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OCEAN WAVE NON-LINEARITY AND WIND INPUT IN DIRECTIONAL SEAS – ENERGY INPUT DURING WAVE-GROUP FOCUSING

Thomas A.A. Adcock

Department of Engineering Science
University of Oxford
Oxford, Oxfordshire, OX1 3PJ, UK
thomas.adcock@eng.ox.ac.uk

Paul H. Taylor

School of Civil, Environmental and Mining Engineering
University of Western Australia
Stirling Highway, Perth, Western Australia.

ABSTRACT

There has been speculation that energy input (wind) can play an important role in the formation of rogue waves in the open ocean. Here we examine the role energy input can play by adding energy to the modified non-linear Schrödinger equation. We consider NewWave type wave-groups with spectra which are realistic for wind waves. We examine the case where energy input is added to the group as the wave-group focuses. We consider whether this energy input can cause significant non-linear effects to the subsequent spatial and spectral evolution. For the parameters considered here we find this to have only a small influence.

INTRODUCTION

Wind causes gravity waves to occur in the open ocean. However, in the short term analysis of wave evolution this is usually neglected – in part because this is small on a wave-by-wave basis but also because the details of the energy transfer from wind to water are not thoroughly understood. In this paper we use a very simple model for energy input and explore how this might alter the non-linear physics as wave-groups focus and de-focus.

The local influence of wind on waves has been suggested as a possible cause of rogue waves [1, 2]. In this context the implication is the wind causes more extreme waves than would occur in its absence. That this can happen has yet to be convincingly demonstrated. A second motivation is that wind and non-linear energy transfers are two of the key components of physics in spectral wave models, commonly used for forecasting and hind-

casting. Any improved understanding of the fundamental physics is useful for informing the spectral changes in such models.

Whilst numerous authors have looked at wind input for spectral wave models here we will focus on energy input into phase resolved waves. A common approach, and the one taken in this work, is to use a variant of the non-linear Schrödinger equation and inject into this some energy (for example [3, 4]). Other numerical approaches are limited but attempts have been made to couple CFD wind models with wave models (e.g. Yan & Ma [5]). Many people have also tried to investigate the problem experimentally [6, 7] although such work has difficulties with scaling. With a few exceptions [8] these studies have been on unidirectional seas (and usually with spectra not representative of steep ocean waves). However, in the real ocean, steep waves are always found to have significant directional spreads. This fundamentally changes the non-linear physics which occurs during the formation of an extreme wave in the open ocean (see discussion in Adcock & Taylor [9]). In this paper we therefore focus on waves with a realistic directional spread.

We conducted this study primarily to see if we could induce two effects connected with rogue waves:

1. Fast non-linear physics can change the shape of directionally spread wave-groups on deep water [10–15]. In previous work we found that the expansion of the wave-group in the lateral direction occurs at much lower non-linearity than the contraction in the wave-group [16, 17]. Now to trigger the classic modulation instability we need a wave-group which is (qualitatively), steep, long-crested, and long in the

mean wave direction. In this paper we are interested to see whether, by energy input during the focusing, we could induce a scenario where the lateral expansion occurred but not the contraction of the group and whether this could induce an instability or even just a persistent wave-group.

2. Another possibility was whether inputting energy during the wave focusing could make a major difference to the spectral changes. Some of the changes which occur as a wave-group focuses are reversed as it subsequently defocuses [11, 18]. We are interested to see whether energy input might alter the non-linear spectral changes during focusing (by making focusing much more rapid giving less time for non-linear physics to occur). We were also interested in how the spectrum would subsequently evolve and whether the energy input could lead to different changes in the resulting spectrum.

METHODS

In this study we use the modified non-linear Schrödinger equation (MNLSE) to model the dynamics of ocean waves (see [19, 20]). This is a narrow-banded approximation to the full potential flow equations which describe wave evolution. The conservative equation has been used by numerous authors as a model for non-linear waves [21–23]. The MNLSE has a number of limitations [24–27]. Despite some limitations the model has been shown to capture much of the key non-linear physics of the evolution of deep water waves. The MNLSE used here is

$$\begin{aligned} \frac{\partial U}{\partial t} + \frac{\omega}{2k_0} \frac{\partial U}{\partial x} + i \frac{\omega}{8k_0^2} \frac{\partial^2 U}{\partial x^2} - i \frac{\omega}{4k_0^2} \frac{\partial^2 U}{\partial y^2} \\ - \frac{\omega}{16k_0^3} \frac{\partial^3 U}{\partial x^3} + \frac{3\omega}{8k_0^3} \frac{\partial^3 U}{\partial y^2 \partial x} - i \frac{5\omega}{128k_0^4} \frac{\partial^4 U}{\partial x^4} \\ - i \frac{3\omega}{32k_0^4} \frac{\partial^4 U}{\partial y^4} + i \frac{15\omega}{32k_0^4} \frac{\partial^4 U}{\partial y^2 \partial x^2} + \frac{7\omega}{256k_0^5} \frac{\partial^5 U}{\partial x^5} \\ - \frac{35\omega}{64k_0^5} \frac{\partial^5 U}{\partial y^2 \partial x^3} + \frac{21\omega}{64k_0^5} \frac{\partial^5 U}{\partial y^4 \partial x} = - \frac{i\omega k_0^2}{2} U |U|^2 \\ - \frac{3}{2} \omega k_0 |U|^2 \frac{\partial U}{\partial x} - \frac{1}{4} \omega k_0 U^2 \frac{\partial U^*}{\partial x} - ik_0 U \left. \frac{\partial \phi}{\partial x} \right|_{z=0} + \frac{1}{2} \Gamma U, \quad (1) \end{aligned}$$

