Factor adjustment costs: A structural investigation

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ABSTRACT

This paper assesses various capital and labor adjustment costs functions estimating a general equilibrium framework with Bayesian methods using US aggregate data. The estimation finds that the adjustment costs are convex in both capital and labor and allowing for their joint interaction is important. The structural model enables us to identify the response of factor adjustment costs to exogenous disturbances, and to establish that shocks to technology and the job separation rate are key drivers of adjustment costs. The analysis shows that factor adjustment costs enable the model to explain fluctuations in the firm's market value in the data.

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1. Introduction

An extensive literature finds that capital and labor inputs are costly to adjust.¹ Factor adjustment costs make the asset values of capital and labor fluctuate according to their underlying marginal adjustment costs whereas they would be constant otherwise. Moreover, adjustment costs generate rents when demand rises unexpectedly, whose movements, in principle, may explain fluctuations in the market value of the firm relative to the underlying factor input costs. In this respect, a structural investigation on the size and dynamics of factor adjustment costs is important in order to understand aggregate fluctuations in the price of capital and labor inputs, and the firm's market value.

The contribution of this paper is to assess factor adjustment costs by estimating a dynamic stochastic general equilibrium (DSGE) model for several competing adjustment costs functions using US aggregate data. This is the first study that conducts the analysis in a general equilibrium framework and that uses a system approach estimated using Bayesian methods. Our approach has several advantages. First, the theoretical setting is microfounded and based on a prototype, production-based model enriched with labor market frictions and factor adjustment costs. Second, rather than estimating asset price functions in a single-equation setting, we pursue a multivariate approach by estimating the entire structural model. The system approach optimally adjusts the estimation of the asset price equations' coefficients for the endogeneity of the variables.

¹ See Bond and Van Reenen (2007) and references therein for a recent review on the topic.
Moreover, we are able to exploit cross-equation restrictions that link agent’s decision rules with the coefficients in the asset price equations. To conduct the estimation we assign prior distributions to the parameters of the adjustment costs function and exogenous disturbances and use Bayesian inference. Posterior distributions are used to determine the functional form of the adjustment costs functions and posterior odds ratio to assess their empirical adequacy. To the best of our knowledge, this is the first time that such a methodology has been applied to investigate factor adjustment costs.

To establish the empirically suitable adjustment costs function, the theoretical model allows, but does not require, capital and labor adjustment costs to include linear and convex cost components, and it also lets capital and labor adjustment costs interact. This formulation encompasses a broad range of adjustment costs functions. In this way, the theoretical model allows for both investment and hiring decisions to simultaneously affect the asset prices of capital and labor, and consequently the firm’s market value. The posterior odds ratio shows that the data prefer the adjustment costs function that includes both linear and convex cost components, and that also accounts for the joint interaction between capital and labor costs. Specifications with capital adjustment costs only (as in the investment literature) or with labor adjustment costs only (as in the labor demand literature) are rejected by the data. The econometric estimation finds that adjustment costs are small for both input factors. According to the theoretical framework, total adjustment costs represent 3.3% of total output per quarter. In addition, the cost of hiring an additional worker amounts to 1.4 weeks of wages, whereas the cost of an extra unit of investment equals 0.22% of average output per unit of capital. Such estimates are within the range of values estimated using disaggregated data as in Shapiro (1986) and Gilchrist and Himmelberg (1995), and in line with Bloom (2009).

The use of a structural approach enables additional interesting results. The estimation identifies structural disturbances in the data based on the dynamic effects that they have on the model’s observable variables. The model’s reduced form enables us to extend the identification of shocks to the model’s unobservable variables, and we are therefore able to map the response of key macroeconomic variables and factor adjustment costs to the exogenous disturbances to technology, labor supply, job and capital destruction rates and tax changes. We find that total factor adjustment costs are pro-cyclical for all the shocks, except for shocks to the job and capital destruction rates. We also detect that the asset prices of capital and labor mirror one-for-one the reaction of the marginal costs of investing and hiring, which in turn determine the firm’s market value. Forecast error variance decompositions show that technology shocks play a prime role on output, factor adjustment costs and the firm’s market value in the short run, whereas shocks to the job separation rate compete with technology shocks to explain the bulk of fluctuations of factor adjustment costs in the long run.

