

The three generator nilpotent Lie algebras with immediate descendants of dimension 8

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$$\langle a, b, c \mid cb = 0, \text{ class } 2 \rangle. \quad (5.4)$$

$$\langle a, b, c \mid \text{ class } 2 \rangle. \quad (6.11)$$

$$\langle a, b, c \mid ca, cb, \text{ class } 3 \rangle. \quad (6.12)$$

$$\langle a, b, c \mid ca - bab, cb, \text{ class } 3 \rangle. \quad (6.13)$$

$$\langle a, b, c \mid ca - baa, cb, \text{ class } 3 \rangle. \quad (6.14)$$

$$\langle a, b, c \mid ca - bab, cb - \omega baa, \text{ class } 3 \rangle. \quad (6.15)$$

$$\langle a, b, c \mid baa, ca - babb, cb, \text{ class } 4 \rangle. \quad (6.22)$$

$$\langle a, b, c \mid baa - babb, ca - babb, cb, \text{ class } 4 \rangle. \quad (6.23)$$

$$\langle a, b, c \mid baa, ca, cb, \text{ class } 4 \rangle. \quad (6.24)$$

$$\langle a, b, c \mid baa - babb, ca, cb, \text{ class } 4 \rangle. \quad (6.25)$$

$$\langle a, b, c \mid cb, caa, cab, cac, \text{ class } 3 \rangle. \quad (7.65)$$

$$\langle a, b, c \mid cb - baa, caa, cab, cac, \text{ class } 3 \rangle. \quad (7.66)$$

$$\langle a, b, c \mid cb, bab, bac, cac, \text{ class } 3 \rangle. \quad (7.67)$$

$$\langle a, b, c \mid cb - baa, bab, bac, cac, \text{ class } 3 \rangle. \quad (7.68)$$

$$\langle a, b, c \mid cb, bac, caa - bab, cac, \text{ class } 3 \rangle. \quad (7.69)$$

$$\langle a, b, c \mid cb - baa, bac, caa - bab, cac, \text{ class } 3 \rangle. \quad (7.70)$$

$$\langle a, b, c \mid cb, baa, bac, cac, \text{ class } 3 \rangle. \quad (7.71)$$

$$\langle a, b, c \mid cb - caa, baa, bac, cac, \text{ class } 3 \rangle. \quad (7.72)$$

$$\langle a, b, c \mid cb, bac, caa, cac - bab, \text{ class } 3 \rangle. \quad (7.73)$$

$$\langle a, b, c \mid cb, bac, caa, cac - \omega bab, \text{ class } 3 \rangle. \quad (7.74)$$

$$\langle a, b, c \mid cb - baa, bac, caa, cac - bab, \text{ class } 3 \rangle. \quad (7.75)$$

$$\langle a, b, c \mid cb - baa, bac, caa, cac - \omega bab, \text{ class } 3 \rangle. \quad (7.76)$$

$$\langle a, b, c \mid cb, bac, caa - baa, cac + bab, \text{ class } 3 \rangle. \quad (7.77)$$

$$\langle a, b, c \mid cb - baa, bac, caa - baa, cac + bab, \text{ class } 3 \rangle. \quad (7.78)$$

$$\langle a, b, c \mid cb, baa, bac, caa, \text{ class } 3 \rangle. \quad (7.79)$$

$$\langle a, b, c \mid cb, bac, caa, cac - baa, \text{ class } 3 \rangle. \quad (7.80)$$

$$\langle a, b, c \mid cb, bac, caa - bab, cac - baa, \text{ class } 3 \rangle. \quad (7.81)$$

$$\langle a, b, c \mid cb, bac, caa - \omega bab, cac - baa, \text{ class } 3 \rangle. (p = 1 \bmod 3) \quad (7.82)$$

$$\langle a, b, c \mid cb, baa, caa, cac, \text{ class } 3 \rangle. \quad (7.83)$$

$$\langle a, b, c \mid cb, baa, caa - bab, cac, \text{ class } 3 \rangle. \quad (7.84)$$

$$\langle a, b, c \mid cb, bab - baa, caa, cac, \text{ class } 3 \rangle. \quad (7.85)$$

$$\langle a, b, c \mid cb, baa, caa, cac - \omega bab, \text{ class } 3 \rangle. \quad (7.86)$$

