). The two-dimensional SWEs consist of the depth-averaged continuity equation and the x and y momentum equations written here in conservative form:

$$\frac{\partial \zeta}{\partial t} + \frac{\partial}{\partial x}(Hu) + \frac{\partial}{\partial y}(Hv) = 0 \quad (1)$$

$$\frac{\partial}{\partial t}(uH)+\frac{\partial}{\partial x}(Hu^2+\frac{1}{2}g(H^2-h^2))+\frac{\partial}{\partial y}(Huv) = g\zeta\frac{\partial h}{\partial x}-c_f u\sqrt{u^2+v^2}+fv+F_x \quad (2)$$

$$\frac{\partial}{\partial t}(vH)+\frac{\partial}{\partial y}(Hv^2+\frac{1}{2}g(H^2-h^2))+\frac{\partial}{\partial x}(Huv) = g\zeta\frac{\partial h}{\partial y}-c_f v\sqrt{u^2+v^2}-fu+F_y \quad (3)$$

where H is the total depth of the water column ($H = h + \zeta$), which is equivalent to the sum of the free surface elevation (ζ) and the bathymetric depth, h. The variables u and v represent the depth-averaged velocity components in $x-$ and $y-$ directions. g is the gravitational acceleration, c_f is the bottom drag coefficient, f is the Coriolis "force", and F_x and F_y represent additional forces in the system such as tidal potential forces, wind or wave radiation stresses (Falconer, 1993). Due to the large tidal range and relatively small river discharge, the flow in the Severn Estuary and Bristol Channel does not display any significant stratification, which justifies the use of the shallow water equations in the hydrodynamic analyses (Liang et al., 2014).

Discontinuous Galerkin Finite Element Method

The discontinuous Galerkin, DG, is a class of finite element methods which makes use of the same function space as the continuous method, but with relaxed continuity at inter-element boundaries. A comparison between continuous Galerkin (CG) discretisation method and discontinuous Galerkin (DG) method has been made by Kubatko et al. (2006). It was found that the DG model has a number of favourable properties that make them a more reliable choice than CG model, including better convergence rates, better efficiency on an error-per-computational-cost basis, and better parallel performance. However, the DG model was about four times slower than the CG model, which is one of the downsides of DG method (Yeager, 2014).

Despite the longer simulation time for a DG model compared to a CG model on a single processor, the model used in the present study was very small, only focusing on the area of the Bristol Channel. Thus the DG method was used to solve the shallow water equations in this project.

Model Set Up

The model was built from Serhadlıoğlu's study (2014) whose modelling region includes the Irish Sea, the Celtic Sea and the Bristol Channel. The coastline of the overall region of interest was determined from bathymetric data provided by SeaZONE Ltd., which was input to the meshing software SMS (Militello and Zundel, 1999). The bathymetric data consists of mean sea level and intertidal level contour data. An unstructured computational mesh was then constructed by SMS from the mean sea level contour data. A mesh convergence study has been conducted to evaluate the necessary level of resolution required in the numerical model to obtain a converged M_2 response using five unstructured triangular finite element meshes (Serhadlıoğlu, 2014).

On the open boundary, the water depths were prescribed and were the best estimates based on previous work (Serhadlıoğlu et al., 2013), and no

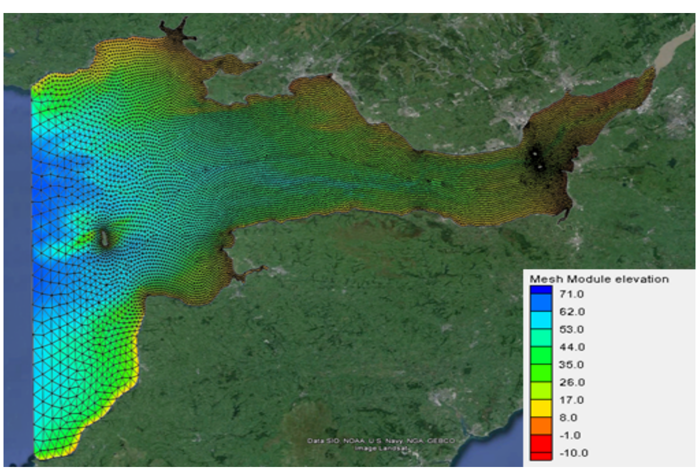

Fig. 2 Model mesh shown in Google Earth.

current was specified on the boundary. There is slip boundary condition at the coastline. In the real fluid the slip boundary condition cannot happen, but the boundary layer thickness here in the model is much smaller than the mesh size, therefore we used a slip boundary condition.