In addition, the structural model allows us to estimate the unobservable shocks using a Kalman smoothing algorithm that uses the information contained in the full sample of the data. By feeding the estimated structural shocks into the theoretical model we generate time series for the firm’s market value that can be compared against the actual series in the data. We find that the adjustment costs function that allows for both linear and convex capital and labor adjustment costs, and that also allows for their joint interaction, is able to replicate more closely the fluctuations in the firm’s market value in the data.

Before proceeding, we discuss the context provided by related studies. As mentioned, one contribution of the paper is to estimate the adjustment costs function that fits aggregate data. In general, estimates of factor adjustment costs are based on disaggregated firm-level data, as surveyed by Bond and Van Reenen (2007), and only a few studies focus on aggregate data. Of these, the majority estimates either capital adjustment costs, or labor adjustment costs individually, assuming that the other factor is flexible. In particular, Ireland (2003), Christiano et al. (2005) and Smets and Wouters (2007) use DSGE models to estimate capital adjustment costs in a frictionless labor market. On the other hand, Cogley and Nason (1995), Chang et al. (2007) and Janko (2008) estimate labor adjustment costs in the absence of capital adjustment costs. Our paper uses a similar methodology but it assesses the adequacy of various adjustment costs functions that allow for both capital and labor adjustment costs.

Similar to our approach, Dib (2003) estimates a DSGE model using maximum likelihood methods that allows for simultaneous capital and labor adjustment costs. However, the model abstracts from the joint interaction between capital and labor costs, and the analysis focuses neither on the size of adjustment costs, nor on their implication for the model’s dynamics. Merz and Yashiv (2007), Bloom (2009) and Yashiv (2013) develop partial equilibrium models to study the interaction of capital and labor adjustment costs. They estimate asset pricing equations in a single-equation setting, using the generalized method of moments and instrumental variables. Instead, we use a fully-defined DSGE model that uses the same asset price equations and also exploits the cross-equation restrictions of the entire structural model, thereby overcoming the identification issues encountered in single-equation estimates.

The remainder of the paper is organized as follows. Section 2 lays out the model. Section 3 presents the econometric methodology and the data. Section 4 presents the estimation results, illustrates the steady-state and dynamics properties of the model and assesses the empirical fit of alternative adjustment costs functions. Section 5 concludes.

2. The model

In our model the standard production-based model by Cochrane (1991) is enriched with labor market frictions as in Blanchard and Gali (2010) and a factor adjustment costs function as in Merz and Yashiv (2007) and Bloom (2009). This framework relies on the assumption that the process of job search and recruitment is costly for both the firm and the worker. Job creation takes place when a firm and a job seeker meet and agree to form a match at a negotiated wage, which
depends on the parties' bargaining power. The match continues until the parties exogenously terminate the relationship.

When this occurs, job destruction takes place and the worker moves from employment to unemployment, and the firm can either withdraw from the market or hire a new worker. The wage splits the surplus from working between the firm and the household.

The model economy consists of a representative firm and household. The rest of this section describes the agents' preferences, technologies and the structure of the labor market.

2.1. The representative firm

During each period \( t = 0, 1, 2, \ldots \), the representative firm employs \( n_t \) units of labor and \( k_t \) units of capital from the representative household, in order to manufacture \( y_t \) units of good according to the constant returns to scale production technology:

\[
y_t = f(a_t, k_t, n_t),
\]

where \( a_t \) is the neutral technology process \( a_t = \Gamma(a_{t-1}, \varepsilon_{at}) \), and \( \varepsilon_{at} \) is an i.i.d. shock. The firm's real profits, \( \pi_t \), equal the difference between revenues net of factor adjustment costs, \( g(i_t, k_t, h_t, n_t) \), which depend on the firm's new investment \( i_t \), the installed capital \( k_t \), the number of new hires \( h_t \), the stock of labor \( n_t \), and total labor compensation, \( w_i h_t \):

\[
\pi_t = f(a_t, k_t, n_t) - g(i_t, k_t, h_t, n_t) - w_i h_t,
\]

where \( w_i \) is the real wage. The problem for the firm is to maximize its total real market value, \( v_t \), given by

\[
E_0 \sum_{t=0}^{\infty} \beta^t \lambda_t d_t,
\]

where \( d_t \) is the firm's real cash flow payments (defined below), and \( \beta^t \lambda_t \) measures the marginal utility value (defined below) to the representative household of an additional dollar in value during period \( t \). The firm's real cash flow payments, \( d_t \), equals profits minus purchases of investment goods:

