Flexible prices, labor market frictions and the response of employment to technology shocks☆

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Abstract
Recent empirical evidence establishes that a positive technology shock leads to a decline in labor inputs. Standard RBC models fail to replicate this stylized fact, while recent papers show that augmenting the model with implementation lags, or habit formation, or shock persistence in growth rates among others accounts for this fact. In this paper, we show that a standard flexible price model with labor market frictions that allows hiring costs to depend on technology shocks may also lead to the same negative impact on labor inputs. Labor market frictions are therefore able to account for the fall in labor inputs. However, the elasticity of hiring costs to technology shocks is large, suggesting that additional extensions to the model are needed.

1. Introduction

Galí (1999) and a number of subsequent studies show that technology shocks have a contractionary effect on employment. In a standard flexible price model, a positive technology shock increases employment since output rises on impact and additional labor inputs are required to keep pace with higher technology.

This paper investigates whether a standard flexible price model enriched with labor market frictions is able to generate the negative response of employment to a technology shock.3 In order to investigate this issue, we set up a standard flexible price model that allows, but does not require, labor market frictions to generate a negative response of employment to technology shocks. We estimate the model using Bayesian methods and find that the data strongly prefer the version of the model in which labor market frictions generate a negative response of employment to technology shocks.

As mentioned, the presence of labor market frictions overturns the positive reaction of employment to a technology shock in the standard flexible price model. The intuition is straightforward. In the standard flexible price model, households supply labor up to the point where the marginal disutility from working equals its marginal contribution to output. An increase in productivity induces the household to supply more labor in response to a technology shock. In a labor market characterized by search and matching frictions, workers and firms

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2 See Mumtaz and Zanetti (2012a) and references therein for a recent review of the literature on this topic.
3 As detailed below, a number of recent studies propose alternative mechanisms to generate the negative response of employment to a positive technology shock in the context of flexible price models. This paper is the first study that addresses the issue using labor market frictions, modeled as in Thomas (2008) and Blanchard and Gali (2010), which are empirically relevant and theoretically appealing.
face a cost in forming a match, and therefore the optimal choice of labor units also depends on the cost of hiring an additional worker. Depending on how the cost of hiring reacts to productivity, the response of employment to a technology shock can be either positive or negative. For instance, if hiring costs co-move positively with productivity, a technology shock increases the marginal product of labor (as in the standard flexible price model), but it also increases the cost of recruiting an extra worker. If the latter effect is sufficiently strong, employment reacts negatively to a technology because hiring costs reduce the marginal contribution to production of an additional unit of labor. In principle, as Yashiv (2000) and Rotemberg (2006) point out, hiring costs can be either pro- or counter-cyclical. On one hand, recessions represent times of low opportunity costs, thereby implying more re-structuring of the workforce so that firms devote more resources to screening and lead to counter-cyclical hiring costs. On the other hand, recessions also are times when, due to the high availability of workers looking for jobs, the cost of advertising is low, encouraging hiring costs to be pro-cyclical. In this paper, we internalize both mechanisms by allowing hiring costs to react directly to productivity and leaving the data to establish whether the reaction is pro- or counter-cyclical. The estimation of the model reveals that labor market frictions enable a flexible price model to generate a decline in labor inputs in response to a positive technology shock.

Before proceeding with the analysis, we relate this work to studies that develop real business cycle (RBC) models able to replicate the negative response of labor input to a positive technology shock and we then position the paper in the broader context of the literature. Hairault et al. (1997) embed implementation lags in the adoption of new technology and labor is employed through hiring, a costly process. Each household are either employed or searching for a job while unemployed. Members of the household produce goods by employing labor. Individuals that produce goods by employing labor. Members of the household are employed through hiring, a costly process. Each household maximizes the utility function:

$$E_{t} \sum_{t=0}^{\infty} \beta^{t} \ln C_{t} + \frac{N_{t+1}^{1-\psi}}{1 + \phi}$$

where $C_{t}$ is consumption, $N_{t}$ is the fraction of household members who are employed, $\beta$ is the discount factor such that $0 < \beta < 1$ and $\phi$ is the inverse of the Frisch intertemporal elasticity of substitution in labor supply such that $\phi > 0$. In this model we assume full participation, such that the members of a household can be either employed or unemployed, which implies $0 < N_{t} < 1$. Eq. (1), similar to Smets and Wouters (2003), contains two preference shocks: $\varepsilon_{t}$ represents a shock to the discount rate that affects the intertemporal rate of substitution between consumption in different periods, and $\bar{\varepsilon}_{t}$ represents a shock to the labor supply. Both shocks are assumed to follow a first-order autoregressive process with i.i.d. normal error terms such that

$$\bar{\varepsilon}_{t+1}^{b} = \bar{\varepsilon}_{t}^{b} \exp(\eta_{b,t+1}),$$

where $0 < \rho_{b} < 0, \eta_{b} - N(0,\sigma_{b})$, and similarly,

$$\varepsilon_{t+1}^{b} = \varepsilon_{t}^{b} \exp(\eta_{b,t+1}),$$

with $0 < \rho_{b} < 0, \eta_{b} - N(0,\sigma_{b})$.