As seen in Fig. 2, the model domain stretches from the outer Bristol Channel, close to Lundy Island, to Caldicot and thereby includes the entire expanse of water from the open sea to the tidal limit. The domain is approximately 160 km long, narrowing down dramatically towards the head of the Estuary, from around 112 km at the seaward boundary to about 1.4 km at the landward boundary. In addition, the water depth also decreases greatly, from 65 m near the seaward boundary to about 9 m above mean sea level near Avonmouth. The model domain was divided into 14606 unstructured triangular cells and allows a large variation in the scales of regions of interest, which in this study varies from 500 m to 5000 m.

From tidal analysis, the dominant constituent in the Bristol Channel is the M_2, followed by the S_2, N_2 and μ_2 (Hashemi et al., 2008), but in this study attention is restricted to the dominant harmonic constituent, the M_2 tide. The open seaward boundary was forced with a single sinusoidal constituent with an amplitude distribution across the boundary given by that of the M_2 constituent interpolated from the same model (Serhadlıoğlu et al., 2013). The DG-ADCIRC modelling parameters were set as constant throughout the study: most of the parameters were attained as the default values recommended by the ADCIRC model developers, some are same with that of Serhadlıoğlu's study; while the rest, such as time step, bottom friction, wetting and drying were determined using equations and a parameter sensitivity analysis for the area of focus (Table 2). Initially, the simulations were run without any meteorological input, in order to observe the general flow of tides though the region.

Table 2 . Key model parameters.

Parameters	Values	Descriptions
NOLIFA	2	Finite amplitude terms are included in the model run and wetting and drying function is enabled
NTIP	1	Tidal potential forcing is used
G	9.80665	Gravitational acceleration (ms^{-2})
TAU0	-1	Weighting factor that weights the relative contribution of the primitive and wave portions of the GWCE
DT	1	ADCIRC time step (in seconds)
H0	0.25	Minimum water depth (units of length)
ESLM	8	Spatially constant horizontal eddy viscosity for the momentum equations (units of length^{2}time^{-1})

Table 3 . Comparison of the observed M_2 tidal elevations and phases against model results using various bed friction coefficients.

Station	Amplitude (m)					Phase (°)				
	Obs.	0.0025	0.0035	0.004	0.005	Obs.	0.0025	0.0035	0.004	0.005
Stackpole Quay (51.63,4.85)	2.51	2.52	2.52	2.51	2.51	168	172	172	172	172
Mumbles (51.57,4)	3.18	3.10	3.05	3.04	3.01	171	173	176	176	178
Swansea (51.62,3.93)	3.19	3.15	3.12	3.10	3.03	173	174	176	177	179
Port Talbot (51.58,3.78)	3.13	3.17	3.15	3.12	3.07	173	174	176	177	179
Barry (51.4,3.28)	3.92	3.90	3.80	3.74	3.62	185	182	187	190	193
Steep Holm island (51.33,3.1)	3.87	4.04	3.94	3.88	3.75	186	183	189	191	196
Cardiff (51.48,3.17)	4.01	4.07	3.95	3.89	3.79	191	186	190	193	199
Weston-super-Mare (51.35,2.97)	3.95	4.17	4.01	4.00	3.87	181	184	189	193	199
Hinkley Point (51.2,3.13)	3.8	4.00	3.89	3.83	3.71	195	181	186	188	193
Minehead (51.2,3.47)	3.59	3.70	3.64	3.59	3.49	183	176	180	182	186
Porlock Bay (51.22,3.6)	3.42	3.54	3.47	3.43	3.36	179	173	176	178	181
Ilfracombe (51.2,4.12)	3.04	3.00	2.99	2.97	2.94	162	165	166	167	168
Appledore (51.05,4.18)	2.57	2.62	2.59	2.57	2.52	165	170	172	173	174
Port Isaac (50.58,4.82)	2.47	2.42	2.42	2.42	2.41	144	151	151	151	151

