Counting p-groups and Lie algebras using PORC polynomials

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The files data.m, lie6, lie7, group5, group6, group7 contain the data for the functions $L_{d,k}(q)$ and $G_{d,k}(q)$ defined in our paper. You can, of course, examine the files to investigate the data, but data.m is a MAGMA program which reads the data and defines two functions WedgeProduct(d, k, q) and WedgeProductPlus(d, k, q) giving the values of $L_{d,k}(q)$ and $G_{d,k}(q)$ for $d \leq 7$. To use the program you should copy these 6 files to your computer, and at the MAGMA prompt enter

```
load "data.m";
```

The data is then read in, and the two functions are defined.

The parameters d, q must be integers, with $2 \leq d \leq 7$, and with $q = 0$ or q a prime power. For the function WedgeProduct, k must be an integer in the range $1 \leq k \leq \binom{d}{2}$, and for WedgeProductPlus, k must be an integer in the range $1 \leq k \leq \binom{d+1}{2}$. If $q = 0$ then the functions return a string which gives the PORC formula with symbolic q. If you set a value of q you can use `eval` to evaluate this formula at q. For example:

```
a:=WedgeProduct(6,5,0);
for q in [2,3,4,5] do
   eval a;
end for;
```

If q is a prime power then the functions return the numerical value at q.

WedgeProduct(d, k, q) gives the number of class two d-generator Lie algbras of dimension $d+k$ over GF(q). If p is an odd prime then WedgeProduct(d, k, p)

gives the number of class two d-generator groups of order p^{d+k} with exponent p, and WedgeProductPlus(d, k, q) gives the number of d-generators groups of p-class two with order p^{d+k}.