where ω and k_0 are the frequency and wavenumber of the carrier wave. U is the complex wave envelope. The individual waves and the carrier wave are moving in the positive x direction. The left hand side of this equation is a high order approximation to linear evolution. The right hand side contains non-linear and energy input terms. For linear evolution we simply set the non-linear terms in the right hand side of equation 1 to zero. The final term in equation 1 simulates the interaction with the induced local surface current driven by spatial variations in the wave en-

velope. The return current term, ϕ , can be found from

$$\left. \frac{\partial \phi}{\partial z} \right|_{z=0} = \frac{\omega}{2} \frac{\partial |U|^2}{\partial x}, \quad (2)$$

and within the fluid $\nabla^2 \phi = 0$, with ϕ tending to zero as z goes to infinity. We note that even for the deep ocean the water depth is still finite and that the length-scale of the return current is influenced by this [28].

The local interactions of wind and waves are exceptionally complex. One theory is the mechanism of Miles [29] which is unquestionably over simplistic but, even in situations when formally it should not apply, does seem to agree to some degree with observations. A number of authors have attempted to include this mechanism in the NLSE [3, 30, 31] where, to the leading order, it appears as a simple input of energy proportional to the envelope. Studies have also been made of dissipation [32, 33] where the equations take a similar form. In this study we use this approach. Thus we include an extra term in the governing equations given by $\frac{1}{2} \Gamma U$ where Γ is a constant of proportionality controlling the rate of energy input. To set our values of Γ in context, the highest value we consider ($\Gamma = 0.008 \text{s}^{-1}$) equates to a $\sim 5\%$ increase in amplitude over a wave period.

Using this simple form has the added benefit that the equations can be rewritten (following [32]) by making the transformation $U = Q \exp(\Gamma t/2)$. The governing equation then becomes

$$\begin{aligned} \frac{\partial Q}{\partial t} + \frac{\omega}{2k_0} \frac{\partial Q}{\partial x} + i \frac{\omega}{8k_0^2} \frac{\partial^2 Q}{\partial x^2} - i \frac{\omega}{4k_0^2} \frac{\partial^2 Q}{\partial y^2} - \frac{\omega}{16k_0^3} \frac{\partial^3 Q}{\partial x^3} \\ + \frac{3\omega}{8k_0^3} \frac{\partial^3 Q}{\partial y^2 \partial x} - i \frac{5\omega}{128k_0^4} \frac{\partial^4 Q}{\partial x^4} - i \frac{3\omega}{32k_0^4} \frac{\partial^4 Q}{\partial y^4} \\ + i \frac{15\omega}{32k_0^4} \frac{\partial^4 Q}{\partial y^2 \partial x^2} + \frac{7\omega}{256k_0^5} \frac{\partial^5 Q}{\partial x^5} - \frac{35\omega}{64k_0^5} \frac{\partial^5 Q}{\partial y^2 \partial x^3} + \frac{21\omega}{64k_0^5} \frac{\partial^5 Q}{\partial y^4 \partial x} = \\ k_0 e^{(\Gamma t)} \left[- \frac{i\omega k_0}{2} Q |Q|^2 - \frac{3\omega}{2} |Q|^2 \frac{\partial Q}{\partial x} - \frac{\omega}{4} Q^2 \frac{\partial Q^*}{\partial x} - i Q \left. \frac{\partial \phi_Q}{\partial x} \right|_{z=0} \right], \quad (3) \end{aligned}$$

where ϕ_Q can be found in a similar way to the above. Although we do not solve this equation directly here it is useful for helping to interpret results and the inter-play of energy input and non-linearity. Simplistically, the ‘Q-form’ might suggest that the non-linear part of the equation scales with energy input, suggesting that energy input can enhance the non-linear dynamics.

In this study we examine the evolution of an isolated ‘NewWave’ wave-group which focuses and then de-focuses. The ‘NewWave’ is the average shape of an extreme event in a Gaussian random process – it is equal to the scaled auto-correlation function (see [34]). This has the advantage of being straightforward to interpret. Adcock *et al.* [14] found that the average

change to wave-groups due to non-linear physics was well captured by the NewWave model. Past studies [11, 12] have used isolated wave-groups to examine spectral changes. One might consider a real random sea to be made up of many such focusing and de-focusing events which lead, in the end, to cumulative changes to the underlying wave spectrum. Thus by studying the changes to the spectrum of a wave-group one gets some, although not perfect, insight into global spectral changes in the ocean.

