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Housing and relative risk aversion☆

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HIGHLIGHTS

- This paper derives closed-form and numerical solutions for relative risk aversion in the presence of housing.
- Housing enables the household to hedge against unexpected shocks and may decrease relative risk aversion.
- Housing may generate state-dependent, time-varying risk aversion.

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ABSTRACT

This paper derives closed-form and numerical solutions for relative risk aversion in a standard consumption-based model enriched with housing. The presence of housing enables the household to hedge against unexpected shocks and may decrease relative risk aversion. In addition, housing may generate state-dependent, time-varying risk aversion.

1. Introduction

Since the seminal contributions of Arrow (1965) and Pratt (1964), measures of relative risk aversion are obtained from models that abstract from housing.[1] However, recent studies by Iacoviello (2005), Silos (2007), and Rubio (2011) show that housing is an important component for the household's consumption decisions, which can either dampen or magnify the response of macroeconomic aggregates to shocks. These findings suggest that housing may play a critical role in understanding risk aversion. The goal of this paper is to use an otherwise standard consumption-based model enriched with housing to derive closed-form and numerical solutions of relative risk aversion and to outline the relevance of housing for the household's attitude towards risk.

The analysis shows that accounting for housing significantly affects risk aversion. In particular, when fluctuations in the housing stock have a milder effect on utility compared to movements in consumption, housing provides the household with an additional margin to cushion against unexpected shifts in wealth. It therefore reduces relative risk aversion. On the other hand, relative risk aversion remains unchanged if movements in the stock of housing have a stronger effect on utility than fluctuations in consumption. In addition, the analysis shows that accounting for housing may generate state-dependent, time-varying risk aversion.

Section 2 of the paper sets up the model. Section 3 shows how to derive analytical, closed-form solutions for relative risk aversion and discusses how they change in the presence of housing. Section 4 provides a quantitative assessment of the results and further discusses the issues.

2. The model

The theoretical framework is based on the standard consumption-based model that allows for housing investment, as in Iacoviello and Pavan (2013). During each period, $t = 0, 1, 2, \ldots,$

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[1] The relationship between housing and risk aversion has been hinted in the previous work. For example, Grossman and Laroque (1990) and Flavin and Nakagawa (2008) point out that housing affects the agents' attitude towards risk, despite their analyses do not explicitly focus on risk aversion.

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the representative household maximizes the von Neumann–Morgenstern expected utility function:

$$W(c_t, h_t) = E_0 \sum_{t=0}^{\infty} \beta^t \left[\alpha U(c_t) + (1-\alpha)V(h_t)\right], \tag{1}$$

where E_0 is the expectation at period $t = 0$, c_t is consumption, h_t is the housing stock, β is the discount factor, α and $1-\alpha$ are the share of consumption goods and housing stock, respectively. The representative household's end-of-period assets, a_{t+1}, are equal to the beginning-of-period assets, a_t, augmented for a gross return $(1+r_t)$, a net of lump-sum net transfer payments, π_t, purchases of consumption goods, c_t, and investment in the stock of housing, h_t. Hence, the household's budget constraint is:

$$a_{t+1} = a_t(1+r_t) - \pi_t - c_t - h_t + (1-\delta)h_{t-1}, \tag{2}$$

where δ is the depreciation rate of the housing stock. In addition, the non-Ponzi scheme condition holds: $\lim_{T\to\infty} \prod_{t=0}^{\infty} \frac{a_{T+1}}{(1+r_t)} \geq 0$. Thus, the household chooses $\{c_t, h_t, a_{t+1}\}_{t=0}^{\infty}$ to maximize its utility (1) subject to the budget constraint (2) for all $t = 0, 1, 2, \ldots$. The optimality conditions for this problem are

$$\alpha U'(c_t) = (1-\alpha)V'(h_t) + \beta(1-\delta)\alpha E_t U'(c_{t+1}) \tag{3}$$

and

$$U'(c_t) = \beta E_t U'(c_{t+1})(1+r_{t+1}), \tag{4}$$

where $U'(c_t)$ and $V'(h_t)$ denote the marginal utility of consumption and housing stock, respectively. Eq. (3) states that the marginal utility of consumption equates the direct utility gain from an additional unit of housing stock at time t, plus the discounted gain that the additional unit of housing stock brings into the next period, $t+1$, for the remaining fraction $(1-\delta)$. Eq. (4) is the standard Euler equation for consumption that equates the marginal utility of consumption at time t with the expected, discounted, marginal utility of consumption at time $t+1$.

