Numerical modelling of two-scale flow dynamics

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Summary: Two-scale actuator disc theory is shown to predict qualitatively the performance of a lateral array of tidal turbines in a depth-averaged channel model. The agreement between the numerical and analytical models is improved by extrapolating the numerical results to zero background roughness. Increasing background roughness is shown to reduce the optimal turbine spacing and increase the peak power coefficient of the array.

Introduction

To maximise power output, a tidal turbine array should be designed to sweep the largest permissible fraction of the flow cross-section [1]. Since it would be impractical to fill a cross-section with turbines, developers must consider the variation of power output with array width (array blockage, B_A) and the size and spacing of individual turbines (local blockage, B_L). The two-scale ADT [2] provides a useful description of the basic effects of B_A and B_L on the performance of a lateral array of turbines. Specifically, it predicts an optimal lateral spacing (optimal B_L) to maximise the maximum power coefficient ($C_{PG(max)}$) of the array. Subsequent studies employing three-dimensional RANS simulations [3] and physical experiments [4] have shown this theory to predict qualitatively the power and thrust behaviour of lateral rows of porous discs. The present work shows the theory to predict qualitatively the power performance of a lateral turbine array in a depth-averaged channel model, and investigates the effects of background roughness (C_F) on array performance.

Model

The discontinuous Galerkin version of ADCIRC is used to simulate steady flow through an idealised tidal channel (fig. 1). This flow is driven by a head difference between the flow boundaries, which, following [5], are situated in deep water far upstream and downstream of the channel. Following [6] and [7], turbines are modelled as line discontinuities based on the finite Froude number ADT [8]. This method accounts for local-scale mixing in a sub-grid scale model whilst array-scale mixing is simulated directly by the shallow water equations. For a single row of identical turbine edges, B_A is varied and B_L adjusted to maintain a global blockage (B_G) of 0.1. A constant B_G ensures that, even though the number of edges varies with B_A, this is analogous to varying the lateral spacing between a fixed number of turbines. For each B_L, the edges are tuned to produce $C_{PG(max)}$. In this way, the effect of lateral spacing on $C_{PG(max)}$ is calculated numerically (NM) and compared with the predictions of the theory (AM).

Discussion

In the simple case of fixed B_L and constant inflow, the power extracted by a tidal turbine initially increases with reducing throughflow velocity before decreasing as the applied thrust increasingly diverts flow around the turbine. The optimal thrust may be defined as that produced by the optimal tuning to maximise the useful power output. For two-scale flow, $C_{PG(max)}$ also depends on the balance between local-scale slowing and array-scale choking [2]. As the lateral spacing between turbines is reduced (B_L is increased), $C_{PG(max)}$ initially increases as the decreasing potential for flow diversion around each turbine permits higher optimal thrusts. As B_L is further increased, $C_{PG(max)}$ reduces as the increasing potential for diversion around the entire array necessitates lower optimal thrusts. The value of B_L that maximises $C_{PG(max)}$ is that for which the optimal thrust is maximised.

Fig. 2a shows qualitative agreement between the models, which is improved by extrapolating the numerical results to $C_F = 0$. Contrary to the findings of [3], the results of the NM are lower than those of the AM for low B_L and higher for high B_L. In this case, since the effects of free surface deformation [9] in the NM are expected to have been small, the disparity between the models is thought to be due primarily to two-dimensional flow effects in the NM and the approximate extrapolation of the numerical results to $C_F = 0$. Nevertheless, the present NM provides a suitable basis for qualitative analysis of the effects of C_F on array performance. It is worth noting, however, that due to the low C_F selected, the array mixing length-scale in the NM is not short compared to the channel length (fig. 1), as is assumed in the AM. The significance of this will be considered in future work.

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As reported in [10], for a given B_L, $C_{PG(max)}$ increases with increasing C_F (fig. 2b). This is because, firstly, increasing the resistance of the bypass flow relative to the core flows improves the performance of the turbines, even for a fixed tuning, and secondly, this change in relative resistance increases the optimal thrust, allowing the turbines to be retuned to extract even more power. Further, the amount by which an increase in C_F enhances $C_{PG(max)}$ increases with B_L because, for optimally tuned turbines, the difference between core and bypass velocities increases with B_L, which also means that, after increasing C_F, the additional thrust that can be applied by retuning also increases with B_L. This explains why, for two-scale flow, increasing C_F increases both the peak $C_{PG(max)}$ and the optimal B_L (fig. 2b).

It is important to note, however, that measurement of performance in terms of power coefficient neglects the effect of power extraction on the channel flow rate [1]. In this case, the assumption of constant inflow also conceals the fact that increasing C_F increases frictional dissipation, reducing the amount of power available for extraction.

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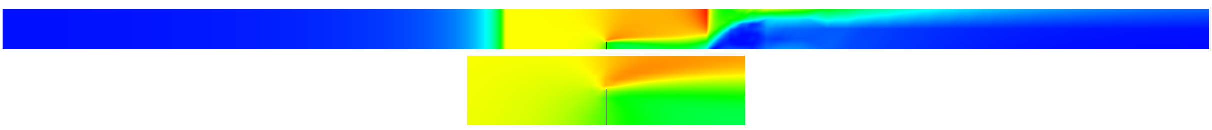

Fig. 1: Plan view of domain showing flow around the turbine array (black line) within the channel. A close-up of the flow around the array is shown below. Contours are of depth-averaged velocity and of arbitrary scale.

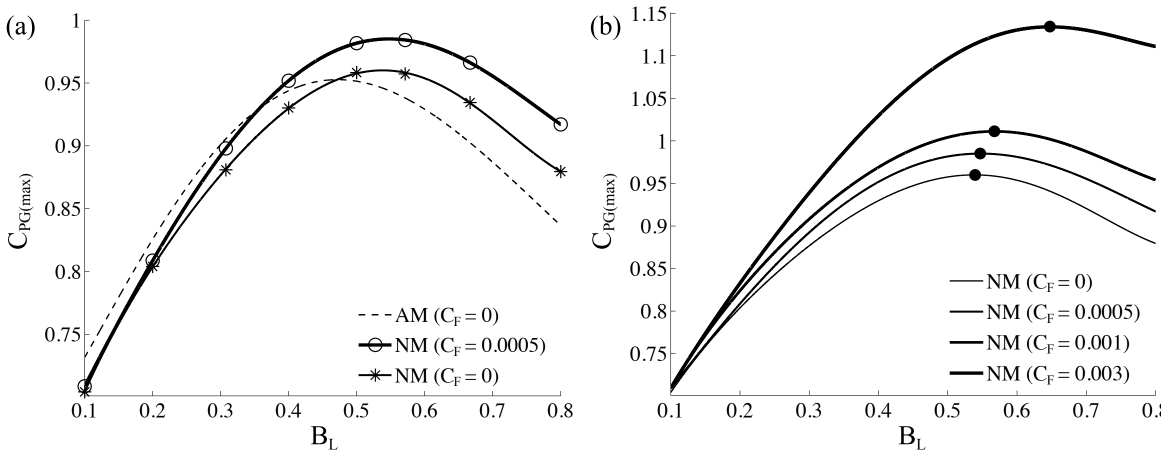

Fig. 2: The effect of lateral spacing (a) and background roughness (b) on $C_{PG(max)}$.