We consider a wavegroup which has an envelope at focus given by a Gaussian $U(x, y, t = 0) = A \exp(-\frac{1}{2}s_x^2 x^2) \exp(-\frac{1}{2}s_y^2 y^2)$. This is an excellent approximation the average shape for an extreme event in the ocean for a Gaussian spectrum (see [35]). In this study we use $s_x = 0.0046\text{m}^{-1}$ and $s_y = 0.0073\text{m}^{-1}$ and take the period of the carrier wave as 12 s. The bandwidth in the mean wave direction, s_x is chosen to be a narrow banded approximation to a JONSWAP spectrum with $\gamma = 3.3$. The lateral bandwidth is chosen to give a wrapped normal directional spreading with rms of 15° . These values have been used previously by [11, 13, 14].

Starting with a focussed Gaussian we run the model ‘back’ in time under linear evolution for 16 periods ($t = -16$). When not considering energy extraction we do this by setting the right-hand side of equation 1 to zero and changing the sign of the time derivative (see also [27]). However, when considering wind excitation we extract energy from this so that if the group re-focused under linear evolution, but with energy input, it would have the same energy and amplitude that we started with. This is done by adding the term $-\frac{1}{2}\Gamma U$ to the right hand side of equation 1. This does not imply that the energy extracted when going back in time is exactly the same as that when we run the model forward using the non-linear model – energy input is proportional to amplitude and in the non-linear simulation amplitude will be different – indeed this is the point of this study. At the point at which the wave-group would have focussed had the evolution been linear ($t = 0$) we turn off energy input. The subsequent evolution during the de-focusing is driven only by the standard non-linear terms in equation 1 without energy input (except in the final section of the paper). We halt the simulation 16 periods after linear focusing time ($t = 16$). This process is summarised in Figure 1.

RESULTS

We analyse two wave steepnesses to explore the effect of non-linearity. We classify our simulation cases based on the steepnesses: we consider a 5 m crest amplitude at linear focus ($ak_0 = 0.17$) and also a group with amplitude at linear focus of 8 m ($ak_0 = 0.25$).

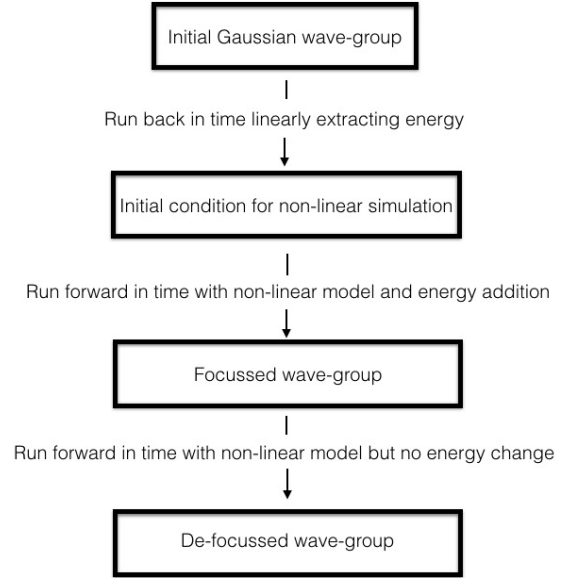


FIGURE 1. Schematic of approach taken in this paper.

Elevation and shape of the group

We start by examining the spatial maximum of the envelope over the course of the simulations. Figure 2 presents these. The initial amplitude of the simulations with greatest energy input is of course lower, since these have had the energy removed whilst the group has run backwards. For the lower steepness case, all the groups refocus to very nearly the same amplitude as under linear evolution (see also Figure 3a described later). For the steeper case we find the amplitude is slightly lower for the high energy input – this is presumably due to slight de-focusing of the wavegroup.

After linear focus ($t = 0$), energy input is stopped and simulations continue under energy conserving non-linear evolution. The subsequent evolution of all groups, at least as far as the maximum elevation is concerned, is very similar. There is a small asymmetry with time. The group persists slightly longer after focus (an effect which is far more dramatic in uni-directional waves [36]) but this appears to be consistent regardless of the energy input (which could also be thought of as regardless of the shape of the group at focus for the parameters considered here). There are no strongly nonlinear processes being activated that might cause a group to persist for many periods after focus.