\[
d_t = (1 - \tau_t)\pi_t - i_t,
\]

where \( \tau_t \) is the corporate income tax rate \( \tau_t = \Gamma(\tau_{t-1}, \varepsilon_{\tau t}) \), and \( \varepsilon_{\tau t} \) is an i.i.d. shock. During each period \( t = 0, 1, 2, \ldots \), by investing \( i_t \) units of output during period \( t \), the firm increases the capital stock \( k_{t+1} \) available during period \( t+1 \) according to

\[
k_{t+1} = (1 - \delta_t)k_t + i_t,
\]

where \( \delta_t \) is the capital depreciation rate \( \delta_t = \Gamma(\delta_{t-1}, \varepsilon_{\delta t}) \), and \( \varepsilon_{\delta t} \) is an i.i.d. shock. Similarly, by hiring \( h_t \) new workers during period \( t \), the firm increases the employment stock \( n_{t+1} \) available during period \( t+1 \) according to

\[
n_{t+1} = (1 - \psi_t)n_t + h_t,
\]

where \( \psi_t \) is the exogenous separation rate \( \psi_t = \Gamma(\psi_{t-1}, \varepsilon_{\psi t}) \), and \( \varepsilon_{\psi t} \) is an i.i.d. shock. Thus the firm chooses \( \{n_{t+1}, k_{t+1}, h_t, i_t\}_{t=0}^{\infty} \) to maximize its market value (3) subject to the law of capital and employment accumulation (5) and (6) for all \( t = 0, 1, 2, \ldots \). By letting \( q^k_t \) and \( q^n_t \) denote the non-negative Lagrange multiplier on the law of capital accumulation (5) and the law of employment accumulation (6), the first-order conditions for this problem are

\[
q^k_t = E_t \beta_{t+1} \left[ (1 - \tau_t)(f_{k,t+1} - g_{k,t+1}) + q^k_{t+1}(1 - \delta_{t+1}) \right],
\]

\[
q^n_t = E_t \beta_{t+1} \left[ (1 - \tau_t)(f_{n,t+1} - g_{n,t+1} - w_{t+1}) + q^n_{t+1}(1 - \psi_{t+1}) \right],
\]

\[
q^k_t = 1 + g_{k,t},
\]

and

\[
q^n_t = g_{h,t},
\]

where \( E_t \) is the expectation conditional on information available in period \( t \), \( \beta_{t+1} = \beta \lambda_{t+1} / \lambda_t \) is the stochastic discount factor, \( f_{x,t+1} \) denotes the marginal product of factor \( x \) at time \( t+1 \), \( g_{x,t+1} \) denotes the marginal cost of changing variable \( x \) at time \( t+1 \), and \( w_{t+1} \) the real wage at time \( t+1 \). Eq. (7) equates the contribution of an additional unit of investment to the firm's market value (left-hand side, LHS) to the expected marginal productivity of capital net of adjustment costs, plus the expected marginal contribution of investment during period \( t+1 \) (right-hand side, RHS). Eq. (8) equates the contribution of an additional hired worker to the firm's market value (LHS) to the expected marginal product of labor, net of total labor compensation, plus the expected future saving if the worker is retained during period \( t+1 \) (RHS). Finally, Eqs. (9) and (10) are the standard marginal \( q \) equations for investment and hiring respectively, which equate the contribution of an additional unit of investment or worker (LHS) to the firm’s costs generated by the additional unit of investment or the cost of recruiting (RHS).
To conclude the description of the representative firm, we specify the firm’s market value. The firm’s ex dividend market value in period \( t \) is defined as
\[
s_t = E_t \beta_{t+1} (s_{t+1} + d_t). \tag{11}
\]

As shown in Merz and Yashiv (2007), the firm’s market value can be decomposed into the sum of the value due to physical capital and the stock of employment, such that Eq. (11) can be written as
\[
s_t = k_{t+1} q^n_k + n_{t+1} + q^n_u. \tag{12}
\]

Eq. (12) shows that the market value of the firm depends on the present expected value of capital as well as the present expected value of labor.