2. The model

A standard flexible price model is enriched to allow for labor market frictions of the Diamond–Mortensen–Pissarides model of search and matching, as in Thomas (2008) and Blanchard and Gali (2010). As in Gali’s (1999) original study, our setting abstracts away from investment and capital accumulation and, in addition, assumes that the processes of job searching and recruitment are costly for both the firm and the worker.\(^5\) The economy is populated by a continuum of infinite-living identical households that produce goods by employing labor. Members of the household are either employed or searching for a job while unemployed. During each period, a constant fraction of jobs is destroyed and labor is employed through hiring, a costly process. Each household maximizes the utility function:

\(^{4}\) The appendix discusses the role of investment-specific technology shocks.

\(^{5}\) This paper does not focus on investment-specific technology shocks for two reasons. First, there is no clear consensus on their importance. For instance, Fisher (2006) finds them important in the context of a SVAR model. However, Schmitt-Grohe and Uribe (2012) and Mandelman et al. (2011) find that they play a minor role when the full-information Bayesian approach strategy is implemented to estimate business cycle models with investment-specific technology shocks. Second, and more important, the focus of the paper is different. Our objective is not to replicate the SVAR facts, or to take a particular stance on the importance of investment-specific technology shocks. We instead aim to show that a standard flexible price real business cycle model is compatible with Gali’s original results once it incorporates labor market frictions. It would be certainly be valuable extension for future research.
\[ e_{t+1} = \left( e_t \right)^{\gamma_t} \exp \left( \eta_{t+1}, \right), \] where \( 0 < \rho_e < 0, \) and \( \eta_e \sim N(0, \sigma_e^2). \)

During each period, output, \( Y_t, \) is produced according to the production function:

\[ Y_t = A_t N_t, \] (2)

where \( A_t \) is an exogenous technology shock that follows a first-order autoregressive process with i.i.d. normal error terms such that \( A_t = (A_1)^\rho_t \exp \left( \eta_{1,t+1}, \right), \) where \( 0 < \rho_e < 0, \) and \( \eta_e \sim N(0, \sigma_e^2). \) During each period, total employment is given by the sum of the number of workers who survive the exogenous separation and the number of new hires, \( H_t. \) Hence, total employment evolves according to:

\[ N_t = (1 - \delta)N_{t-1} + H_t, \] (3)

where \( \delta \) is the job destruction rate and \( 0 < \delta < 1. \) Accounting for job destruction, the pool of household’s members unemployed and available to work before hiring takes place is:

\[ U_t = 1 - (1 - \delta)N_{t-1}. \] (4)

It is convenient to represent the job finding rate, \( \gamma_t, \) by the ratio of new hires over the number of unemployed workers such that:

\[ \gamma_t = H_t/U_t, \] (5)

with \( 0 < \gamma_t < 1, \) given that all new hires represent a fraction of the pool of unemployed workers. The job finding rate, \( \gamma_t, \) may be interpreted as an index of labor market tightness. This rate also has an alternative interpretation: from the viewpoint of the unemployed, it is the probability of being hired in period \( t, \) or in other words, the job-finding rate. The cost of hiring a worker is equal to \( G_t \) and, as in Blanchard and Gali (2010), is a function of \( \gamma_t \) and the state of technology:

\[ G_t = A_t^\alpha B_t^{\beta}, \] (6)

where \( \gamma \) determines the extent to which, if any, hiring costs co-move with technology; \( \alpha \) is the elasticity of labor market tightness with respect to hiring costs; and \( B \) is a scale parameter. Hence, \( \gamma \in \mathbb{R}, \alpha \geq 0, \) and \( B \geq 0. \) As pointed out in Yashiv (2000) and Rotemberg (2006), this general formulation captures the idea that, in principle, hiring costs may be either pro- or counter-cyclical.\(^7\) Note that given the assumption of full participation, the unemployment rate, defined as the fraction of household members left without a job after hiring takes place, is defined as:

\[ u_t = 1 - N_t. \] (7)

The aggregate resource constraint:

\[ Y_t = C_t + G_t H_t, \] (8)

completes the description of the model.