We now consider the shape of the groups at the maximum point in their evolution. Figure 3 presents these along with the linear focussed NewWave shape. For the case without energy input we observe the expected difference between linear and non-linear evolution ([11, 13, 14, 17]). Thus we see (i) very small amounts of extra elevation; (ii) broadening of the wave-group in the lateral direction; (iii) narrowing of the wave-group in the mean wave direction; (iv) movement of the large wave to the

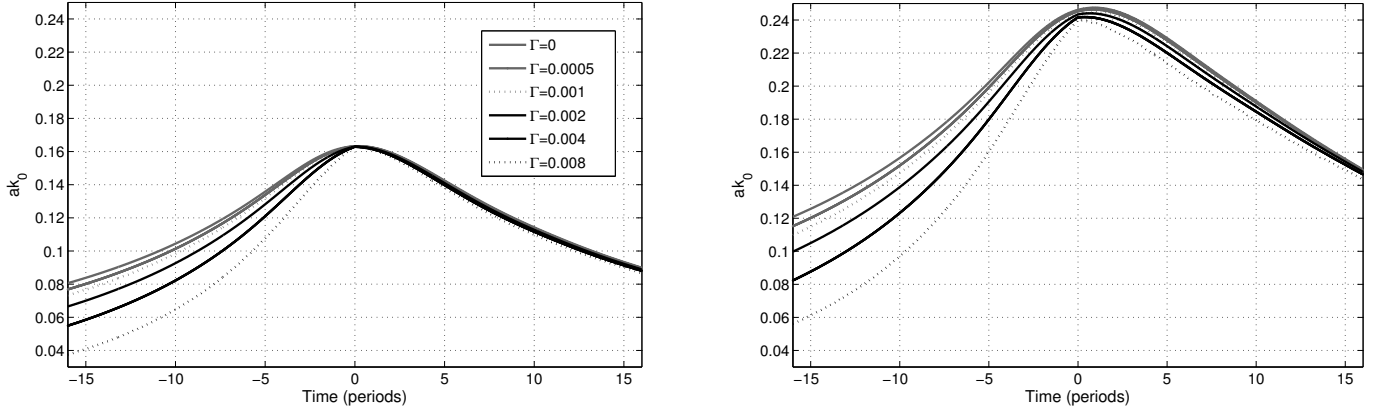


FIGURE 2. Maximum elevation of the group over the course of the simulation. Left – $ak = 0.17$; right – $ak = 0.25$.

front of the wave-group relative to linear evolution. As expected these changes are more significant for the more non-linear case. The change in each of these is examined below.

The basic shape is the same for all the non-linear runs – the differences between the groups are very difficult to see visually. The higher the energy input the smaller the changes to the group shape. This is presumably because the group has a shorter time when the amplitude is large and the non-linear physics can take effect.

To quantify the changes to the wave-group we ‘fit’ a Gaussian to the wave-packet. We define these simply by finding where the amplitude of the group drops below $2/3$ of its maximum value (consistent with previous studies [16]). To evaluate the change in shape we equate this to the width of an equivalent Gaussian to find the ‘bandwidth’ in the x and y directions. We also use this to evaluate the asymmetry of the wave-group envelope in space. We define asymmetry as the ratio of distances from the crest to the points at which the wave envelope drops below $2/3$ of the crest behind and in front in the mean wave direction. Thus an asymmetry of 1 implies a symmetrical group whereas a value greater than 1 implies the crest has moved towards the front of the group. In all cases we present the ratio of the bandwidth at non-linear focus to the bandwidth at linear focus. Thus a bandwidth ratio greater than one would imply the group has contracted in this direction (and *vice versa*). These parameters are shown, as well as the normalised elevation at non-linear focus, in Figure 4.

As expected from visual inspection of Figure 3, energy input does not make major difference to the non-linear changes to the shape of the extreme wave-groups. The contraction of the wave-group is known to be very sensitive to the non-linearity [16, 17] and we find this again here. For the high energy input cases there is insufficient time for the non-linear changes to take place and the contraction of the group in the mean wave direction is very significantly reduced. The lateral expansion of the group is less

sensitive to non-linearity – whilst there are smaller changes for the high energy input cases these differences are less than for the contraction of the group in the mean wave direction. The movement of the crest to the front of the group is similar for different values of Λ suggesting that this change occurs in the last few periods before focusing.

Spectral changes

Non-linear evolution will lead to spectral changes. Initial spectra (which remain unchanged in shape under linear evolution and simply scale with energy input) are given in Figure 5. Figures 6 and 7 present the spectra at non-linear focus and at the end of the simulation where the group is dispersing and steepness reduced so that there is little further evolution of the spectrum.

As expected the non-linear changes to the group are larger for the steeper case. However, the form of the spectral changes are the same for both the steepnesses considered. The dominant processes are slightly different during focusing and de-focusing (see [11, 12]). As the group is focusing, energy is transferred to higher wave-numbers which are close to the mean wave direction. This is coupled with a small loss of energy from components travelling at large angles to the mean wave direction. During de-focusing this is partially reversed but there is also a downshift of the spectral peak and movement of energy to high wavenumbers at about 25° to the peak of the spectrum.

The effect of energy input in our simulations is to suppress the non-linear changes before focus as the group is significantly less steep compared to the energy input case. Consequently, during the de-focusing process the changes to the spectrum are larger since normally the de-focusing would reverse some of the changes during focusing.

