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APPLICATION OF PHASE DECOMPOSITION TO THE ANALYSIS OF RANDOM TIME SERIES FROM WAVE BASIN TESTS

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ABSTRACT

Many ocean engineering problems involve bound harmonics which are slaved to some underlying assumed close to linear time series. When analyzing signals we often want to remove the bound harmonics so as to "linearise" the data or to extract individual bound harmonic components so that they may be studied. For even moderately broadbanded systems filtering in the frequency domain is not sufficient to separate components as they overlap in frequency. One way to overcome this difficulty is to use input signals with the same linear envelope but with different phases and then use simple addition and subtraction of the resulting signals to extract different harmonics. This approach has been established for the analysis of wave groups. In this paper we examine whether this approach can be used on random time series as well. We analyse random wave time series of wave elevation from the towing tank in Shanghai Jiao Tong University and force measurements on a cylinder taken in the Kelvin tank at the University of Strathclyde.

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INTRODUCTION

Ocean waves are fundamentally non-linear due to the nature of the free surface boundary condition. One approach for analysing waves is to carry out a perturbation expansion around the mean water level – this approach naturally gives rise to higher harmonics of the fundamental sinusoids – a so called Stokes expansion. When analysing data, either numerical, experimental or field measurements, we often want to separate a timeseries into freely propagating and bound harmonics.

A number of approaches exist for separating out harmonics from wave records. The simplest is direct frequency filtering. However, most signals in the ocean are sufficiently broadbanded that the frequency range of different harmonics overlaps and it becomes impossible to cleanly separate the different harmonics using this alone. An alternative approach is to assume initially that the signal is dominated by free waves and use known physics to calculate an estimate for the higher-order components which may then be subtracted from the original signal. Examples of this for second order random waves are [1, 2] or using the Creamer transform [3]. In principle this could be done iteratively.

In the open ocean we have no control over the phase of

waves. However, in the laboratory or in a numerical simulation the phase of an experiment can (often) be controlled. This allows one to make a timeseries which has a fixed phase-shift relative to another. The simplest is a 180 degree shift where the original input linear timeseries to the paddle is simply negated. If the propagation of the waves is independent of the phase of the signal then these can be manipulated to remove different harmonics of the signal. Alternatively with a random timeseries one can examine the average shape of both large crests and troughs which allows a similar analysis to be conducted. To our knowledge this approach was first used by Jonathan & Taylor [4] who effectively used a two-phase approach. This approach was extended to four phases by Fitzgerald *et al.* [5] and has been extended to include even more different phases [6]. This approach has been widely used on studies of focussed wave-groups [7–13]. – in one study even the 14th harmonic of the fundamental linear input has been cleanly extracted [14].

The above studies all use focussed wave-groups or the average shape of extreme events in a timeseries. In this paper we aim to explore the applicability and limitations of this approach in random sea-states. We analyse data from two experimental facilities. We look at both the harmonic structure of wave elevation and forces on a surface piercing cylinder. We note that recent unpublished work by Sarkar *et al.* has adapted the four-phase approach to make it applicable to random timeseries which have not been phase-manipulated – although we can only extract averaged information using Sarkar's method. Until this is published we leave it as an open question whether this approach is better than manipulating the phase of the input signal as investigated here, noting that thus requires multiple repeats of the same experiment (with different phases).

FORMULATION OF THE TWO AND FOUR PHASE APPROACH

Let us consider a signal which can be written in the form

$$
\begin{aligned}
\zeta = {} & A f_{11} \cos\phi + A^2 (f_{20} + f_{22}\cos 2\phi) + \\
& A^3 (f_{31}\cos\phi + f_{33}\cos 3\phi) + \\
& A^4 (f_{40} + f_{42}\cos 2\phi + f_{44}\cos 4\phi) + \mathcal{O}(A^5),
\end{aligned} \tag{1}
$$

where A is an amplitude which is slowly varying relative to the phase function ϕ. It is straightforward to examine what happens when a phase shift is introduced into this in the form $\phi + \xi$. So if we introduce a 90 degree phase shift ($\xi = \pi/2$) we get

$$
\begin{aligned}
\zeta_{90} = {} & -A f_{11}\sin\phi + A^2 (f_{20} - f_{22}\cos 2\phi) + \\
& A^3 (-f_{31}\sin\phi - f_{33}\sin 3\phi + \\
& A^4 (f_{40} - f_{42}\cos 2\phi + f_{44}\cos 4\phi) + \mathcal{O}(A^5).
\end{aligned} \tag{2}
$$

We can then combine together different phase shifts (along with their Hilbert transforms e.g. $\hat{\zeta}_{90}$) in such a way that many of the terms cancel. Thus we get the standard four phase approach as given in Fitzgerald et al.