The resource allocations can be characterized by solving the social planner’s problem. The social planner chooses \( \{Y_t, C_t, H_t, G_t, \gamma_t, \alpha_t, U_t, N_t, N_{t-1}, A_t\} \) to maximize the household's utility subject to the aggregate resource constraints, represented by Eqs. (2)–(8). To solve this problem, it is convenient to use Eq. (8), with the other constraints to obtain the aggregate resource constraint of the economy expressed in terms of consumption and employment. The aggregate resource constraint of the economy therefore can be written as:\(^8\)

\[ A_t N_t = C_t + A_t^\beta B \left( \frac{N_t - (1 - \delta)N_{t-1}}{1 - (1 - \delta)N_{t-1}} \right)^{1 - \alpha}. \] (9)

In this way, the social planner chooses \( \{G_t, N_t\} \) to maximize the household’s utility (1), subject to the aggregate resource constraint (9). Letting \( A \) be the non-negative Lagrange multiplier on the resource constraint, the first order condition for \( C_t \) is:

\[ A_t = \frac{\partial}{\partial C_t} \] (10)

and the first order condition for \( N_t \) is:

\[ \frac{\partial}{\partial N_t} \] (11)

Eq. (10) is the standard Euler equation for consumption, which equates the Lagrange multiplier to the marginal utility of consumption. Eq. (11) equates the marginal rate of substitution to the marginal rate of transformation. The marginal rate of transformation depends on productivity, \( A_t, \) as in the standard flexible price model, but also, due to the presence of labor market frictions, on foregone present and future costs of hiring. More specifically, the three terms composing the marginal rate of transformation are as follows. The first term, \( A_t, \) corresponds to the additional output generated by the marginal employed worker. The second term represents the cost of hiring an additional worker, and the third term captures the savings in hiring costs resulting from the reduced hiring needs in period \( t + 1. \) In the standard flexible price model, only the first term appears.

3. Bayesian estimation

Eqs. (2)–(8), (10) and (11) describe the behavior of the endogenous variables \( \{Y_t, C_t, H_t, G_t, \gamma_t, \alpha_t, u_t, N_t, \} \) and persistent autoregressive processes describe the exogenous shocks \( \{e_t, \varepsilon_t, \varepsilon_t^2\}. \) The equilibrium conditions do not have an analytical solution. For this reason, the system is approximated by loglinearizing Eqs. (2)–(8), (10) and (11) around the stationary steady state. In this way, a linear dynamic system describes the path of the exogenous variables’ relative deviations from their steady state value, accounting for the exogenous shocks. The solution to this system takes the form of a state-space representation and is solved using the method in Klein (2000). The latter can be conveniently used to compute the likelihood function in the estimation procedure.\(^9\)

We estimate the model using Bayesian methods, as described in An and Schorfheide (2007). This approach allows one to formalize the use of prior information coming either from microeconometric studies or previous macroeconometric studies and thereby makes an explicit link with the previous calibration-based literature. Second, the use of prior densities over the parameters space makes the maximization of the log-likelihood computationally more stable since the model is estimated with the previous calibration-based literature.
for the parameters of the model. This approach is particularly valuable when only relatively small samples of data are available, as in the case of small- and medium-size macroeconomic models. Third, there is an asymptotic justification for choosing the Bayesian procedure. Fernandez-Villaverde and Rubio-Ramírez (2005) prove consistency of both the point estimates and the posterior odds ratio. In addition, the small sample performance of Bayesian estimates tends to outperform classical methods even when evaluated by frequentist criteria, as shown in Jacquier et al. (1994) and Geweke et al. (1997). Finally, the Bayesian approach allows evaluating the models’ misspecification by using the marginal likelihood of the model, as described in Section 4.

The estimation uses U.S. quarterly data for output, unemployment and the job finding rate for the sample period 1951:1 through 2007:4. Output is defined as real gross domestic product in chained 2000 dollars taken from the Bureau of Economic Analysis. The unemployment rate is defined as the civilian unemployment rate and is taken from the Bureau of Labor Statistics. The job finding rate is taken from Shimer (2012). The data for output are logged and HP filtered prior to estimation, and the unemployment and job finding rate series are demeaned.