Energy input does not make a dramatic difference to the spectral changes during the simulation. It certainly does not fundamentally change these. The dominant difference in the high energy input is simply to reduce the overall non-linear changes

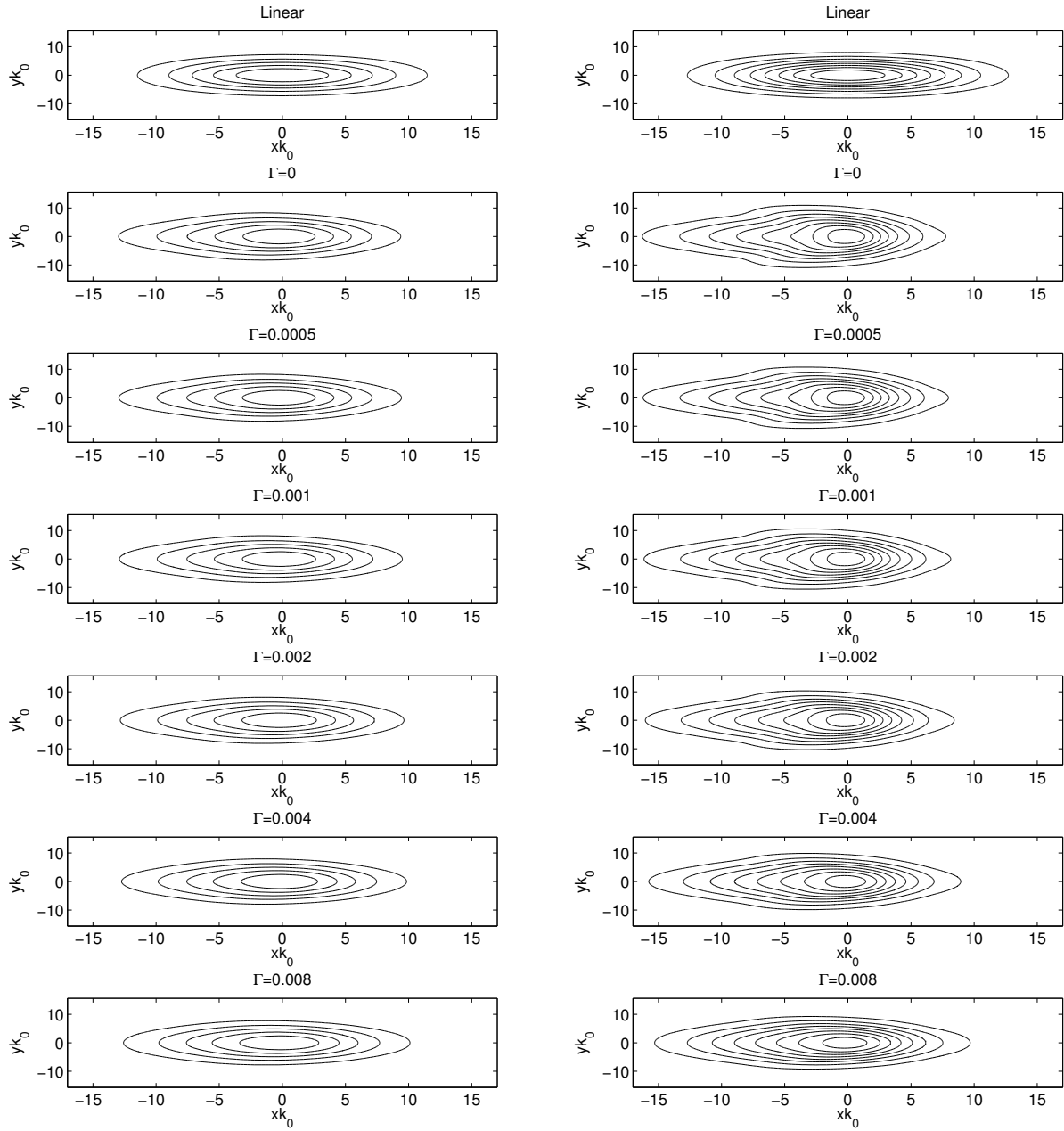


FIGURE 3. Envelope of wave-groups at non-linear focus. Left – $ak = 0.17$; right – $ak = 0.25$. Contours at 1 m intervals. Envelopes centered on maximum point.