2.2. The representative household

During each period \( t = 0, 1, 2, \ldots \), the representative household maximizes the expected utility function:
\[
E_0 \sum_{t=0}^{\infty} \beta^t \left[ \ln c_t - \chi c_t^{1+ \phi} / (1 + \phi) \right], \tag{13}
\]

where the variable \( c_t \) is consumption, \( n_t \) is units of labor, \( \beta \) is the discount factor \( 0 < \beta < 1 \), \( \phi \) is the inverse of the Frisch elasticity of labor supply \( \phi > 0 \), and \( \chi \) is the degree of disutility of labor \( \chi = \Gamma(\chi, \epsilon, \epsilon) \), and \( \epsilon_{xt} \) is an i.i.d. shock. The representative household enters period \( t \) with the firm’s cash flow payments \( d_t \). The household supplies \( n_t \) units of labor at the real wage rate \( w_t \) to the firm during period \( t \). The household uses its income for consumption, \( c_t \), subject to the budget constraint:
\[
c_t = w_t n_t + d_t, \tag{14}
\]

for all \( t = 0, 1, 2, \ldots \). Thus, the household chooses \( \{c_t\}_{t=0}^{\infty} \) to maximize its utility (13) subject to the budget constraint (14) for all \( t = 0, 1, 2, \ldots \). Letting \( \lambda_t \) denote the non-negative Lagrange multiplier on the budget constraint (14), the first-order condition for \( c_t \) is
\[
\lambda_t = 1 / c_t. \tag{15}
\]

According to Eq. (15), the Lagrange multiplier equals the household’s marginal utility of consumption.

The wage splits the total surplus from working. As in Pissarides (2000), the wage is set according to the Nash bargaining solution. In what follows we describe the structure of the labor market to explicitly derive the wage-setting equation.

At the beginning of each period \( t \) there is a pool of unemployed household members who are available for hire, and whose size we denote by \( u_t \). As in Blanchard and Gali (2010), we refer to the latter variable as the beginning of period unemployment. The pool of household’s members unemployed and available to work before hiring takes place is
\[
u_t = 1 - (1 - \psi_t) n_t. \tag{16}
\]

It is convenient to represent the job creation rate, \( x_t \), by the ratio of new hires over the number of unemployed workers such that
\[
x_t = h_t / u_t, \tag{17}
\]

with \( 0 < x_t < 1 \), given that all new hires represent a fraction of the pool of unemployed workers.

Let \( W^m_t \) and \( W^n_t \) denote the marginal value of the expected income of an employed, and unemployed worker respectively. The employed worker earns a wage, suffers disutility from work, and might lose her job with probability \( \psi_t \). Hence, the marginal value of a new match is
\[
W^m_t = w_t - \chi n_t^\phi / \chi + \beta E_{t+1} \left( [1 - \psi_{t+1} (1 - x_{t+1})] W^n_{t+1} + \psi_{t+1} (1 - x_{t+1}) W^m_{t+1} \right). \tag{18}
\]

This equation states that the marginal value of a job for a worker is given by the wage less the marginal disutility that the job produces to the worker, plus the expected-discounted net gain from being either employed or unemployed in period \( t+1 \).

The unemployed worker expects to move into employment with probability \( x_t \). Hence, the marginal value of unemployment is
\[
W^n_t = \beta E_{t+1} \left[ x_{t+1} W^m_{t+1} + (1 - x_{t+1}) W^n_{t+1} \right]. \tag{19}
\]

This equation states that the marginal value of unemployment is made up of the expected-discounted capital gain from being either employed or unemployed in period \( t+1 \).

As mentioned, the share of the surplus from establishing a job match is determined by the wage level, which is set according to the Nash bargaining solution. The worker and the firm split the surplus of their matches with the absolute share \( 0 < \eta < 1 \). The difference between Eqs. (18) and (19) determines the worker’s economic surplus. The firm’s surplus is simply given by the real cost per additional hire, \( g_{0,t} \), as in Blanchard and Gali (2010) and Muntaz and Zanetti (2012). Hence,
the total surplus from a match is the sum of the worker’s and firm’s surpluses. The Nash wage bargaining rule for a match is
\[ s_{W,t} = 1 - \eta(W_{t} - W_{t+1}). \]

Substituting Eqs. (18) and (19) into this last equation produces the agreed wage:
\[ w_{t} = \gamma n^{2} / \lambda_{t} + \zeta_{g,b,t} - \beta(1 - \gamma_{t+1})E_{t}(\lambda_{t+1}/\lambda_{t})(1 - \xi_{t+1})\zeta_{g,b,t+1}. \tag{20} \]
where \( \zeta = \eta/(1 - \eta) \) is the relative bargaining power of the worker. Eq. (20) shows that the wage equals the marginal rate of substitution between consumption and leisure (first term on the RHS) plus current hiring costs (second term on the RHS), minus the expected savings in terms of the future hiring costs if the match continues in period \( t + 1 \) (third term on the RHS). Eq. (20) is the standard wage equation with Nash bargaining.