3. Relative risk aversion with housing

Relative risk aversion, R_t, is a measure of the household's willingness to accept risk as a function of the fraction of the household's assets that are exposed to risk. As shown in Swanson (2012), the coefficient of relative risk aversion with respect to the beginning-of-period assets, a_t, can be derived from the household indirect utility:

$$R_t = -\frac{W''(a_t)}{W'(a_t)} a_t, \tag{5}$$

where $W'(a_t)$ and $W''(a_t)$ represent the first and second derivatives of the indirect utility function over wealth, $W(a_t)$, with respect to a_t, respectively. Eq. (5) shows that the value of the coefficient of relative risk aversion crucially depends on the definition of beginning-of-period assets, a_t. We define beginning-of-period assets as the present discounted stream of consumption and housing stock, as implied by the household budget constraint (2).

Given Eq. (5), we are able to derive closed-form solutions for relative risk aversion, R_t, by determining explicit functional forms for $W'(a_t)$ and $W''(a_t)$ from the indirect utility function $W(a_t)$. In particular, use the beginning-of-period assets to express the utility function (1) as:

$$\begin{aligned} W(a_t) &= \alpha U\left\{a_t r_t - [h_t(a_t) - (1-\delta)h_{t-1}(a_{t-1})]\right\} \\ &\quad + (1-\alpha)V\left\{h_t(a_t)\right\}. \end{aligned} \tag{6}$$

Differentiating Eq. (6) with respect to a_t and imposing the household's optimal condition with respect to h_t, reported in Eq. (3), yield:

$$W'(a_t) = \alpha U'(c_t) r_t, \tag{7}$$

which, once differentiated with respect to a_t, yields:

$$W''(a_t) = \alpha U''(c_t) r_t \frac{\partial c_t}{\partial a_t}. \tag{8}$$

We can use the model comprising Eqs. (2)–(4), and the non-Ponzi scheme condition to obtain an explicit functional form for the derivative $\partial c_t/\partial a_t$ in Eq. (8). In particular, differentiating Eq. (3) with respect to a_t yields:

$$\begin{aligned} \alpha U''(c_t)\frac{\partial c_t}{\partial a_t} &= (1-\alpha)V''(h_t)\frac{\partial h_t}{\partial a_t} \\ &\quad + \beta(1-\delta)\alpha E_t U''(c_{t+1})\frac{\partial c_{t+1}}{\partial a_t}, \end{aligned} \tag{9}$$

and differentiating Eq. (4) with respect to a_t yields:

$$U''(c_t)\frac{\partial c_t}{\partial a_t} = \beta E_t U''(c_{t+1})(1+r_{t+1})\frac{\partial c_{t+1}}{\partial a_t},$$

which, by imposing the steady state condition, $\beta = 1/(1+r)$, implies:

$$\frac{\partial c_t}{\partial a_t} = \frac{\partial c_{t+1}}{\partial a_t}. \tag{10}$$

Eq. (10) holds for each period $t = 0, 1, 2, \ldots$, implying that, in the steady state, changes in the current household's consumption are the same across any future change in consumption. Imposing Eq. (10) into the long-run equilibrium of Eq. (9) yields:

$$\alpha U''(c)\left[1-\beta(1-\delta)\right]\frac{\partial c}{\partial a} = (1-\alpha)V''(h)\frac{\partial h}{\partial a}. \tag{11}$$

In steady state, Eq. (9) must hold, such that

$$\frac{V'(h)}{U'(c)} = \alpha\frac{[1-\beta(1-\delta)]}{(1-\alpha)}, \tag{12}$$

and inserting Eq. (12) into Eq. (11) yields:

$$\gamma\frac{\partial c}{\partial a} = \chi\frac{\partial h}{\partial a}, \tag{13}$$

where $\gamma = -cU''(c)/U'(c)$ is the elasticity of $U'(c)$ with respect to c, and $\chi = -hV''(h)/V'(h)$ is the elasticity of $V'(h)$ with respect to h. We now can differentiate the household's budget constraint (2) with respect to a_t and evaluate it at the steady state to obtain:

$$r = \frac{\partial c}{\partial a} + \delta\frac{\partial h}{\partial a}. \tag{14}$$

Hence, using Eq. (13) to solve for $\partial h/\partial a$ and substituting the outcome into Eq. (14), yields:

$$\frac{\partial c}{\partial a} = \frac{r}{1+\delta\frac{\gamma}{\chi}\frac{h}{c}}. \tag{15}$$

Eq. (15) shows that consumption increases in response to a unitary increase in the assets. In particular, consumption rises by the extra asset income r, but it decreases by the amount, $1+\delta\frac{\gamma}{\chi}\frac{h}{c}$, that accounts for the effect of housing. We can now derive a closed-form solution for the long-run coefficient of relative risk aversion, R.