$$
\frac{\left(\zeta + \hat{\zeta}_{90} - \zeta_{180} - \hat{\zeta}_{270}\right)}{4} = (A f_{11} + A^3 f_{31})\cos\phi + \mathcal{O}(A^5). \tag{3}
$$

$$
\frac{(\zeta - \zeta_{90} - \zeta_{180} - \zeta_{270})}{4} = (A^2 f_{22} + A^4 f_{42})\cos 2\phi + \mathcal{O}(A^6). \tag{4}
$$

$$
\frac{\left(\zeta - \hat{\zeta}_{90} - \zeta_{180} + \hat{\zeta}_{270}\right)}{4} = A^3 f_{33}\cos 3\phi + \mathcal{O}(A^5). \tag{5}
$$

$$
\frac{(\zeta + \zeta_{90} + \zeta_{180} + \zeta_{270})}{4} = A^2 f_{20} + A^4 f_{40} + A^4 f_{44}\cos 4\phi + \mathcal{O}(A^6). \tag{6}
$$

The two-phase approach is somewhat simpler. The linear paddle signal is simply inverted (equivalent to a phase change of π). The resulting timeseries are then simply added to give even harmonics and subtracted to give odd harmonics.

This approach relies on near perfect control of the phase. If this does not happen then 'leakage' occurs as some components fail to cancel exactly.

Limitations

The method assumes that that the 1st harmonic components travel at the same speed regardless of the phase of the component. Where this is invalid the approach in this paper becomes invalid. The obvious example of where this is the case is for shallow water waves where a wave trough will move differently to a crest. Thus this method would not be expected to work well in water depths less than $kd \sim 0.8$. Though we note the shallow water analysis of Whittaker et al. [15] who pushed this limit down to $kd \sim 0.5$, where k is the wavenumber and d the water depth, for wave buoy data from the field.

The phase separation method assumes that the signal is 'narrow banded'. In practice, most standard spectra typically found in ocean engineering seem to have sufficiently narrow spectra that the method is generally applicable. For situations where there are a wide range of frequencies (e.g. cases with wind sea and swell waves) the approach cannot be used. A simple reason for this is that the shorter wave travelling over the longer wave will be shifted horizontally in space (see for instance [16]) meaning that the different phases will not line up in time (or space) invalidating the approach.

Not all terms can be extracted using phase manipulation. The most problematic one is the third order '3-1' term – i.e. the bound component resulting from a third order interaction with frequency given by $\omega_{3,1} = \omega_1 \pm \omega_2 \mp \omega_3$. We have not been able to find any combination of terms which allows this to be separated from the fundamental linear terms and it obviously falls in

the same part of the spectrum as these (although with a slightly different spectral shape). This can be a significant source of contamination to the results for highly non-linear cases. Note that this term does not effect the method, it just means that it cannot be separated from the linear signal.

EXPERIMENTAL SET-UP
Multifunctional Ship Model Towing Tank (Shanghai Jiao Tong)

The first experiment was carried out in the multifunctional ship model towing tank at Shanghai Jiao Tong University, China. The total length of the wave tank was 300 meters and the width of the tank was 16 meters. The tank had a flat bed with a water depth of 7.5 meters, which gives a non-dimensional water depth greater than $kd > 3$. There were 40 hinged-flap type wave makers at one end of the flume. First-order wave generation theory was applied and the impact of second-order error wave was analysed carefully and found not to affect results. There was a parabolic beach at the far end of the flume, which was opposite to the wave makers. Reflection analysis using the least squares method [17] estimates that less than 10% of the energy is reflected. The wave surface elevation was measured by 10 capacitance probes at 100 Hz with excellent calibration characteristics. However, due to facility limitations, the wave probes could only be installed on the carriage. To track the wave evolution over a wider range, the experiment was repeated with different carriage positions. The facility is shown in Figure 1.

In the Shanghai tank only two phase decomposition was used. This was partly due to experimental time constraints but also because the spectra considered were very narrow banded. The three spectra considered were based on the classic work of Onorato et al. [18] where the object had been to investigate modulation instabilities.

Kelvin tank (Strathclyde)

The second experimental campaign was undertaken at the Kelvin Hydrodynamic Laboratory in the University of Strathclyde, Glasgow, United Kingdom. The tests were carried out in the lab's 76 m by 4.6 m towing tank with a constant water depth of 1.8 m over a flat bottom. The tank is equipped with a 'flap-type' wavemaker consisting of four paddles with force-feedback at one end, and a sloping beach acting as a passive absorber at the other end. A single surface-piercing vertical cylinder of diameter 0.315 m was placed at the centre of the tank, 35.315 m away from the wavemaker. The location of the cylinder was positioned by a laser rangefinder. The draft of the cylinder is 1.6 m. The cylinder was supported on top by a stiff frame which was attached to a load cell capable of measuring 6 degrees-of-freedom forces and moments. The load cell was fixed to the sub-carriage. The support frame of the sub-carriage is about 2.0 m

above the still water surface, to allow the possible high runup of the large focussed waves. A second load cell was installed under water at the bottom of the cylinder measuring both wave loads and overturning moment. A snapshot of the experimental setup is displayed in Figure 1.