The data do not contain enough information to estimate all of the model’s parameters; some must be fixed prior to estimation. This assumption is common in estimated general equilibrium models, as detailed in Ireland (2004), Fernandez-Villaverde and Rubio-Ramírez (2004) and Smets and Wouters (2007). In particular, we fix three parameters $\beta$, $\delta$ and $B$. As explained in Altig et al. (2011) and Ireland (2004), it is necessary to calibrate the discount factor to successfully estimate the remaining parameters of the real business cycle model. This is particularly relevant in this setup with no capital accumulation. The quarterly discount factor $\beta$ is thus set at 0.99, which is the standard value in the literature. Without data on the job destruction rate, it is difficult to estimate the parameter $\delta$, and therefore we fix this parameter equal to 0.12, as estimated by den Haan et al. (2000) and Fujita and Ramey (2009). Similarly, it is also difficult to estimate the scale parameter of hiring costs $B$ without high-frequency data on the cost of posting vacancies. We therefore fix this parameter at 0.11, which implies that hiring costs approximately represent 1% of total output, as in Blanchard and Galí (2010).

The parameters to estimate are: $\{\alpha, \phi, \gamma, \rho_a, \rho_b, \alpha_\sigma, \alpha_\gamma, \gamma_\rho\}$ Columns (1)–(4) in Table 1 present the mean and standard deviation of the prior distributions, together with their respective densities and ranges. For the elasticity of labor market tightness with respect to hiring costs, $\alpha$, we assume a normal distribution with prior mean and standard deviation equal to 1 and 0.3 respectively. The prior mean is equal to the

### Table 1
Summary statistics for the prior and posterior distribution of the parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Prior mean</th>
<th>Prior SE</th>
<th>Density</th>
<th>Range</th>
<th>Posterior 2.5%</th>
<th>97.5%</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>1</td>
<td>0.3</td>
<td>Normal</td>
<td>$\mathbb{R}$</td>
<td>1.4058</td>
<td>0.9496</td>
</tr>
<tr>
<td>$\phi$</td>
<td>2</td>
<td>0.75</td>
<td>Normal</td>
<td>$\mathbb{R}$</td>
<td>0.6363</td>
<td>$-0.4948$</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0</td>
<td>7</td>
<td>Normal</td>
<td>$\mathbb{R}$</td>
<td>10.1674</td>
<td>5.8933</td>
</tr>
<tr>
<td>$\rho_a$</td>
<td>0.5</td>
<td>0.2</td>
<td>Beta</td>
<td>[0.1]</td>
<td>0.8973</td>
<td>0.8291</td>
</tr>
<tr>
<td>$\rho_b$</td>
<td>0.5</td>
<td>0.2</td>
<td>Beta</td>
<td>[0.1]</td>
<td>0.7354</td>
<td>0.5326</td>
</tr>
<tr>
<td>$\sigma_a$</td>
<td>0.01</td>
<td>1</td>
<td>Inv gamma</td>
<td>$\mathbb{R}^+$</td>
<td>0.0015</td>
<td>0.0088</td>
</tr>
<tr>
<td>$\sigma_b$</td>
<td>0.01</td>
<td>1</td>
<td>Inv gamma</td>
<td>$\mathbb{R}^+$</td>
<td>0.0006</td>
<td>0.0021</td>
</tr>
<tr>
<td>$\sigma_l$</td>
<td>0.01</td>
<td>1</td>
<td>Inv gamma</td>
<td>$\mathbb{R}^+$</td>
<td>0.0053</td>
<td>0.0021</td>
</tr>
</tbody>
</table>

Log-likelihood 937.81

Notes: results based on 200,000 draws of the Metropolis Algorithm. Description of parameters: $\alpha$, elasticity of labor market tightness with respect to hiring costs; $\phi$, inverse of the Frisch intertemporal elasticity of substitution in labor supply; $\gamma$, elasticity of hiring costs to technology; $[\rho_a, \rho_b, \sigma_a, \sigma_b, \sigma_l]$, volatility parameters of technology, preference and labor supply shocks.