$$\langle a, b, c \mid cb, baa, caa - bac, cac - \omega bab, \text{ class } 3 \rangle. \quad (7.87)$$

$$\langle a, b, c \mid cb, baa, caa - kbab - bac, cac - \omega bab, \text{ class } 3 \rangle \ (p = 2 \bmod 3), \quad (7.88)$$

where k is not a value of

$$\frac{\lambda(\lambda^2 + 3\omega\mu^2)}{\mu(3\lambda^2 + \omega\mu^2)}.$$

$$\langle a, b, c \mid bab, caa, cab, cac, cba, cbb, cbc, \text{ class } 3 \rangle, \quad (7.89)$$

$$\langle a, b, c \mid bab - baa, bac, caa - baa, cab, cac + baa, cbb + baa, cbc - baa, \text{ class } 3 \rangle. \quad (7.97)$$

$$\langle a, b, c \mid ca, cb, baaa, baab, \text{ class } 4 \rangle. \quad (7.98)$$

$$\langle a, b, c \mid ca - babb, cb, baaa, baab, \text{ class } 4 \rangle. \quad (7.99)$$

$$\langle a, b, c \mid ca, cb, baab, babb + baaa, \text{ class } 4 \rangle. \quad (7.100)$$

$$\langle a, b, c \mid ca, cb, baab, babb + \omega baaa, \text{ class } 4 \rangle. \quad (7.101)$$

$$\langle a, b, c \mid ca + bab, cb, baaa, baab, \text{ class } 4 \rangle. \quad (7.102)$$

$$\langle a, b, c \mid ca + bab, cb, baab, babb, \text{ class } 4 \rangle. \quad (7.103)$$

$$\langle a, b, c \mid ca + bab, cb - baaa, baab, babb, \text{ class } 4 \rangle. \quad (7.104)$$

$$\langle a, b, c \mid ca + bab, cb, baab, babb + \frac{1}{2}baaa, \text{ class } 4 \rangle. \quad (7.105)$$

$$\langle a, b, c \mid ca + bab, cb, baab, babb + \frac{\omega}{2}baaa, \text{ class } 4 \rangle. \quad (7.106)$$

$$\langle a, b, c \mid ca + bab, cb, baaa, babb, \text{ class } 4 \rangle. \quad (7.107)$$

$$\langle a, b, c \mid ca + baa, cb, baaa, baab, \text{ class } 4 \rangle. \quad (7.108)$$

$$\langle a, b, c \mid ca + baa - babb, cb, baaa, baab, \text{ class } 4 \rangle. \quad (7.109)$$

$$\langle a, b, c \mid ca + baa, cb, baab - baaa, babb - baaa, \text{ class } 4 \rangle. \quad (7.110)$$

$$\langle a, b, c \mid ca + baa, cb, baab, babb - baaa, \text{ class } 4 \rangle. \quad (7.111)$$

$$\langle a, b, c \mid ca + baa, cb, baab, babb - \omega baaa, \text{ class } 4 \rangle. \quad (7.112)$$

$$\langle a, b, c \mid ca + baa, cb, baaa, babb, \text{ class } 4 \rangle. \quad (7.113)$$

$$\langle a, b, c \mid ca + baa, cb, baaa, babb - baab, \text{ class } 4 \rangle. \quad (7.114)$$

$$\langle a, b, c \mid ca + baa, cb, baab - baaa, babb - \lambda baaa, \text{ class } 4 \rangle \ (\lambda \neq 0, 1). \quad (7.115)$$

$$\langle a, b, c \mid ca + bab, cb + \omega baa, baab, babb, \text{ class } 4 \rangle. \quad (7.106a)$$

$$\langle a, b, c \mid ca + bab, cb + \omega bab, baab = \lambda baaa, babb = \mu baaa, \text{ class } 4 \rangle, \quad (7.107a)$$

with $\lambda^2 - \mu \neq 0$. The algebras are defined by the following pairs (λ, μ) :