Proposition 1. *The long-run coefficient of relative risk aversion is:*

$$R = -\frac{U''(c)c}{U'(c)}\frac{1}{1+\frac{\gamma}{\chi}\frac{h}{c}}\left(1+\frac{h}{c}\right). \tag{16}$$

Proof. Inserting the steady state, beginning-of-period wealth equation, $a = (c+\delta h)(1/r)$, into Eq. (5) together with the expressions for $W'(a)$ and $W''(a)$, as outlined in Eqs. (7) and (8), respectively, and using Eq. (15) to substitute for $\partial c/\partial a$ in Eq. (8) yield to Eq. (16). ■

Proposition 1 shows that including housing in the model has important implications for risk aversion. In particular, relative risk aversion depends on the concavity of both arguments c and h in the

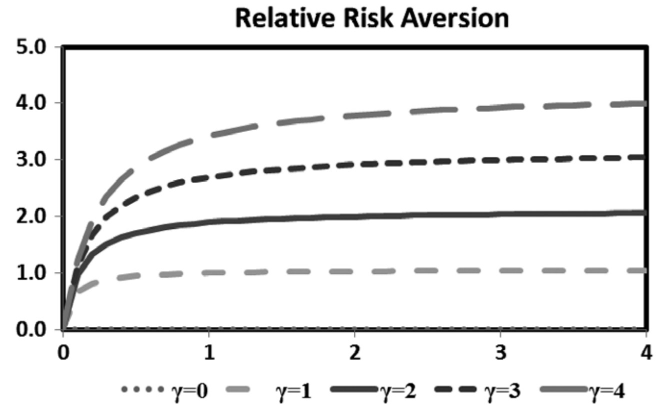

Fig. 1. y-axis: relative risk aversion, R. x-axis: values of the elasticity of utility with respect to the stock of housing, χ. The figure shows values of the coefficient of relative risk aversion in the function of χ, for values of γ equal to 0, 1, 2, 3, and 4.

utility function, as expressed by the ratio γ/χ. Therefore preferences over consumption *as well as* the stock of housing are relevant for the household's attitude towards risk. This result differs from the standard Arrow–Pratt measure of risk aversion that identifies the curvature of the utility function with respect to consumption as the relevant measure to quantify risk aversion. Therefore, the conventional Arrow–Pratt approach to derive measure of relative risk aversion may lead to inaccurate readings of the household's attitude towards risk if the analysis abstracts from housing. In addition, housing makes the measure of relative risk aversion dependent on the ratio between the stock of housing and consumption, whereas it is constant and equal to γ in the standard consumption-based model. Since consumption and the stock of housing fluctuate over the business cycle, relative risk aversion becomes state-dependent and time-varying, which is a robust stylized fact.[2]

4. Quantitative assessment and discussion

To quantitatively assess the implications of Proposition 1, suppose that during each period, $t = 0, 1, 2, \ldots$, the representative household maximizes the Epstein and Zin (1991) utility function, $W(c_t, h_t) = E_0 \sum_{t=0}^{\infty} \beta^t \left[\alpha \frac{c_t^{1-\gamma}}{1-\gamma} + (1-\alpha)\frac{h_t^{1-\chi}}{1-\chi}\right]$. Using Eq. (16), the associated long-run measure of relative risk aversion is:

$$R = \gamma \frac{1}{1 + \frac{\gamma}{\chi}\frac{h}{c}} \left(1 + \frac{h}{c}\right). \tag{17}$$