RESULTS AND DISCUSSION

Model Calibration

In order to achieve the most accurate results, the model was tuned by adjusting the quadratic bottom friction coefficient (c_f) until the model predicted and observed M_2 tidal elevations and phases were in closest agreement. The observational data were from the United Kingdom and Ireland Admiralty tide tables of the year 1997. The final results for fourteen calibration sites are summarised in Table 3. Since all the computed M_2 phases at the north end of the ocean boundary (Stackpole Quay) show a 4° shift from the observed value, the most consistent computed phase are assumed to be 4° larger than the observed phase. It is seen from the table that the model results show a good agreement with the observed data when using a bed friction coefficient c_f=0.004. However, this estimation is an average for the entire area and do not reflect real spatial variability in frictional forces.

Model Validation

The wave forms of M_2 produced by this model (blue curve) were compared at a location, Mumbles (51° 34'N 3° 58'W), to those observed from the British Oceanographic Data Centre BODC (red curve) (Fig. 3). The data presented in the figure is from 01st January 2011 onwards. After the 2-day initial spin-up period, the general wave forms are in close agreement, the consistent pattern of two high/low tides of each day can also be seen. There is a minor discrepancy in the amplitude of the waves observed as the systems approach a spring tide. This is likely to be because only the M_2 tidal constituent was used in the ADCIRC model whereas field measurements are subject to all the harmonic components. This depth averaged model is, of course, a simplification of the complex three dimensional flows that exist in reality and so perfect agreement is not expected.

Quarter-wave Tidal Resonance?

Considering the response of the system using the natural forcing frequency (ω/ω_{M_2}=1.0), it is found that the dominant frequency of the Bristol Channel response is larger than the natural frequency (Fig. 5, Fig. 6, Fig. 8, Fig. 10, Fig. 11), which indicates that the basin length of the Bristol Channel is shorter than the quarter wavelength required for resonance. This is consistent with the result if we compare the M_2 quarter wavelength with the length of the Channel. Tidal waves in the Bristol Channel behave as "long waves" since their wavelength is much greater than the water length implying vertical motion may be neglected. The wavelength of a shallow water wave is given by the formula below:

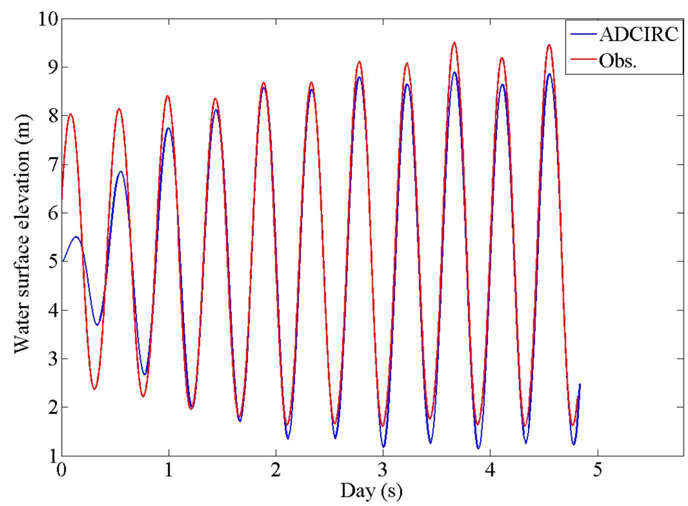

Fig. 3 Comparison of M_2 wave forms between model results and the British Oceanographic Data Centre.