2.3. Aggregate constraint and model solution

The aggregation of the firm’s real cash flow payments (4) and the household’s budget constraint (14) produces the aggregate resource constraint:
\[ \sum_{j} at_{j} = \sum_{j} bt_{j} + \zeta_{g,b,t} - \beta(1 - \gamma_{t+1})E_{t}(\lambda_{t+1}/\lambda_{t})(1 - \xi_{t+1})\zeta_{g,b,t+1}. \]

In order to produce a quantitative assessment of the system we need to parameterize the production technology, the adjustment costs function and the exogenous disturbances. To parameterize the production technology, we use the standard Cobb–Douglas function:
\[ y_{t} = a_{t}k_{t}^{\alpha}n_{t}^{\eta}, \tag{22} \]
where \( 0 < \alpha < 1 \) represents the labor share of production. For the adjustment costs, as in Merz and Yashiv (2007), we use the convex function:
\[ g(i_{t}, k_{t}, h_{t}, n_{t}) = \left[ f_{1}(i_{t}/k_{t}) + f_{2}(h_{t}/n_{t}) + f_{3}(i_{t}/n_{t}) \right]^{\gamma}, \tag{23} \]
where parameters \( f_{1}, f_{2}, \sigma_{1}, \sigma_{2}, \sigma_{3}, \sigma, \sigma \) express scale, and \( \sigma_{1}, \sigma_{2}, \sigma_{3}, \eta \), express the elasticity of adjustment costs with respect to the different arguments. Eq. (23) expresses the idea that the disruption in the production process increases with the size of the factor adjustment relative to the size of production, and adjustment costs increase in the investment and hiring rates. Importantly, the sign of the interaction term, \( \sigma \), determines the complementarity between investment and hiring. A positive value induces an increase in the net value of capital (labor), which triggers an increase in the hiring (investment) rate, whereas the effect is the opposite for a negative estimate. As detailed below, this term is important for the model’s dynamics. It is worth noting that Eq. (23) encompasses a wide range of convex adjustment costs functions.

The processes for \( a_{t}, \chi_{t}, \tau_{t}, \delta_{t}, \psi_{t} \) evolve according to
\[ \begin{align*}
\ln(a_{t}) &= (1 - \rho_{a})\ln(a_{t}) + \rho_{a}\ln(a_{t-1}) + \varepsilon_{at}, \tag{24} \\
\ln(\chi_{t}) &= (1 - \rho_{x})\ln(\chi_{t}) + \rho_{x}\ln(\chi_{t-1}) + \varepsilon_{xt}, \tag{25} \\
\ln(\tau_{t}) &= (1 - \rho_{\tau})\ln(\tau_{t}) + \rho_{\tau}\ln(\tau_{t-1}) + \varepsilon_{\tau t}, \tag{26} \\
\ln(\delta_{t}) &= (1 - \rho_{\delta})\ln(\delta_{t}) + \rho_{\delta}\ln(\delta_{t-1}) + \varepsilon_{\delta t}, \tag{27} \\
\ln(\psi_{t}) &= (1 - \rho_{\psi})\ln(\psi_{t}) + \rho_{\psi}\ln(\psi_{t-1}) + \varepsilon_{\psi t}, \tag{28}
\end{align*} \]
where \( a, \alpha, \tau, \delta, \psi \) are the steady-state levels of technology, disutility of labor, the corporate tax rate, the capital depreciation rate and the separation rate, respectively, with \( 0 < \rho_{\alpha}, \rho_{x}, \rho_{\tau}, \rho_{\delta}, \rho_{\psi} < 1 \), and where the zero-mean, serially uncorrelated innovations \( \varepsilon_{at}, \varepsilon_{xt}, \varepsilon_{\tau t}, \varepsilon_{\delta t}, \varepsilon_{\psi t} \) are normally distributed with standard deviation \( \sigma_{a}, \sigma_{x}, \sigma_{\tau}, \sigma_{\delta}, \sigma_{\psi}, \) respectively.