Fig. 1. Prior and posterior densities of the estimated parameters, benchmark model. Notes: Each entry shows the prior (gray line) and posterior (dark line) densities associated with the estimated parameter. Results based on 200,000 draws of the Metropolis Algorithm.
that the estimation delivers a sizable reading for costs to technology, (1999). Of special interest is the estimate for the elasticity of hiring values used in the macro literature, as reported in King and Rebelo (2007) and more generally is in line with the economic estimates, as in Pencavel (1986), and the macro literature, as in Rogerson and Wallenius (2007). For the elasticity of technology shocks to hiring costs, \( \gamma \), we assume a Normal distribution with prior mean and standard deviation equal to 0.4 and 0.15, respectively. Such priors cover the range of values in between the microeconomic estimates, as in Pencavel (1986), and the macro literature, as in Rogerson and Wallenius (2007). For the elasticity of technology shocks to hiring costs, \( \gamma \), we assume a Normal distribution with prior mean and standard deviation equal to 0 and 7 respectively. In this way, we impose very flat priors that allow for a wide range of plausible values. For the parameters related with the structural shocks, we allow for a wide range of values and use prior distributions commonly found in the literature, as in Smetts and Wouters (2007) and Justiniano et al. (2011). In particular, for the autoregressive parameters of the shocks \( \{\rho_a, \rho_b, \rho_c\} \), we assume a Beta distribution with prior mean and standard deviation equal to 0.5 and 0.2 respectively. Finally, for the variance of the stochastic components \( \{\sigma_a, \sigma_b, \sigma_c\} \), we assume an Inverse Gamma distribution with prior mean and standard deviation equal to 0.01 and 1, respectively.

Columns (5)–(7) in Table 1 present the posterior mean and the 95% probability interval of the parameter estimates. The posterior mean of the elasticity of labor market tightness with respect to hiring costs, \( \alpha \), equals 1.41, which is a value close to 1, commonly used in the literature. The posterior mean of the inverse of the Frisch intertemporal elasticity of substitution in labor supply, \( \phi \), equals 0.84, which implies an elasticity of labor supply equal to 1.2. This value is consistent with that in Rogerson and Wallenius (2007) and more generally in line with the values used in the macro literature, as reported in King and Rebelo (1999). Of special interest is the estimate for the elasticity of hiring costs to technology, \( \gamma \). The posterior mean of \( \gamma \) is 10.17, which, as detailed below, supports the fact that the data prefer a positive response of hiring costs to technology shocks. Furthermore, it is worth noticing that the estimation delivers a sizable reading for \( \gamma \) despite its loose prior. This positive and sizeable estimate corroborates the findings in Yashiv (2000), who establishes that hiring costs respond strongly and positively to technology. However, the estimate of the elasticity of hiring costs to technology shocks is large, since a unitary change in technology implies a change in hiring costs ten times larger, suggesting that additional extensions to the model are needed. Turning now to the stochastic processes, the posterior mean of the persistence of technology shocks, \( \rho_t \), is 0.9, which shows that technology shocks are highly persistent. The posterior mean of the persistence of preference shocks, \( \rho_p \), is 0.73, and the estimate of the persistence of labor supply shocks, \( \rho_\sigma \), is 0.65. The posterior mean of the volatility of technology shocks, \( \sigma_t \), is 0.01, as in King and Rebelo (1999). The posterior mean of the volatility of preference shocks, \( \sigma_p \), is 0.0068, and the posterior mean of the volatility of labor supply shocks, \( \sigma_\sigma \), is 0.0053. Finally, Fig. 1 shows that the prior and posterior densities of the estimated parameters are different in general, providing evidence that the data are informative for the estimation of the model.

![Fig. 2. Impulse responses to a one-standard-deviation technology shock, unconstrained model with labor market frictions. Notes: Impulse responses to a one-standard-deviation technology shock of the unconstrained model with labor market frictions. Each entry depicts the median impulse response (solid line) of each variable together with the 10–90% posterior intervals (dotted line). The horizontal axes measures the time, expressed in quarters.](image-url)

To check the robustness of the results to the assumptions on the prior distribution of \( \gamma \), we have estimated the model using different means and standard deviations on the prior of this parameter. This has a limited effect on the results, which are available on request. In addition, to check whether our choice of prior drives the estimation results, we have estimated the model using uninformative priors and established that the posterior distribution of the parameters remains substantially unchanged. An appendix that details the alternative estimation is available upon request from the authors.