Fig. 1 shows measures of relative risk aversion for values of the elasticity of the marginal utility of consumption with respect to consumption (χ) between 0 and 4, and each line is associated with values of the elasticity of the marginal utility of housing with respect to housing (γ), which equals 0, 1, 2, 3, and 4 respectively.[3] The entries show that the values of γ and χ are critical to determine measures of relative risk aversion. When $\gamma \leq \chi$, the coefficient of relative risk aversion is constant and equal to $-U''(c)c/U'(c) = \gamma$, the same value as the standard Arrow–Pratt measure of relative risk aversion. For instance, when γ is equal to 2, for values of $\chi \geq 2$, the coefficient of relative risk aversion remains equal to 2. However, when the household's preference has a higher elasticity to consumption than the stock of housing (i.e. $\gamma > \chi$), the coefficient of relative risk aversion is lower than the standard Arrow–Pratt measure. In this case, the contribution of an additional unit of housing stock to the household's marginal utility is lower than the contribution of an additional unit of consumption. For instance, when γ is equal to 2, for any value of $\chi < 2$, relative risk aversion is lower than 2, the standard Arrow–Pratt value. The intuition for this result is straightforward. When movements in the stock of housing have a more limited effect on utility than fluctuations in consumption, the housing stock provides the household with an additional margin to cushion against unexpected shocks and therefore reduces relative risk aversion. Eq. (17) also shows that relative risk aversion crucially depends on movements in the housing-consumption stock, which fluctuate over the business cycle. Thus, relative risk aversion becomes state-dependent and time varying if the model is enriched with housing, whereas it is constant in a standard model consumption-based model. In summary, this analysis shows that housing may significantly affect relative risk aversion. Furthermore, fluctuations in relative risk aversion are tightly linked with movements in consumption and the housing stock.

The findings in this paper call for two interesting extensions. First, the analysis assumes that the housing-consumption ratio, h/c, remains constant over variations in χ and γ. It would be interesting to establish whether the quantitative result continues to hold in a general equilibrium model, where the long-run values of consumption and housing stock also depend on the curvature of the utility function with respect to χ and γ. In principle, if the contribution of an additional unit of housing to the marginal utility (χ) is higher than the contribution of an additional unit of consumption goods (γ), changes in the housing stock generate strong movements in utility, making it optimal for the household to hold a high housing stock and thus dampen the effect of marginal movements in housing on the stock of household. Such a mechanism would increase the h/c ratio and therefore potentially increase risk aversion. Second, the analysis has focused on the long-run properties of relative risk aversion, but the underlining theoretical framework can be used to investigate to what extent housing affects the dynamic properties of relative risk aversion. Such an extension would be particularly interesting since the analysis shows that fluctuations in relative risk aversion are related tightly with movements in the housing-consumption ratio. These investigations are open for future research.

References

Arrow, K.J., 1965. Aspects of the theory of risk-bearing. In: Yrjoĺ Jahnsson Lectures. Essays in the Theory of Risk Bearing.
Brunnermeier, M.K., Nagel, S., 2008. Do wealth fluctuations generate time-varying risk aversion? Micro-evidence on individuals. Amer. Econ. Rev. 98, 713–736.
Epstein, L.G., Zin, S.E., 1991. Substitution, risk aversion, and the temporal behavior of consumption and asset returns: an empirical analysis. J. Political Econ. 99, 263–286.
Flavin, M., Nakagawa, S., 2008. A model of housing in the presence of adjustment costs: a structural interpretation of habit persistence. Amer. Econ. Rev. 98, 474–495.
Grossman, S.J., Laroque, G., 1990. Asset pricing and optimal portfolio choice in the presence of illiquid durable consumption goods. Econometrica 58, 25–51.
Guiso, L., Sapienza, P., Zingales, L., 2013. Time varying risk aversion. Chic. Booth Res. Pap. 13–64.
Iacoviello, M., 2005. House prices, borrowing constraints, and monetary policy in the business cycle. Amer. Econ. Rev. 95, 739–764.
Iacoviello, M., Pavan, M., 2013. Housing and debt over the life cycle and over the business cycle. J. Monetary Econ. 60, 221–238.
Ouysse, R., Quin, M., 2013. New Evidence on the Time-Varying Risk Aversion from a Dynamic Multinominal-Logit Augmented Consumption CAPM. University of New South Wales.
Pratt, J., 1964. Risk aversion in the small and in the large. Econometrica 32, 122–136.
Rubio, M., 2011. Fixed and variable-rate mortgages, business cycles and monetary policy. J. Money Credit Bank. 43, 657–688.
Silos, P., 2007. Housing, portfolio choice and the macroeconomy. J. Econom. Dynam. Control 31, 2774–2801.
Swanson, E.T., 2012. Risk aversion and the labor margin in dynamic equilibrium models. Amer. Econ. Rev. 102, 1663–1691.

2 Guiso et al. (2013) and Ouysse and Quin (2013) provide an extensive empirical support to time-varying risk aversion. Brunnermeier and Nagel (2008) show that time-varying relative risk aversion linked to individuals' responses to changes in household assets is supported by the data.

3 Note that in this application the steady-state value of h/c is set equal to 0.066 to match the ratio between real consumption and real residential fixed investment from the BEA data.