Hence, Eqs. (1)–(10), (15)–(17), (20), (21) describe the behavior of the 20 endogenous variables \( \{ y_{t}, c_{t}, k_{t}, i_{t}, h_{t}, x_{t}, u_{t}, w_{t}, s_{t}, d_{t}, \pi_{t}, q_{t}, q_{t}, \lambda_{t}, a_{t}, \tau_{t}, \psi_{t}, \chi_{t} \} \). The equilibrium conditions do not have an analytical solution. Consequently, the system is approximated by loglinearizing its equations around the stationary steady state. In this way, a linear dynamic system describes the path of the endogenous variables’ relative deviations from their steady-state value, accounting for the exogenous disturbances. The solution to this system is derived using Klein (2000).

3. Econometric methodology, data and prior distributions

In this section we first present the econometric methodology and then we describe the data and the prior distributions for the Bayesian analysis.

We estimate the model using Bayesian methods. To describe the estimation procedure, define \( \Theta \) as the parameter space of the DSGE model, and \( Z'_{t} = [z_{t}]_{t=1}^{T} \) as the data observed. According to Bayes’ Theorem the posterior distribution of the
parameter is of the form \( P(\Theta | Z_i) \propto P(Z_i | \Theta) P(\Theta) \). This method updates the a priori distribution using the likelihood contained in the data to obtain the conditional posterior distribution of the structural parameters. In order to approximate the posterior distribution, we employ the random walk Metropolis–Hastings algorithm. We use 600,000 replications and discard the first 100,000 as burn-in. We save every 25th remaining draw. The sequence of retained draws is stable, providing evidence on convergence.\(^2\) The posterior density \( P(\Theta | Z_i) \) is used to draw statistical inference on the parameter space \( \Theta \). An and Schorfheide (2007) provides a detailed description of Bayesian simulation techniques applied to the DSGE models.

The econometric estimation uses US quarterly data for the period 1976:1–2002:4. We use data for output, \( y \), gross investment rate, \( i/k \), gross hiring rate, \( h/n \), the labor share of income, \( wn/y \), the gross depreciation rate of capital, \( \delta \), and the firm’s market value, \( s \). The series are from the NIPA data, except those on gross worker flows and the firm’s market value, which are from the BLS data and Hall (2001) respectively. We demean the stationary series for \( i/k, h/n, wn/y \) while we detrend the non-stationary series for \( y, \delta \) and \( s \) using a HP filter with a smoothing parameter of 1600 prior to the estimation.\(^3\) A detailed description of the data sources and construction is in the appendix. The solution to the model takes the form of a state-space representation that involves an observation equation and a state equation. In order to implement the estimation, we assume that the firm’s market value in the observation equation is enriched with a measurement error, \( \eta_s \), which is normally distributed with standard deviation \( \sigma_{\eta_s} \). As detailed below, such an assumption enables us to assess the ability of alternative adjustment costs functions to fit the data on the firm’s market value by comparing the two-sided filtered estimate of the series from the competing models against the observed data series.