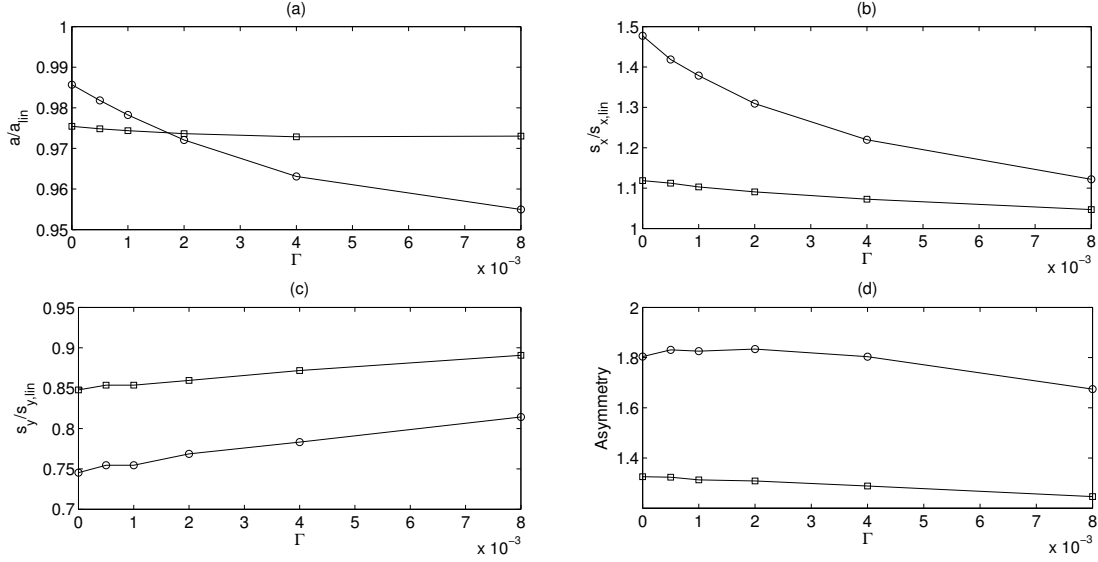


FIGURE 4. Parameters describing the shape of the wave-group at non-linear focus. (a) amplitude; (b) bandwidth in mean wave direction; (c) bandwidth in the lateral direction; (d) asymmetry. Squares – $-ak_0 = 0.17$; circles – $-ak_0 = 0.25$.

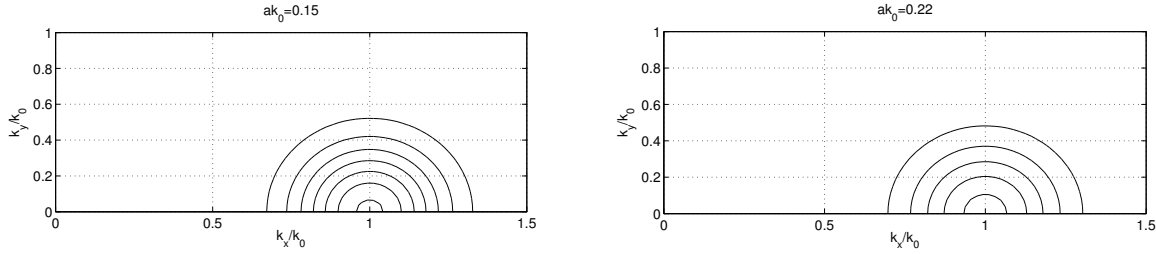


FIGURE 5. Initial (linear) wavenumber amplitude spectra for the two cases. Left has contours at 0.005 m intervals; right at 0.01 m intervals. Amplitude spectrum based on area of ocean of $2.1 \times 10^7 \text{ m}^2$.

(movement of energy to high wavenumbers and downshift of the peak) with no obvious additional redistribution of energy.

DISCUSSION

In this paper we set out to examine the interplay between non-linear wave physics and energy input to the wave-group. We have chosen a somewhat arbitrary model (energy input just during focusing) to explore whether this could trigger some significant interactions. We have found that, even for unrealistic high energy input rates into the wave-group, we do not generate either any dramatic non-linear effects, nor major differences to how the spectrum evolves.

In different scenarios there may of course be significant interactions between energy input and non-linear physics. For instance, we could consider what happens if we do not turn off energy input half way through the simulation. Figure 8 presents

the peak amplitude in this scenario. The first half of the simulation is, of course, identical to Figure 2. After focussing the cases of small energy input do not behave significantly differently. However, the very high energy input cases do either decay slowly or continue growing. Comparing the two plots helps us assess the role of non-linear physics in this. The dominant effect is that energy input is simply directly increasing the amplitude of the group – however, the more non-linear case does appear to increase in amplitude more rapidly suggesting, for these very extreme cases, some interplay between energy input and non-linearity.

CONCLUSIONS

In this paper we have explored whether energy input during the focusing of a wave-group can lead to significant non-linear physics being triggered for wavegroups with spectral bandwidths