$$L = \frac{2\pi\sqrt{gH}}{\sigma} \tag{4}$$

where g is the gravitational acceleration (ms^{-2}), H is the water depth (m) and σ is the angular frequency of the tidal component (radians s^{-1}) (Godin, 1993). If we take $H \approx$ 40m for the Bristol Channel, since σ = $1.405\times10^{-4}s^{-1}$ for the component M_2, we find $\frac{1}{4}L$=221km for the Bristol Channel. This model shows that the length of the Channel is around 160km, therefore somewhat shorter than an M_2 quarter wavelength. It is presented in the theory of tidal resonance that the basin must satisfy the well-known quarter wavelength requirement. Godin (1993) found that the restricted concept was a necessary but not sufficient condition for the phenomenon to occur, especially for deeper embayments. Serhadlıoğlu (2014) found the Bristol Channel is shorter than the quarter wavelength for the M_2 tidal period. The results presented in this paragraph agree with previous findings and suggest the quarter wavelength requirement may as well become more relaxed for shallow basins.

Resonant Periods

The M_2 response curves of the Bristol Channel have been investigated by exciting the model using artificially altered M_2 forcing frequencies (ω), and applying a ratio (ω/ω_{M_2}) varying between 0.5175 and 5.175. The simulated results of M_2 elevations were recorded at 16 stations along the Bristol Channel (Fig. 4). The response curves of several stations taken along the Bristol Channel are plotted in Fig. 5, in which four stations (ST3/5/7/9) were chosen to represent the inner section of the Channel while three stations (ST11/13/15) represent the outer Channel. In Fig. 5, the response curves follow a similar pattern at all the stations considered, but differ in magnitude due to the location of the observation station. It is shown that all the stations show a peak in response at around a ratio of 1.2-1.4 which indicates the quarter wavelength resonance of the system with a period of approximately 8.6-10 hours. A second peak is observed at ω/ω_{M_2}=3.1-3.6 in the outer Channel, and the peak variation is seen to be very site-dependent. According to equation (4), this might be a resonance occurring at the Channel entrance.

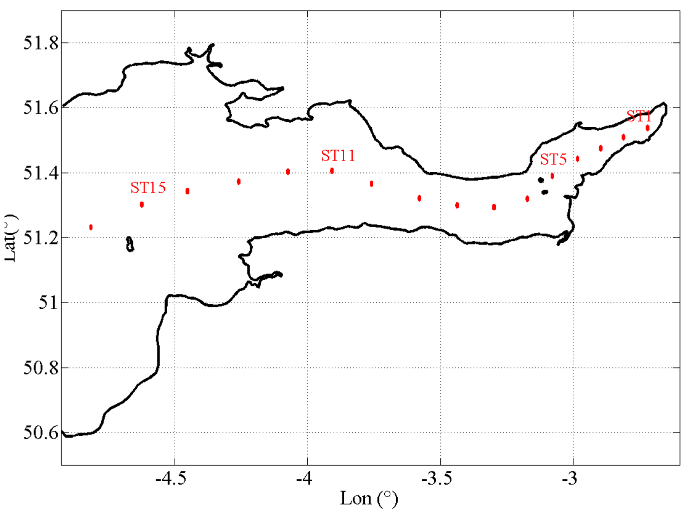

Fig. 4 Model and the 16 observation stations.

The resonant period calculated from this model result agrees well with the previous studies, which suggested a resonant period between 7.3 to 11.3 hours (Table 1). It should be noticed that the resonant period is always longer when the Celtic Sea is included. The method used in this study was same with Serhadlıoğlu (2014) but the dominant tidal period obtained here is slightly lower than what has been found in her study. This subtle difference might be due to the coupled nature of two different modelling systems and this reason was also suggested by Serhadlıoğlu (2014). In the present study the model domain only includes the Bristol Channel while in Serhadlıoğlu's study the Irish Sea, the Celtic Sea, English Sea and Bristol Channel are all included; additionally, the present model applies a coarser mesh than the one of Serhadlıoğlu's. These may both contribute to the peak shift of the resonant response.