Our empirical strategy consists in estimating the 28 parameters in the model that are related to the preferences, technology, adjustment costs function, exogenous disturbances and the measurement error \( \{\alpha, \beta, \phi, \eta, f_1, f_2, e_1, e_2, e_3, \eta_1, \eta_2, \eta_3, a, \chi, \delta, \psi, \tau, \rho_{\alpha}, \rho_{\delta}, \rho_{\psi}, \rho_{\tau}, \sigma_{\alpha}, \sigma_{\delta}, \sigma_{\psi}, \sigma_{\tau}, \sigma_{\eta_1}, \sigma_{\eta_2}, \sigma_{\eta_3}\} \). Table 1 provides a summary of the parameters’ names. Tables 2 and 3 report the prior distributional forms, means, standard deviations and 95% confidence intervals, for the complete set of parameters. Naturally, each constrained model uses a subset of these priors. We choose priors for these parameters based on several considerations. Table 1 reports the prior distributions of the structural parameters \( \{\alpha, \beta, \phi, \eta, f_1, f_2, e_1, e_2, e_3, \eta_1, \eta_2, \eta_3\} \). The priors for the parameters \( \alpha, \beta, \phi \) and \( \eta \) are relatively tight in order to match important stylized facts in the data. In particular, the production labor share, \( \alpha \), is normally distributed with a prior mean equal to 0.66, a value commonly used in the literature and a standard error of 0.05. Similarly, the discount factor, \( \beta \), is normally distributed with a prior mean equal to 0.99 that generates an annual real interest rate of 4%, as in the data, and a standard error equal to 0.001. The inverse of the Frisch intertemporal elasticity of substitution in labor supply, \( \phi \), is normally distributed with a prior mean equal to 1, which is in line with micro- and macro-evidence as detailed in Card (1994) and King and Rebelo (1999), and a standard error equal to 0.01. The steady-state wage bargaining parameter, \( \eta \), is normally distributed with prior mean equal to 0.5, as standard in the search and matching literature, and standard error equal to 0.1. The priors for the parameters of the adjustment costs functions allow for a wide range of values. The linear parameters \( f_1 \) and \( f_2 \) are normally distributed with a prior mean of 0 and a prior standard deviation of 1.5. The priors for the coefficients in front of the convex terms \( e_1, e_2, e_3 \) are assumed to be normally distributed around a mean of 0 with a sizeable standard error of 3. The priors for \( \eta_1, \eta_2, \eta_3 \) are assumed to be gamma distributed with a prior mean of 2 and a standard deviation of 1.

Table 3 reports the prior distributions of the shock parameters \( \{a, \chi, \delta, \psi, \tau, \rho_{\alpha}, \rho_{\delta}, \rho_{\psi}, \rho_{\tau}, \sigma_{\alpha}, \sigma_{\delta}, \sigma_{\psi}, \sigma_{\tau}, \sigma_{\eta_1}, \sigma_{\eta_2}, \sigma_{\eta_3}\} \). In particular, the steady-state technological progress, \( a \), and the disutility of labor, \( \chi \), are assumed to be normally distributed with prior means conveniently set equal to 1 and standard error equal to 0.01. The steady-state capital destruction rate, \( \delta \), and the job destruction rate, \( \psi \), are assumed to be normally distributed with prior means set to match the NIPA data as described in Merz and Yashiv (2007), and therefore equal to 0.015 and 0.086 respectively, with standard errors equal to 0.005. The steady-state corporate tax rate, \( \tau \), is assumed to be normally distributed with prior mean equal to 0.39, as in the data, and a standard error equal to 0.001. Finally, the priors on the autoregressive components and standard errors of the stochastic processes are harmonized across different shocks. We assume that the persistence parameters \( \rho_{\alpha}, \rho_{\delta}, \rho_{\psi}, \rho_{\tau} \) and \( \rho_{\eta} \) are beta distributed, with a prior mean equal to 0.6 and a prior standard deviation equal to 0.2. The standard errors of the innovations \( \sigma_{\alpha}, \sigma_{\delta}, \sigma_{\psi}, \sigma_{\tau} \) and the measurement error \( \sigma_{\eta} \) follow an inverse-gamma distribution with prior mean 0.08 and a prior standard deviation of 0.1, which corresponds to a rather loose prior.

4. Estimation results

In this section we present the estimation results. We first estimate several adjustment costs functions, assess their empirical fit, and evaluate their plausibility using the general equilibrium model. We use the estimated model to provide some additional insights into the model’s dynamics and compare the simulated series for the firm’s market value from alternative models with their empirical counterparts. Finally, we investigate the dynamics properties of the model by using impulse response functions and forecasting variance decompositions.

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\(^2\) An appendix that details evidence on convergence is available upon request from the authors.

\(^3\) As a robustness check, we have also estimated the model by detrending the series for \( i/k, h/n \) and \( wn/y \) using a HP filter and established that the results hold.
4.1. Prior and posterior statistics