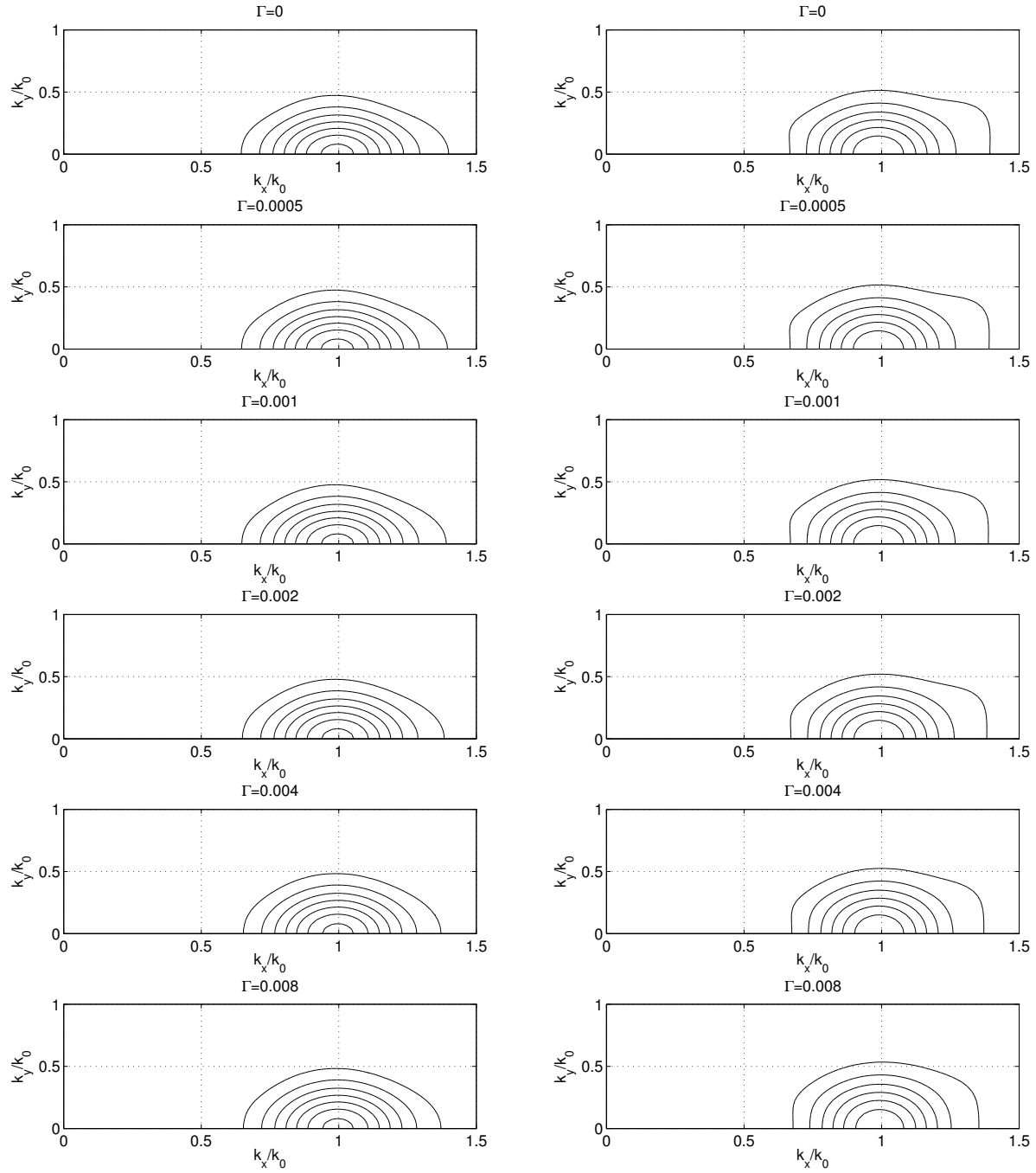


FIGURE 6. Wavenumber amplitude spectra of the free surface for $ak = 0.17$ case. Groups at focus (left) and at end of simulation (right). For $\Gamma = 0.008$ the group at $t = 0$ is used instead of focus. Contours at 0.005 m intervals. Amplitude spectrum based on area of ocean of $2.1 \times 10^7 \text{m}^2$.