The impulse responses of the model to the preference and labor supply shocks are available in a companion appendix to this paper, available upon request from the authors. Note that the reaction of vacancies displays a hump-shape response to a technology shock in the data, as shown in Ravn and Simonelli (2008). In the model, the reaction of new hires decays quickly in the aftermath of the shock. This response is generated by our stylized hiring cost function that does not include any lagged term. Enriching the functional form of hiring adjustment costs to match this important stylized fact in the data would certainly be a useful extension for future research.
Table 2 reports autocorrelation functions of key macroeconomic variables with output based on the data and the mode of the model’s posterior distribution, respectively. In general, the model’s results are in line with the empirical evidence. For instance, the model’s simulations deliver a positive contemporaneous correlation of output with consumption and labor market tightness as well as a negative correlation with the unemployment rate. Moreover, the model matches the sign of correlations at different leads and lags relatively well. Table 3 shows asymptotic (i.e. infinite horizon) forecast error variance decompositions into percentages for each of the model’s shocks. The variance decompositions indicate that productivity and discount factor innovations mostly account for the bulk of macroeconomic variability in the long run. Technology shocks account for nearly 85% of the unconditional variance in detrended output and consumption while they contribute approximately 40% to movements in unemployment and labor market tightness. The rest of the fluctuations are shared between preference and labor supply shocks, similar to the findings in Kydland and Prescott (1991) and Ireland (2001).

To conclude this section, we use the identification test in Ikskrev (2010) to evaluate whether the Bayesian estimation is able to identify the estimated parameters of the model. In essence, the Ikskrev test checks whether the derivatives of the predicted autocovariogram of the observables with respect to the vector of estimated parameters has rank equal to the length of the vector of estimated parameters. We find that the column rank is full when evaluated at the posterior mean of the Bayesian estimate. To establish whether identification would hold for an appropriate neighborhood of our estimates, we also evaluate the rank for 500,000 draws from the prior distributions, and we establish that full rank condition still holds. According to this test, therefore, the estimated parameters are identifiable in the neighborhood of our estimate.

4. The role of labor market frictions

To investigate the role of labor market frictions, we estimate two versions of the model. First, a version that abstracts away from hiring costs by imposing $B = 0$, so that the theoretical framework nests the first order conditions of a standard flexible price model where labor market frictions are absent. Second, a version that assumes that hiring costs do not react directly to technology shocks, by imposing $\gamma = 0$, so we determine whether the data prefer the version of the model with hiring costs reacting to technology shocks or a more constrained specification where hiring costs do not directly react to technology shocks.

Columns (1)–(3) in Table 4 present the posterior mean and the 95% probability interval of the parameter estimates when $B = 0$. In this instance, the theoretical framework nests the first order conditions of a standard flexible price model where labor market frictions are absent. To be consistent throughout the estimation exercise, the prior distributions of the parameters are the same as those in the baseline model. Estimation results indicate that the posterior mean of the inverse of the elasticity of labor supply, $\phi$, equals 1.09. The persistence of the shocks is slightly lower than in the unconstrained model whereas their volatility is similar across the two specifications. In general, these estimates are in line with the results from standard flexible price models without labor market frictions, as in Bencivenga (1992), De-Jong et al. (2000), Ireland (2001, 2004) and Zanetti (2008).

What lies behind the posterior means of the parameters for the reactions of the variables to technology shocks? Fig. 3 traces the estimated model’s implied impulse responses of each variable to a one-standard-deviation technology shock for both specifications of the model, with and without labor market frictions. The reaction of output and consumption is qualitatively similar across the two models whereas the reaction of employment is negative in the presence of labor market frictions and null in a perfectly competitive labor market, due to the offsetting income and substitution effects on labor supply.

How can the presence of labor market frictions generate a negative reaction of employment? As discussed, the answer lies in the way hiring costs react to productivity shocks. Here the reaction is determined by the elasticity of hiring costs to a technology shock, which is represented by the parameter $\gamma$. The estimation exercise allows the value of this parameter to be either positive, negative or equal to zero and leaves the data to choose the preferred value. The estimation suggests that the data prefer $\gamma$ to be positive, such that hiring costs co-move positively with technology shocks (which is also the assumption in the calibrated model of Blanchard and Gali (2010)). To understand how this movement generates a negative reaction of employment to technology shocks, consider Eq. (11), which represents the labor market equilibrium condition. A productivity shock would increase the marginal product of labor, the first term on the right-hand side of Eq. (11), as in the standard flexible price model, but it also would increase the cost of recruiting an additional worker, the second term on the right-hand side of Eq. (11), and at the same time, reduce the hiring needs in period $t + 1$, the third term on the right-hand side of Eq. (11). The effect on the second term, namely the cost of recruiting an additional worker, dominates the other two and, as a result, the marginal rate of transformation, which is the right-hand side of Eq. (11), is reduced and therefore generates a negative response of employment to technology shocks. In the model without labor market frictions (i.e. $B = 0$), the correspondent equilibrium