In the Kelvin tank four phase decomposition was used. The results presented here are for a JONSWAP spectrum with $\gamma = 3.3$.

RESULTS
Focussed wave-groups

Before proceeding to analyse random timeseries we demonstrate the facilities and the basic technique using focussed wave-groups. Focussed wave-groups were generated in each of the facilities using linear dispersion to approximately focus the groups at the probes (exact focussing was not important in these tests).

We start by looking at water surface elevation in the Shanghai tank. A focussed NewWave (see Tromans [19]) is generated based on an underlying Pierson-Moskowitz spectrum with peak period 3.5 seconds and amplitude at focus 0.1 m and measured at 52.8 m down the tank. Figure 2 presents the timeseries of the crest and trough focussed wave-group as well as the extracted odd and even harmonics. Visually crest and trough focussed waves show good phase alignment and the extraction of odd and even harmonics appears to be clean. A second-order error wave, due to the linear generation at the paddle is observable after the signal which is, of course, in phase for both crest and trough focussed events.

The separation of the harmonics can be more clearly seen if we examine the spectra of the measured 'odd' and 'even' harmonics. Figure 3 presents these data. The separation between odd and even harmonics can clearly be seen – indeed despite the tiny magnitude of these waves the harmonics can still be separated at $f/f_p = 6 - 7$.

Unlike for the free surface, where, building on the fundamental work of Stokes, we expect a harmonic structure to the higher harmonics we do not have a similar theoretical basis, to arbitrary order, for forces on a column. Recent work [5, 11], as well as unpublished work from the De-Risk project [20], does suggest that this may be a good model for non-breaking waves on columns. Clearly for breaking waves the physics is strongly non-linear and a model based on powers of some linear time-series will not be appropriate. In this study we choose sea-states with minimal breaking. But applying this methodology to forces does require us to make more assumptions.

The analysis of the force data from the Kelvin tank works even better. This is because 4-phases have been used and also because force is more non-linear than elevation and there the signal's higher harmonics relative to the linear waves are larger. Figure 4 summaries these results. The incident wave spectrum has a peak frequency 0.429 Hz and the linear wave-group amplitude

FIGURE 1. THE EXPERIMENTAL FACILITIES USED IN THIS STUDY. LEFT: THE KELVIN TANK; RIGHT; MULTIFUNCTIONAL SHIP MODEL TOWING TANK
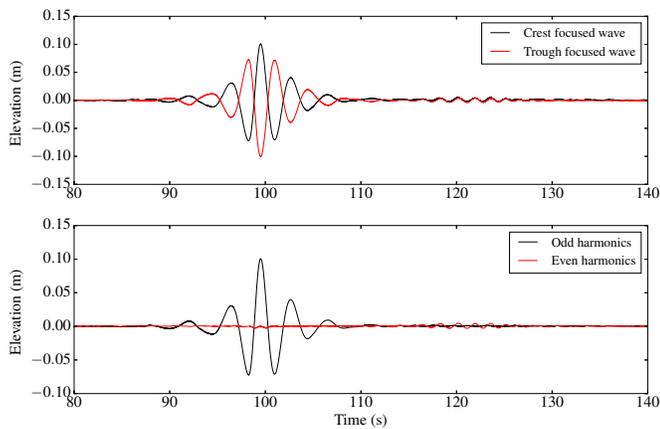
FIGURE 2. WAVE GROUP PROFILES AT THE MULTIFUNC-TIONAL SHIP MODEL TOWING TANK WITH TWO PHASE DE-COMPOSITION
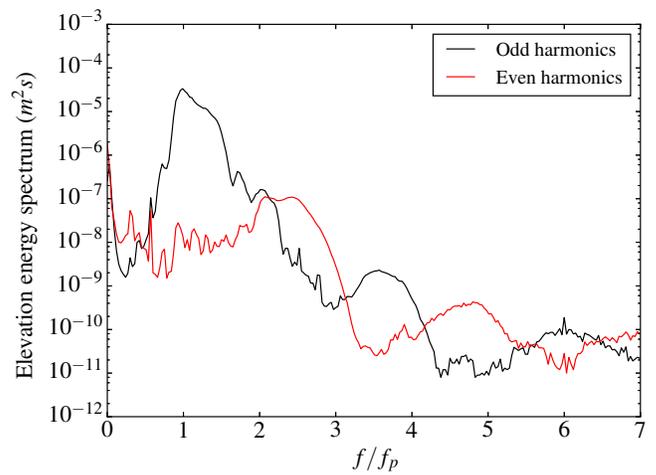

FIGURE 3. WAVE SPECTRUM FOR THE WAVE GROUP AT THE MULTIFUNCTIONAL SHIP MODEL TOWING TANK WITH TWO PHASE DECOMPOSITION

is 0.169 m. The linear force envelope in dash line is estimated by the Morison inertial formula using the linear component envelope of the incident wave group in the absence of the cylinder. The envelopes shown in red at higher harmonics give the predicted shape of each harmonic based on raising the shape of the linear signal to the relevant power. These envelopes are scaled to the maximum of the envelope of the extracted harmonic.