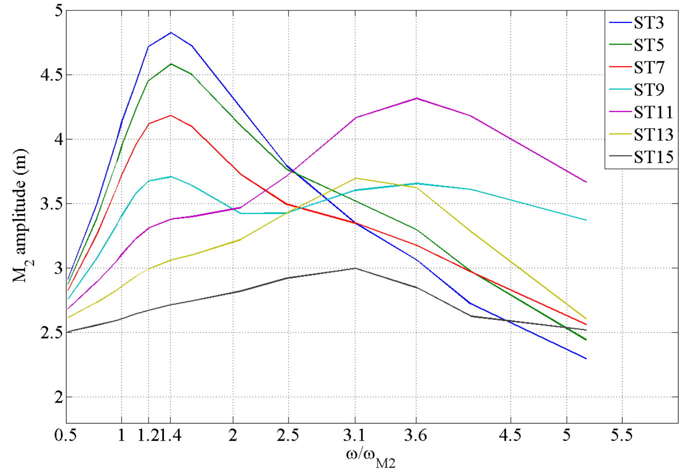

Fig. 5 The response curves of several stations along the Bristol Channel: ST3, ST5, ST7 and ST9 represent the inner Channel response; ST11, ST11 and ST15 represent the outer Channel response.

Fig. 6 shows the amplification of the response along the Bristol Channel by normalising the response of the Channel by the elevation at the Channel mouth (blue curve). The figure also shows the amplification within the inner Channel (red curve) and in the outer Channel (green curve). The inner section of the Bristol Channel (red curve) shows an amplified response over the frequency range ω/ω_{M_2} ratio of 1.2-1.5 while the outer part of the Channel (green curve) exhibits an apparent resonance around the ratio of 4.1. The result indicates a coupled resonant system of the Bristol Channel: the main peak might be the dominant resonant mode of the Bristol Channel, while the second peak might be due to the response of the Channel to the forcing at its mouth. This complicated resonance pattern was also found by Liang et al. (2014). They suggested that some regions in the outer Bristol Channel also experience significant, if not greater, resonances when the ω/ω_{M_2} ratio =3-6.

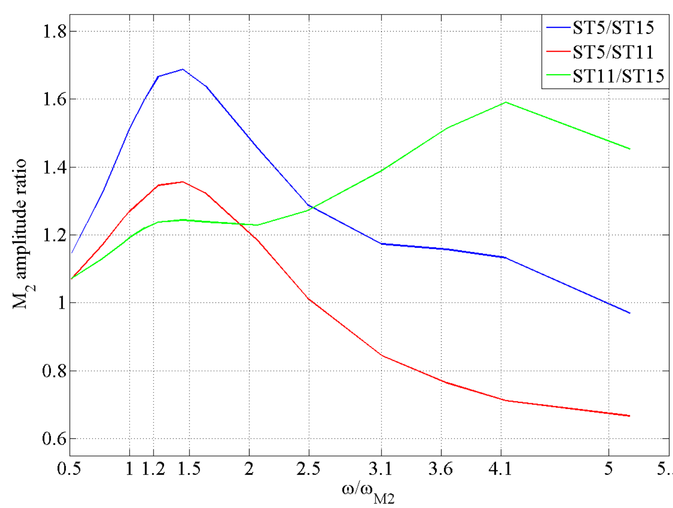

Fig. 6 Amplification of the response observed in the Channel.

Five stations (A, B, C, D and E) were set across the lower Channel from north to south to further investigate the spatial variation of the response with the open boundary (Fig. 7). Among these five stations Station C is overlapped with ST11, which is near the Swansea Bay. It is seen in Fig. 8 that at around the forcing frequency of ω/ω_{M_2}=3-4 there is a significant increase in the M_2 amplitude response. When approaching the coasts, the response slightly increases which is probably due to the decrease in water depth. Increases of response amplitudes in Station A and B are more obvious, this may attribute to the topography of the coast near them compared to the other side.