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Table 2
Descriptive statistics.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Data Corr(Variable$_{t-h}$, Y$_t$)</th>
<th>Model Corr(Variable$_{t-h}$, Y$_t$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>−2</td>
<td>−1</td>
</tr>
<tr>
<td></td>
<td>−2</td>
<td>−1</td>
</tr>
<tr>
<td>Y</td>
<td>0.59</td>
<td>0.84</td>
</tr>
<tr>
<td>u</td>
<td>−0.31</td>
<td>−0.45</td>
</tr>
<tr>
<td>C</td>
<td>0.68</td>
<td>0.83</td>
</tr>
<tr>
<td>x</td>
<td>0.33</td>
<td>0.46</td>
</tr>
</tbody>
</table>

Notes: results based on 200,000 draws of the Metropolis Algorithm. The posterior estimated median is reported.

Table 3
Variance decompositions.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Decompositions</th>
<th>$A_l$</th>
<th>$c_l^2$</th>
<th>$c_l^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y</td>
<td></td>
<td>0.86</td>
<td>0.09</td>
<td>0.05</td>
</tr>
<tr>
<td>u</td>
<td></td>
<td>0.42</td>
<td>0.39</td>
<td>0.19</td>
</tr>
<tr>
<td>C</td>
<td></td>
<td>0.87</td>
<td>0.08</td>
<td>0.05</td>
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<tr>
<td>x</td>
<td></td>
<td>0.40</td>
<td>0.40</td>
<td>0.20</td>
</tr>
</tbody>
</table>

Notes: results based on 200,000 draws of the Metropolis Algorithm. Asymptotic variance decompositions decompose the forecast error variance into percentages due to each of the model’s shocks. The posterior estimated median is reported.

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14 Note that $B = 0$ implies that $C_t = 0$, as from Eq. (6). Hence, the parameters $\alpha$ and $\gamma$ are excluded from the estimation of the model.
condition, equivalent to Eq. (11), is $e^{\gamma N_t + \frac{1}{\phi} t} = 1$, which implies a level of employment invariant to technology shocks as a result of offsetting income and substitution effects on labor supply. Without capital accumulation, such a result is standard in this class of models, as King and Rebelo (1999) point out. Despite the different reactions of employment to a technology shock, the functioning of the two models is qualitatively similar.

Turning to the parameter describing the elasticity of hiring costs to technology shocks, $\gamma$, we now impose the neutral assumption that hiring costs do not react directly to technology shocks. In this way, we determine whether the data prefer the version of the model with hiring costs reacting to technology shocks or a more constrained specification where hiring costs do not directly react to technology. We test which version of the model the data prefer by imposing $\gamma = 0$ on the baseline specification of the model. As before, the prior distributions of the parameters are the same as those in the baseline model. Columns (4)–(6) in Table 4 report the posterior mean and 95% probability interval of the parameters for the constrained model. The posterior mean of the structural parameters for this constrained specification are reasonably close to those where $\gamma$ is allowed to differ from zero. In particular, the posterior mean of the elasticity of labor market tightness with respect to hiring costs, $\alpha$, equals 1.43. The posterior mean of the inverse of the elasticity of labor supply, $\phi$, equals 0.81, and the posterior mean of the autoregressive component of the labor supply shocks are highly persistent. Results indicate that the volatility of the stochastic components is of a similar magnitude to the estimates of the unconstrained model. Overall, the similarity of these estimates to those of the unconstrained model suggests that the underlying model with labor market frictions is consistently estimated across different model specifications.

Fig. 4 shows the model’s implied impulse responses of each variable to a one-standard-deviation technology shock for both the constrained model where $\gamma = 0$ and the baseline model with labor market frictions. Output, consumption and employment positively react to a technology shock, as in the unconstrained specification. When $\gamma = 0$, hiring costs do not directly react to technological innovations. In this case, the effect on the second term on the right-hand side of Eq. (11), namely the cost of recruiting an additional worker, is dominated by the counteracting effect of the two other terms, thus generating a positive response of employment to technology shocks. The positive reaction of employment leads to a positive response in the number of hires and this, coupled with the negative reaction of unemployment, generates an increase in labor market tightness. Consequently, the cost of hiring increases slightly on impact.

Before concluding, to establish whether the data prefer the unconstrained specification of the model, the version without labor market frictions ($B = 0$), or the version in which hiring costs do not directly react to technological innovations ($\gamma = 0$), the last row in Table 4 reports the posterior odds ratio. This metric is computed as the difference between the log marginal likelihood of each model with respect to the constrained specification. For a description of the parameters see notes in Table 1.