There is generally good agreement on the shape of the higher harmonics – they agree reasonably well with the predicted envelopes. The worst fit is for the third harmonic, a result consistent with Fitzgerald et al. [5] where it is suspected additional physics is important.

Figure 5 presents the spectra of the different harmonics. Some leakage is apparent – for instance in the second order sum term there is a small spike in the linear frequency-range – however this is two orders of magnitude smaller than the second order peak itself and can mostly be removed by frequency filtering. We also note that even a small amount of drag on the cylinder would also damage the Stokes like symmetry of the results.

Thus we have established that for individual wave-groups the phase separation method is effective at isolating the harmonics in the two different experiments, for different response and in

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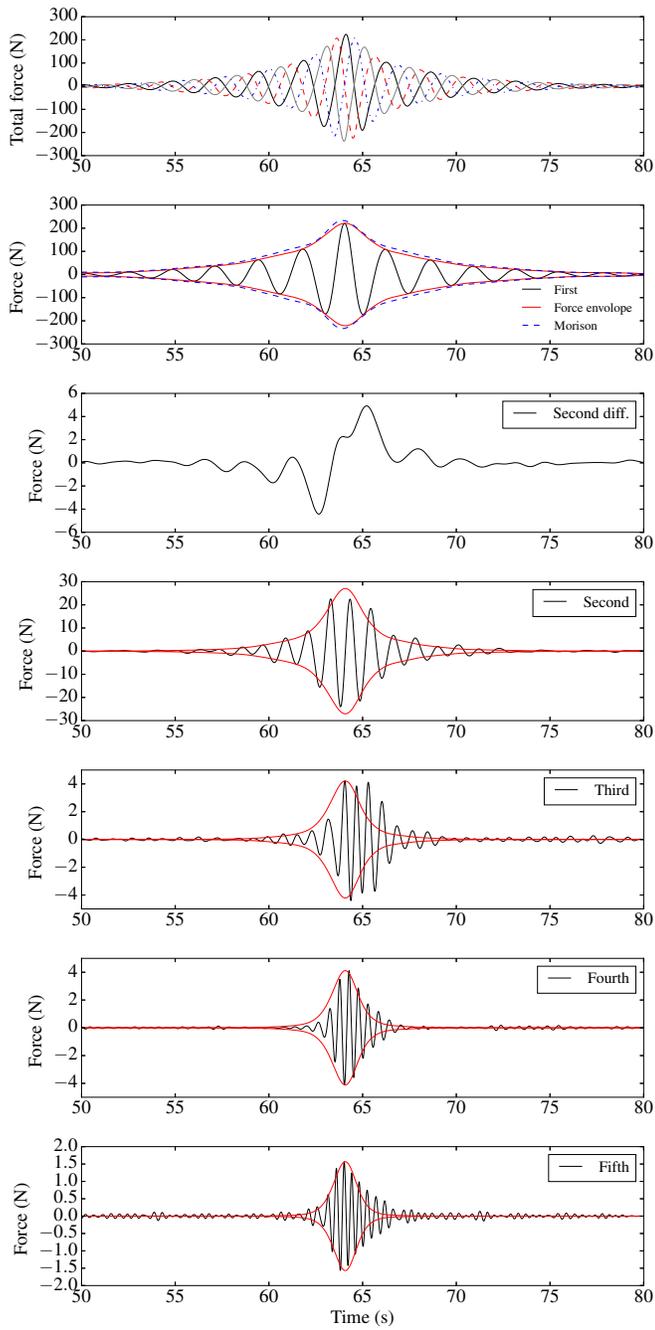
FIGURE 4. HORIZONTAL WAVE FORCES ON THE CYLINDER FOR THE WAVE GROUP IN THE KELVIN TANK WITH FOUR-PHASE DECOMPOSITION. TOP: INDIVIDUAL TOP SERIES OF FORCES FROM THE 4 EXPERIMENTS. BELOW: THE EXTRACTED TIMESERIES (WITH APPROPRIATE FREQUENCY FILTERS APPLIED) FOR DIFFERENT HARMONICS.
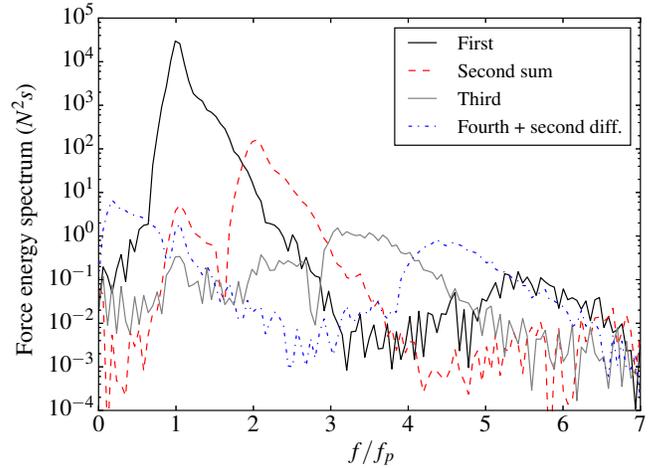
**FIGURE 5**. HORIZONTAL WAVE FORCE SPECTRUM FOR THE WAVE GROUP IN THE KELVIN TANK WITH FOUR-PHASE DECOMPOSITION