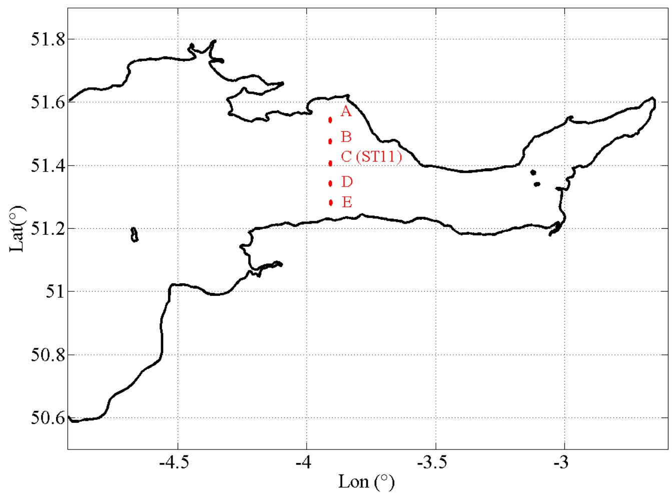

Fig. 7 Five observation stations (A, B, C, D and E) near Swansea Bay.

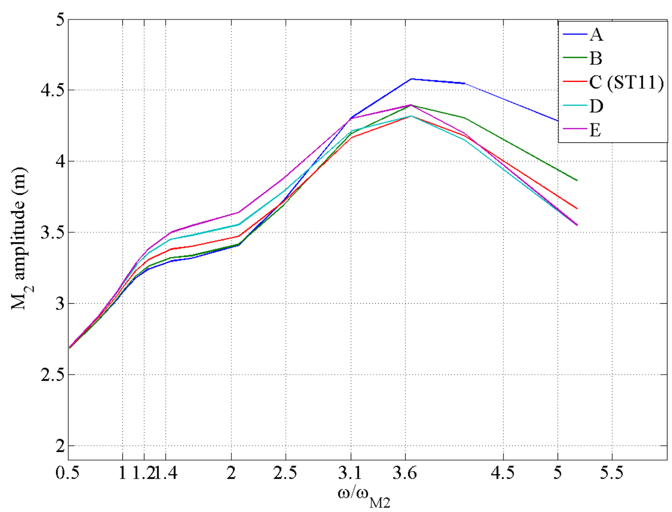

Fig. 8 The response curves of stations across the Bristol Channel.

Fig. 9 and Fig. 10 present the comparison of M_2 response curves obtained from the original model and the results from two different boundary positions. The model with blue boundary (boundary 1) is the original one adopted in this study. It is seen that the peak at around ω/ω_{M_2}=1.2-1.4 is not shifted with changing boundary position, while the second peak is apparently boundary-dependent because it is shifted from around a ratio of 3.1 to around 4.1 with the boundary moving up to the Channel head. This again indicates that the second peak should be the tidal response with the open boundary at the outer Channel. It is also worth noticing that the response amplitudes decrease with the boundary position moving from the ocean to the Channel head. Therefore, in the present study, the dominant resonant mode of the response M_2 in the Bristol Channel are hardly affected by the slight changing of boundary position, but the response amplitudes can be affected.

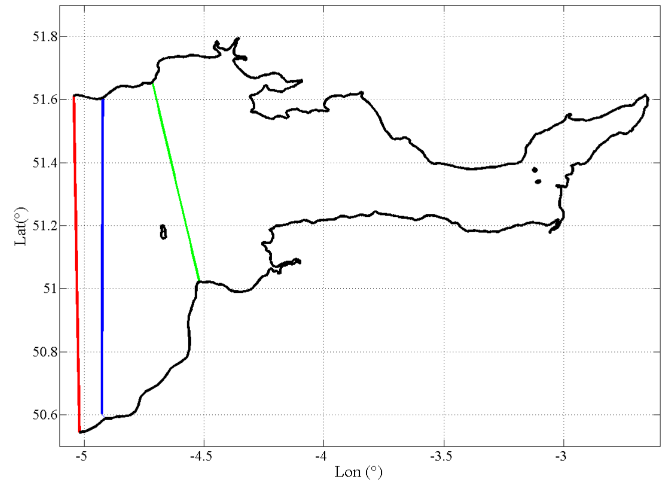

Fig. 9 Model with different boundary positions (red: boundary3; blue:boundary1; green: boundary2).