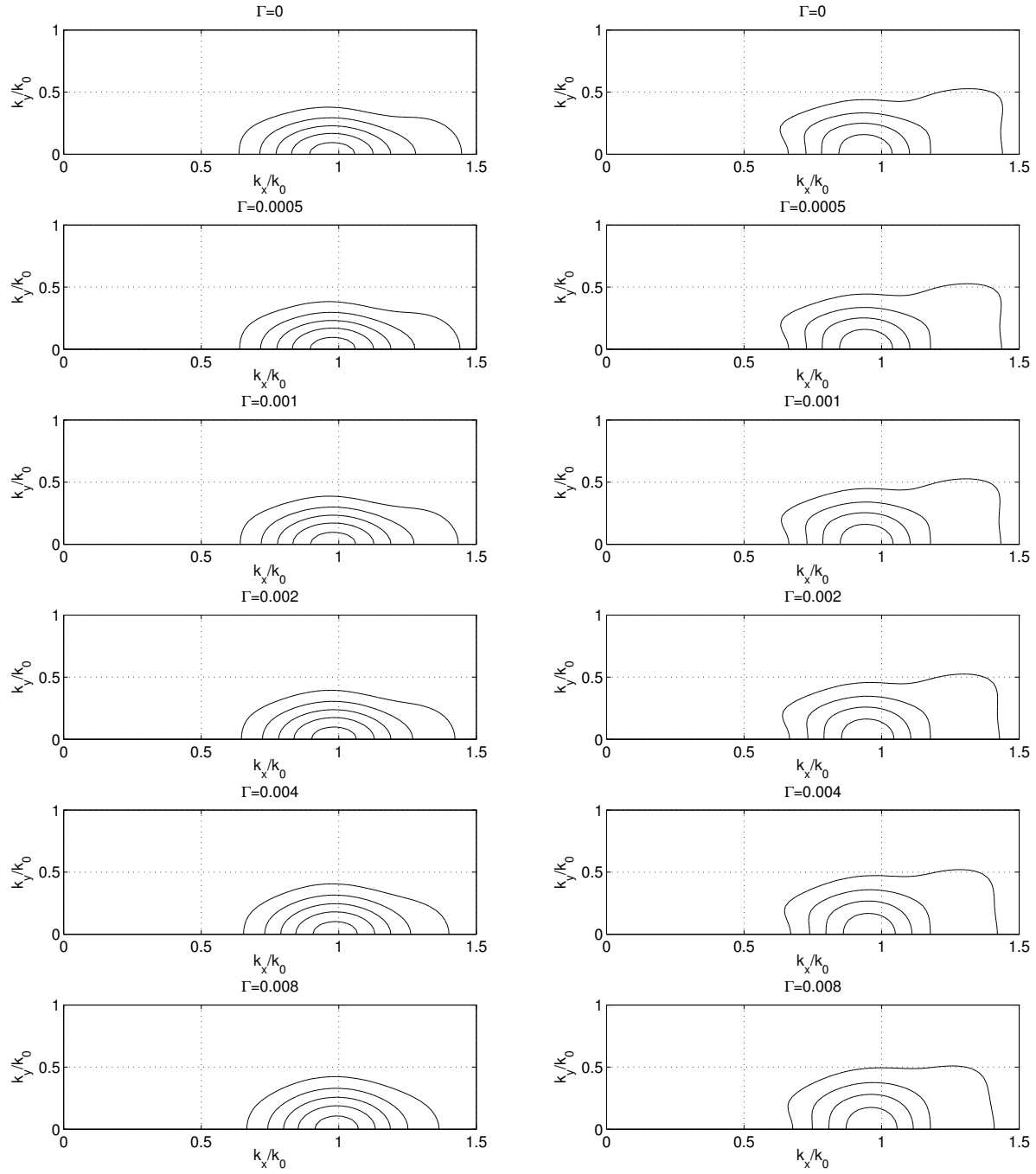


FIGURE 7. Wavenumber amplitude spectra of the free surface for $ak = 0.25$ case. Groups at focus (left) and at end of simulation (right). For $\Gamma = 0.008$ the group at $t = 0$ is used instead of focus. Contours at 0.01 m intervals. Amplitude spectrum based on area of ocean of $2.1 \times 10^7 \text{ m}^2$.

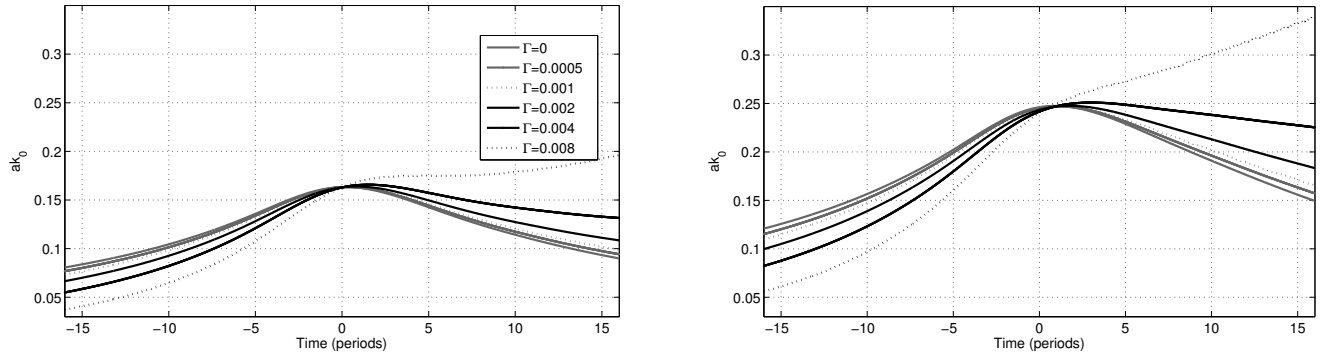


FIGURE 8. Maximum elevation of the group over the course of the simulation when energy continues entering the simulation. Left – $ak = 0.17$; right – $ak = 0.25$.

representative of those in the ocean. We have looked at both whether energy input can cause the wavegroup to focus in such a way that the usual non-linear changes are suppressed, possibly giving a wave-group in the right form to trigger an instability – we have found no evidence that this happens. We have also considered whether energy input during focusing can have a major effect on the permanent spectral changes which occur in non-linear evolution. Our results do not suggest this causes significantly different redistributions of energy compared with the case without energy input.

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