different experimental facilities.

Random sea state

We now turn to the analysis of irregular timeseries. The wave generation is carried out in an identical way to the focus wave-groups with a linear timeseries being provided which is phase manipulated to give the desired signal.

We first consider the water surface elevations measured in the Shanghai tank. To our surprise the results were largely independent of the input spectrum. Relatively close to the wave-maker the phase separation method appears to work well. Figure 6 shows sample timeseries for the most non-linear cases. It also shows the timeseries of the odd and even harmonics after the series have been added and subtracted. This sea-state was based on a JONSWAP spectrum with $\gamma = 6$, $H_s = 0.182$m and $T_p = 1.5$s – this is exceptionally steep and narrowbanded. The phase separation appears to work well visually. Figure 7 shows the spectrum for this case. Due to the irregular nature of the waves the spectral separation is slightly less clear than for the wave-groups – however; clearly the method is essentially working.

However, further down the tank the story is somewhat different. Beyond approximately 20m from the paddle (approximately 6 wavelengths) the phases move out of alignment and the method essentially breaks down. Rather surprisingly this appears to happen at around the same spot for all cases regardless of non-linearity. Note that the method worked perfectly regardless of measurement location in the tank for focussed wave groups. Figures 8 and 9 present example timeseries and spectrum for a typical case. In this case the input spectrum wave based on a Pierson-Moskowitz spectrum with $H_s = 0.162$m and $T_p = 1.5$s. There is
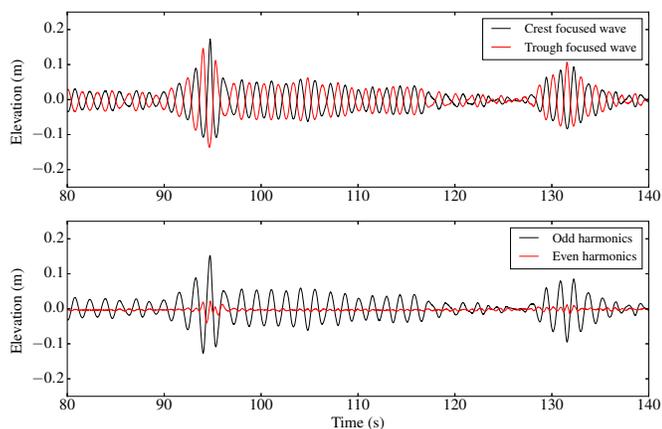
FIGURE 6. RANDOM WAVE PROFILES AT THE MULTIFUNCTIONAL SHIP MODEL TOWING TANK WITH TWO PHASE DECOMPOSITION METHOD AT 11.36 M FROM THE WAVEMAKER
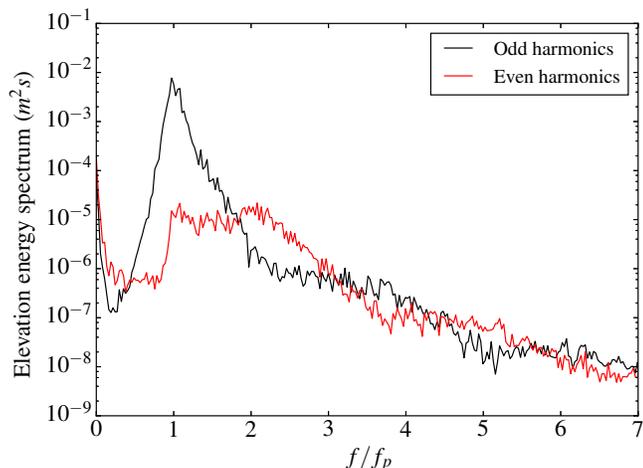
FIGURE 7. WAVE SPECTRUM FOR THE RANDOM WAVES AT THE MULTIFUNCTIONAL SHIP MODEL TOWING TANK WITH TWO PHASE DECOMPOSITION METHOD AT 11.36 M FROM THE WAVEMAKER

clearly some cancellation working – the even harmonics are significantly below the odd harmonics in the linear frequency range. However, the extraction of the different harmonics is clearly not clean enough to be useful without further manipulation.