1. $(0, -\omega)$,
2. (λ, ω) where $\lambda^2 - \omega$ is a square (all these algebras are isomorphic),
3. (λ, ω) where $\lambda^2 - \omega$ is not a square (all these algebras are isomorphic),
4. $(\lambda, 0)$ where $1 \leq \lambda \leq (p-1)/2$ (giving $(p-1)/2$ different algebras),
5. (λ, μ) where $\lambda^2 - \mu$ is not a square, $\mu \neq \omega$, $\lambda \neq 0$ if $\mu = -\omega$; these parameters give $(p-3)/2$ different algebras with two pairs (λ, μ) , (λ', μ') giving isomorphic algebras if $(\lambda, \mu) = (\lambda', \mu')$ or if

$$(\lambda', \mu') = \left(\frac{r^2\lambda + r(\omega + \mu) + \omega\lambda}{r^2 + 2r\lambda + \mu}, \frac{r^2\mu + 2r\omega\lambda + \omega^2}{r^2 + 2r\lambda + \mu} \right)$$

for some $r \in \text{GF}(p)$.

$$\langle a, b, c \mid cb, baa, caa, cab, cac, \text{ class } 4 \rangle. \quad (7.108a)$$

$$\langle a, b, c \mid cb, baa - babb, caa, cab, cac, \text{ class } 4 \rangle. \quad (7.109a)$$

$$\langle a, b, c \mid cb, baa, caa, cab - babb, cac, \text{ class } 4 \rangle. \quad (7.112a)$$

$$\langle a, b, c \mid cb, baa, caa - babb, cab, cac, \text{ class } 4 \rangle. \quad (7.113a)$$

$$\langle a, b, c \mid cb, baa, caa - babb, cab - babb, cac, \text{ class } 4 \rangle. \quad (7.114a)$$

$$\langle a, b, c \mid cb, bab, bac, caa, cac, \text{ class } 4 \rangle. \quad (7.115a)$$

$$\langle a, b, c \mid cb, bab - baaa, bac, caa, cac, \text{ class } 4 \rangle. \quad (7.116)$$

$$\langle a, b, c \mid cb - baaa, bab, bac, caa, cac, \text{ class } 4 \rangle. \quad (7.117)$$

$$\langle a, b, c \mid cb - baaa, bab - baaa, bac, caa, cac, \text{ class } 4 \rangle. \quad (7.118)$$

$$\langle a, b, c \mid cb, bab, bac - baaa, caa, cac, \text{ class } 4 \rangle. \quad (7.119)$$

$$\langle a, b, c \mid cb - baa, bab, bac - \lambda baaa, caa, cac, \text{ class } 4 \rangle. \quad (7.123)$$

$$\langle a, b, c \mid cb - baa, bab - baaa, bac + \frac{1}{2}baaa, caa, cac, \text{ class } 4 \rangle. \quad (7.124)$$

$$\langle a, b, c \mid ca + babb, cb, baa, babba, \text{ class } 5 \rangle. \quad (7.126)$$

$$\langle a, b, c \mid ca + babb - babbb, cb, baa, babba, \text{ class } 5 \rangle. \quad (7.127)$$

$$\langle a, b, c \mid ca + babb, cb, baa - babb, babba, \text{ class } 5 \rangle. \quad (7.130)$$

$$\langle a, b, c \mid ca, cb, baa, babba, \text{ class } 5 \rangle. \quad (7.132)$$

$$\langle a, b, c \mid ca, cb, baa - babbb, babba, \text{ class } 5 \rangle. \quad (7.133)$$

$$\langle a, b, c \mid ca - babbb, cb, baa, babba, \text{ class } 5 \rangle. \quad (7.135)$$

$$\langle a, b, c \mid ca - babbb, cb, baa - babbb, babba, \text{ class } 5 \rangle. \quad (7.136)$$

$$\langle a, b, c \mid ca, cb, baa - babbb, babba, \text{ class } 5 \rangle. \quad (7.138)$$

$$\langle a, b, c \mid ca - babbb, cb, baa - babbb, babba, \text{ class } 5 \rangle. \quad (7.140)$$

$$\langle a, b, c \mid ca + bab - \lambda babbb, cb, baa, \text{ class } 5 \rangle \ (\lambda = 0, 1, \omega). \quad (7.142)$$

$$\langle a, b, c \mid ca + bab, cb, baa - babbb, \text{ class } 5 \rangle. \quad (7.143)$$