Table 4

<table>
<thead>
<tr>
<th>Parameter</th>
<th>No hiring costs ($B = 0$)</th>
<th>No reaction to technology ($\gamma = 0$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Posterior 2.5% 97.5%</td>
<td>Posterior 2.5% 97.5%</td>
</tr>
<tr>
<td></td>
<td>(1) (2) (3) (4) (5) (6)</td>
<td>(1) (2) (3) (4) (5) (6)</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>$-$ $-$ $-$ $-$ $-$ $-$</td>
<td>1.4318 1.0593 1.3697</td>
</tr>
<tr>
<td>$\phi$</td>
<td>1.0872 0.1679 1.9735 0.8073 $-$ $-$</td>
<td>$-$ $-$ $-$ $-$ $-$ $-$</td>
</tr>
<tr>
<td>$\rho_0$</td>
<td>0.8270 0.7652 0.8844 0.8234 $-$ $-$</td>
<td>0.7661 0.8794</td>
</tr>
<tr>
<td>$\rho_1$</td>
<td>0.7892 0.6959 0.8770 0.8550 $-$ $-$</td>
<td>0.7814 0.9444</td>
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<tr>
<td>$\sigma_\alpha$</td>
<td>0.0089 0.0032 0.0096 0.0084 $-$ $-$</td>
<td>0.0080 0.0094</td>
</tr>
<tr>
<td>$\sigma_\phi$</td>
<td>0.0110 0.0056 0.0163 0.0105 $-$ $-$</td>
<td>0.0019 0.0157</td>
</tr>
<tr>
<td>$\sigma_\gamma$</td>
<td>0.0040 0.0023 0.0058 0.0042 $-$ $-$</td>
<td>0.0023 0.0060</td>
</tr>
<tr>
<td>Log-likelihood</td>
<td>855.99</td>
<td>935.72</td>
</tr>
<tr>
<td>Posterior odds ratio</td>
<td>$e^{1.43}$</td>
<td>$e^{0.81}$</td>
</tr>
</tbody>
</table>

Notes: Results are based on 200,000 draws of the Metropolis Algorithm. The prior distributions of the parameters are the same as those in the baseline model, as reported in columns (1)–(4) of Table 1. The posterior odds ratio is computed as the difference between the marginal likelihood of the unconstrained model that allows for labor market frictions, reported in the bottom line of Table 1, and each of the marginal likelihood functions associated with the alternative specification of the model that either abstracts away from labor market frictions by imposing $B = 0$ (i.e. hiring costs are absent), or assumes that hiring costs do not react directly to technology shocks, by imposing $\gamma = 0$. For a description of the parameters see notes in Table 1.
that this metric penalizes overparameterization, models with labor market frictions do not necessarily rank better if the extra friction does not sufficiently help in explaining the data. As from the entries in Table 4, the odds ratio of the flexible price model is $e^{0.182}$ and the odds ratio of the model in which hiring costs do not directly react to technological innovations is $e^{0.09}$. In other words, to choose one of these constrained versions of the model over the unconstrained specification, the Bayes factor requires a prior probability over the constrained versions of $e^{0.182}$ and $e^{0.09}$ times larger than over the unconstrained model. This indicates that the estimation strongly prefers the model that accounts for labor market frictions over and above the alternative specifications based on either a model that abstracts away from these frictions or a model where hiring costs do not directly react to technology shocks.

5. Conclusion

Recent empirical evidence led by Gali (1999) and supported by several subsequent studies finds that a positive technology shock leads to a decline in labor inputs. This paper uses Bayesian methods to establish that labor market frictions can enable a flexible price model to match this stylized fact. We believe that this finding clearly underlines the importance of labor market frictions to accurately characterize the dynamics of labor inputs to technology shocks in the context of estimated general equilibrium models.

The model puts forward some interesting avenues for future research. First, labor market frictions introduce flows in and out of employment. It would be interesting to establish the contribution of each flow to the fall in employment. This task, however, would prove to be non-trivial because it requires introducing endogenous job destruction. Second, it also would be interesting to enrich the model with nominal price rigidities that Gali (1999) identifies as an alternative mechanism to rationalize the fall in employment in the aftermath of a positive technology shock. In this way, it would be possible to establish to what extent labor market frictions and nominal price rigidities compete to account for the observed stylized fact. These investigations offer avenues for future research.

References


Yashiv, E. (2013). ‘The forward looking behavior of hiring and investment’, Tel Aviv University, mimeo.