We do not fully understand why the method works well close to the paddle but then fails further down the tank. The obvious suggestion is that this is due to wave-breaking the exact location at which this happens being strongly linked to the phase of the wave. Close to the paddle very little breaking was observed for all cases (presumably due to breaking at the paddle of very

large waves). However, for the least non-linear case, very little breaking was observed at any point in the tank. Hence, wave breaking does not appear to be a fully satisfactory hypothesis to explain all the results. An alternative explanation might be due to modulation instabilities leading to the phase alignment failing. The tests do show good agreement with the evolution of kurtosis down the tank (see Janssen [21] and Figure 1 in [22]) although these results are not presented here. However, this should not obviously lead to the changes in phase which lead to the breakdown of the method observed experimentally. Third order interactions should not be a problem for this approach however 4th order (5-wave interaction) would break the symmetry pattern as crests and troughs would then start to behave differently.

We next turn to the analysis of forces. Unlike the free surface data from the Shanghai tank, we effectively only sample these at one point along the tank. Figure 10 presents a sample timeseries of the total inline force measured on the column. The four different timeseries can be seen approximately $90°$ out of alignment. These can be combined together to extract timeseries for the different harmonics also shown in the figure. Figure 11 presents the spectrum of the force timeseries. There appears to be a very clean separation of the linear force around the spectral peak. Of the other harmonics the 'third' appears the most questionable, whilst as expected this is the largest component around 3 to 4 f_p this signal is surprisingly large (relative to the other harmonics) for lower frequencies. The reason for this is unclear. The extraction is clearly less clean than for the isolated wave-groups. This may be due to reflections into the tank or, as discussed above, to breaking waves.

CONCLUSIONS

In this paper we have considered whether phase manipulation can be used to extract 'slave' harmonics from random timeseries extending its existing application to wave-groups. We considered both wave elevation measurements as well as forces on a surface piercing column. Our findings are somewhat mixed. The basic technique clearly works in theory, and the fact that we can get acceptable results out implies that contamination from reflections and similar issues which are inevitable in random wave tests are typically not sufficient to invalidate the approach. However, the method clearly broke down for some tests where we would have expected it to work. An advantage of the method is that it is reasonably clear where it breaks down. Further work needs to be carried out to understand why the method failed.

Perhaps the more important question is left open by the present paper. An alternative approach, not presented here or published yet has been developed by Sarkar *et al.* This requires a long timeseries (to reduce sample variability) but allow coefficients describing harmonics to be extracted without running experiments with different phase angles. To some extent this approach is confined to looking at average properties whereas the

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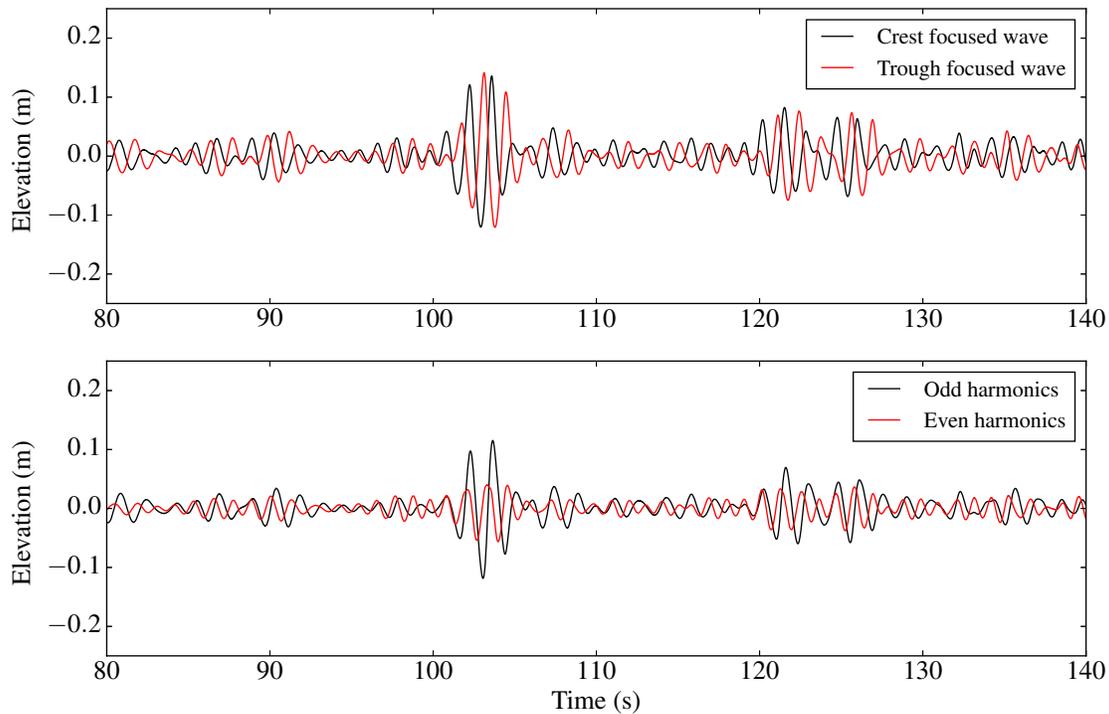
FIGURE 8. RANDOM WAVE PROFILES AT MULTIFUNCTIONAL SHIP MODEL TOWING TANK WITH TWO PHASE DECOMPOSITION METHOD AT 68 M FROM THE PADDLE