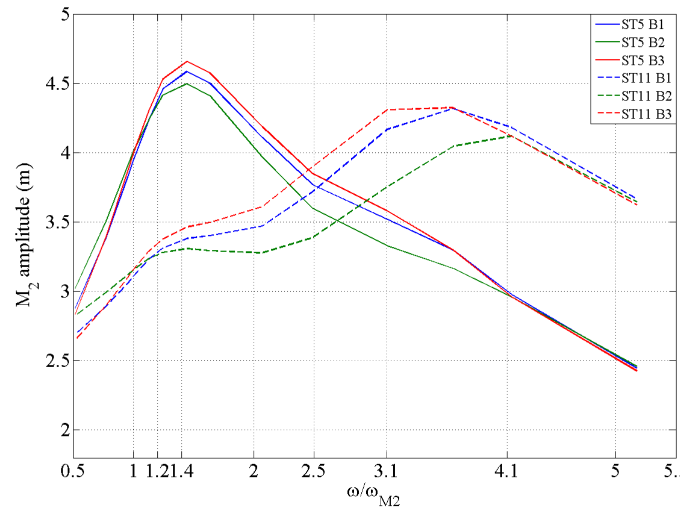

Fig. 10 Response curves of ST5 and STS11 with different boundary positions.

The Bristol Channel is a complex hydrodynamic system and sensitive to small changes, factors such as bed friction can have influences on the resonant period. As seen in Fig. 11, it is obvious that the amplitudes of M_2 response curves drops with increasing bed friction coefficient. However, another surprising find is that the bed friction can affect the resonant period since a shift of the peak can be seen from a ratio of ω/ω_{M_2} =1.3 to ω/ω_{M_2}=1.5. There is obviously a great deal of interest in extracting

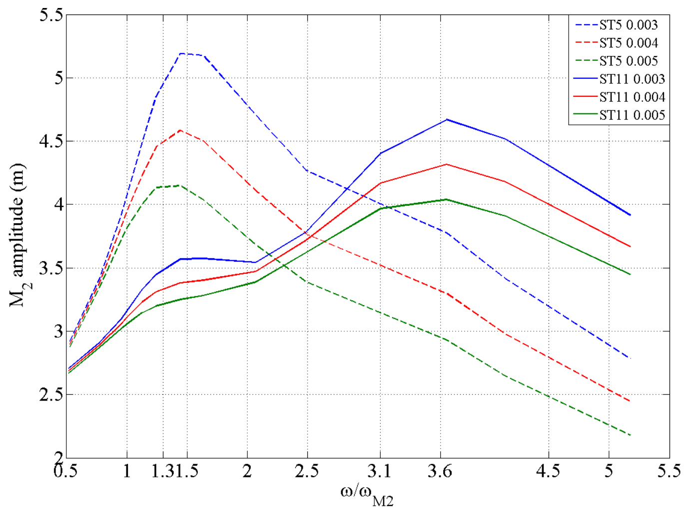

Fig. 11 Response curves of ST5 and ST11 with different bed frictions.

energy from the Bristol Channel, and adding bottom friction would be a simplistic representation of additional dissipation in the model. Therefore, the results here have some implication for what might happen once people started to try extracting power from the system.

Sensitivity Tests

The tidal regime is most sensitive to changes in those estuaries where they are close to resonance. The results of the sensitivity tests on water level, nodal factor and bed friction in both inner (ST5) and outer (ST11) parts of the Channel are shown in Fig. 12. The response tides are driven by tidal forcing of M_2 constituent.

Sea level rise is the dominant influence on any far-field impacts and has influenced the tidal regimes in the past. Some modelling studies show even moderate sea level rise may have significant impact on the tides on the European shelf (Ward et al., 2012). In the present study, however, with water level varying from 4 m lower to 4 m higher than the real situation, the tidal heights do not present visible change in the Bristol Channel. The angle between the plane of the Moon's orbit around the earth and the plane through the equator of the Earth varies with a period of 18.6 years, and the nodal tidal cycle is usually represented as a linear modulating factor in the calculation of the tidal amplitudes (Adcock et al., 2014). Thus for the M_2 constituent:

$$\eta_{M_2} = f_{M_2} \times a_{M_2} cos(\omega_{M_2} t + g) \tag{5}$$

where η_{M_2} is the water level variation at the frequency of M_2, f_{M_2} is the nodal factor, a_{M_2} is the amplitude of the M_2 constituent, ω_{M_2} is the frequency of the M_2 tide and g its phase. M_2 was the only tidal constituent used in this model, therefore f_{M_2} dominates the annual water level variation. Over a period of 9.3 years, the M_2 nodal factor changes from its minimum value 0.96, to its maximum 1.04; however, during this time period the response of M_2 tidal heights almost remain unchanged. This suggest the response is essentially linear.

The bed friction is seen to be a dominant effect on the tidal response, since the M_2 amplitude at ST11 decreases by around 0.2m with the quadratic friction coefficient increasing from 0.0025 to 0.005. The bed friction has even greater influence on the shallower areas: at ST5 the amplitude change reaches more than 0.3m. In nature the forced resonant

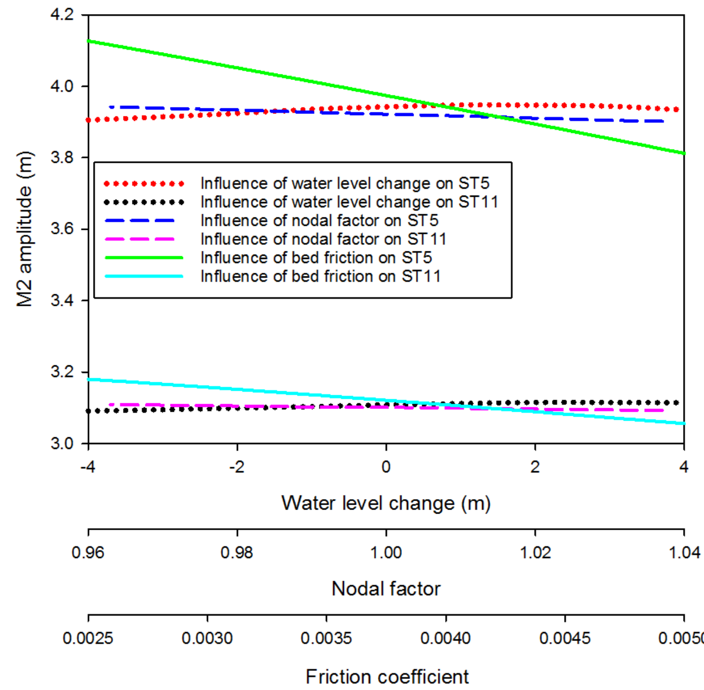

Fig. 12 Sensitivity test results on water level (dotted lines), nodal factor (dash lines) and bed friction (solid lines) at ST5 and ST11.

oscillations cannot grow indefinitely because the leakage of energy due to friction increases more rapidly than the amplitudes of the oscillations themselves. Tidal amplitude is strongly affected by frictional resistance especially in shallow channels. When friction is incorporated, the progressive waves are damped so that the elevation decreases with distance in the wave propagation direction, and the reflected waves may also travel along with lower amplitudes (Allen, 2009).

CONCLUSIONS

A two-dimensional shallow water model was used to study the tidal resonance in the Bristol Channel. The results show that the quarter-wavelength resonant period of the Bristol Channel is close to but shorter than the semi-diurnal tidal band, suggesting that the basin length of the Bristol Channel is shorter than the resonant quarter wavelength. Generally speaking, the main resonance of the channel seems consistent with previous studies, and the second peak at about ω/ω_{M_2}=3.1 might be the resonant response of the outer Channel itself. The main resonance is slightly sensitive to increased bed friction which may have implications for tidal energy extraction. Neither the amplitude on the boundary nor the mean water level has a significant impact on the resonant response, suggesting that the nodal factor of the tide, or any possible sea-level rise, will not influence the tidal response in the Channel dramatically. Further investigation of the idealised 2-D model and consideration of the real situation of tidal dynamics in the Bristol Channel are necessary to take this analysis further.

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