approach described in the present paper gives us full timeseries of the bound harmonics. Nevertheless, it would be interesting to compare the two approaches in detail.

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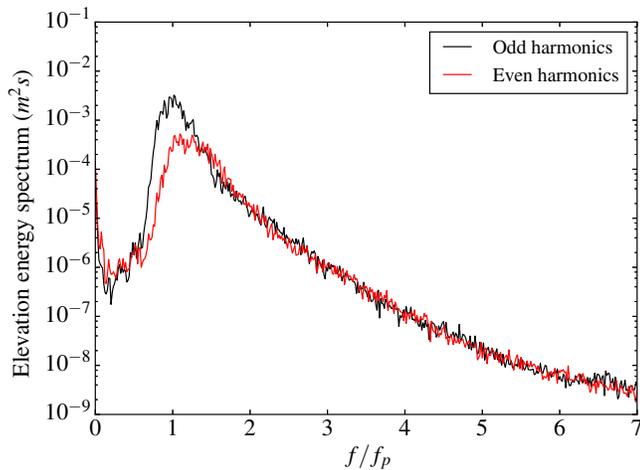
FIGURE 9. WAVE SPECTRUM FOR RANDOM WAVES AT MULTIFUNCTIONAL SHIP MODEL TOWING TANK WITH TWO PHASE DECOMPOSITION METHOD 68 M FROM THE PADDLE

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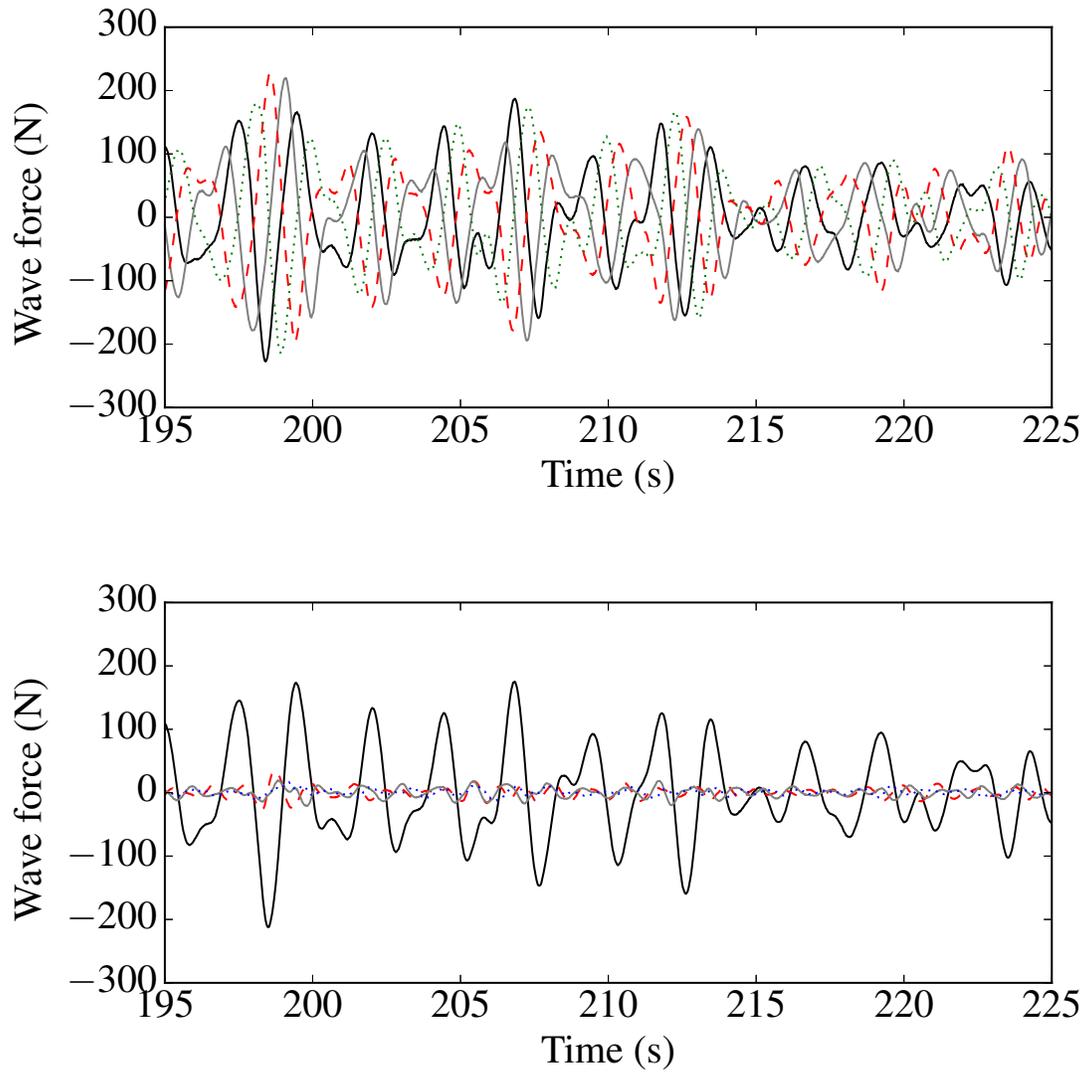
FIGURE 10. HORIZONTAL WAVE FORCES ON THE CYLINDER FOR THE RANDOM SEASTATE AT THE KELVIN TANK WITH FOUR-PHASE DECOMPOSITION. TOP: FORCE TIME HISTORIES FOR ALL FOUR PHASES; BOTTOM: DECOMPOSED FORCES WITH LINEARISED FORCE SHOWN IN BLACK.

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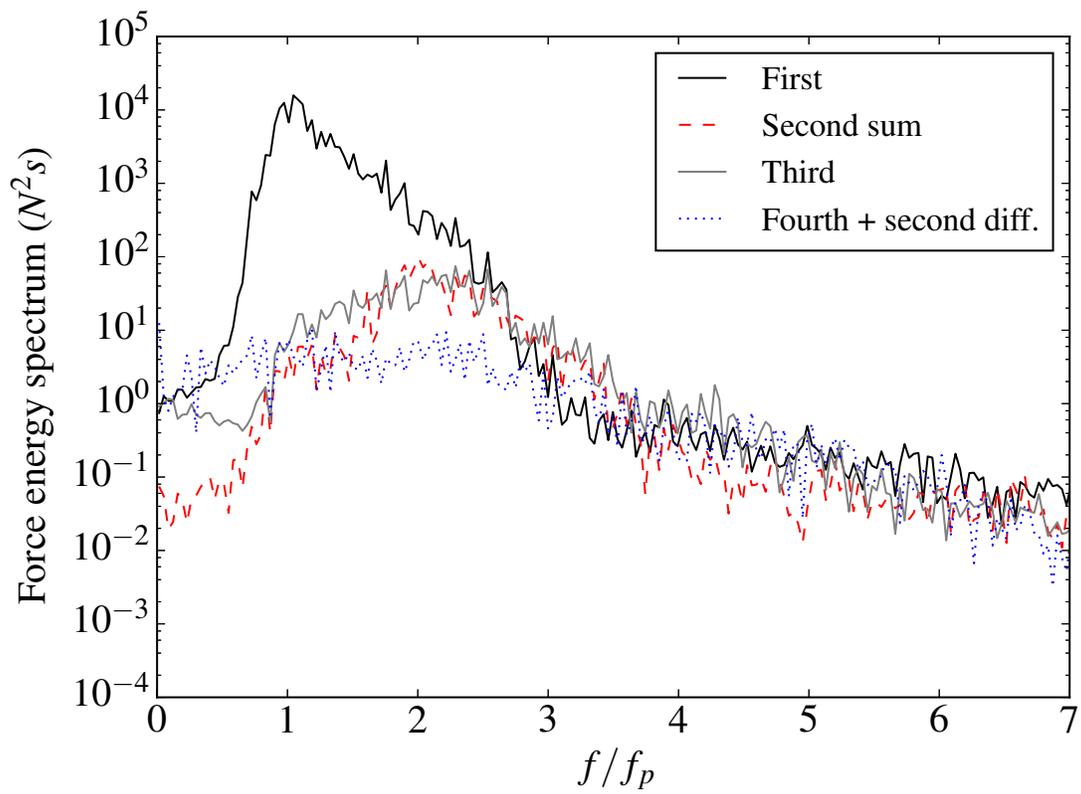
FIGURE 11. HORIZONTAL WAVE FORCE SPECTRUM FOR THE RANDOM SEASTATE AT THE KELVIN TANK WITH FOUR-PHASE DECOMPOSITION