Using the priors we estimate several versions of the model, whose posterior mean estimates and standard errors (in parenthesis) are reported in each column of Tables 4 and 5. The first column shows the adjustment costs function that allows for both linear and convex capital and labor adjustment costs, and that also allows for their joint interaction, as in Eq. (23). The second column shows the adjustment costs function that allows for capital adjustment costs only, assuming labor costs are absent, as typical in the investment literature. The third column shows the adjustment costs function that allows for labor adjustment costs only, assuming capital costs are absent, as typical in the labor demand literature. The fourth column shows the adjustment costs function that allows for quadratic costs only and no interaction between capital and labor adjustment costs, as typical in the convex adjustment costs models. The fifth column shows the adjustment costs function that allows for quadratic costs only and no interaction between capital and labor adjustment costs, as typical in the convex adjustment costs models.
function that allows for quadratic costs only and this also allows for interaction between capital and labor adjustment costs, as typical in convex adjustment costs models. To visually summarize the estimation results, Figs. 1 and 2 plot the prior and posterior distributions for the estimated parameters associated with the unconstrained version of the model, whose mean estimates are reported in the first column of Tables 4 and 5.4

4.2. Model’s fit and posterior estimates

Before looking into the parameters’ estimates we assess the overall fit of the models. In order to establish which theoretical framework fits the data more closely, we use the marginal log-likelihood. The marginal or the integrated log-likelihood represents the posterior distribution is the appropriate density to update econometrician’s prior beliefs over a set of competing models.5 The marginal log-likelihood is approximated using the modified harmonic mean, as detailed in Geweke (1999). Considering that this criterion penalizes overparametrization, the model with the unrestricted adjustment costs function does not necessarily rank better if the extra parameters are not informative in explaining the data. As from the last row of Table 5, the marginal log-likelihood associated with the model that allows for all types of adjustment costs is the highest among the constrained alternatives and equal to 507.07. To econometrically test the extent to which the model with the highest log-likelihood improves the fit of the data over and above the alternative models, we use the posterior odds ratio. Table 6 reports the posterior odds ratios, computed as the difference between the marginal log-likelihood of the model with the unrestricted adjustment costs function and each of the marginal log-likelihoods of the alternative specifications. The posterior odds ratio ranges from $e^{6.96}$ to $e^{4.74}$, which provides very strong evidence in favor of the model that includes both linear and convex cost components, and that also accounts for the joint interaction between capital and labor costs. The rest of the analysis focuses on the unconstrained model, unless otherwise stated.

Tables 4 and 5 display the value of the posterior mean of the structural and shock parameters together with their standard errors in parenthesis. In each table, column 1 reports the model that allows for all types of adjustment costs and the other columns report the alternative models. The posterior mean estimates are remarkably close among models, indicating that parameter estimates are consistently and robustly estimated across the different settings. The posterior means of the taste and technology parameters $\alpha, \beta, \phi$ and $\eta$ equal 0.745, 0.989, 1.012 and 0.656 respectively, which are in line with the estimates in Ireland (2001), Zanetti (2008) and Mumtaz and Zanetti (2013). Similarly, the posterior means of the shock parameters $\alpha, \chi, \delta, \psi$ and $\tau$ are equal to 1.001, 1.005, 0.031, 0.085 and 0.388, respectively, in line with Zanetti (2008) and Merz and Yashiv (2007). The posterior means of the linear parameters $f_1$ and $f_2$ equal 0.211 and 0.271 respectively, showing that the linear components of both labor and capital adjustment costs are small, similar to Bloom (2009). The convex components of the adjustment costs function are more sizeable, as the posterior means of $e_1$ and $e_2$
equal 0.128 and 2.411 respectively. Also, it is interesting to note that the estimation reveals quadratic capital adjustment costs, as the posterior mean of \(\eta_1\) equals 2.597, whereas the degree of convexity of labor adjustment costs component \(\eta_2\) is cubic and equal to 3.116. Interestingly, the posterior mean of the term that allows for the interaction between capital and labor adjustment costs, \(e_3\), is low and equal to 0.073 and the posterior mean of \(\eta_3\) equals 2.018, the latter showing a quadratic degree of convexity. Note that a positive posterior mean of the interaction parameter between capital and labor, \(e_3\), implies that total and marginal costs of investment increase with hiring. As detailed below, this relation is important in establishing the dynamic response of the adjustment costs function to exogenous disturbances to the job and capital destruction rates. The posterior means of the stochastic processes show that shocks have a similar degree of persistence. In addition, the posterior means of the volatilities of the stochastic processes show that shocks to preferences and the job separation rate are more volatile, whereas the volatility of the other shocks is of similar magnitude.

These estimates have important implications for the steady state and the dynamics properties of the model, as we detail below, and they differ from those obtained using a single equation model, as in Merz and Yashiv (2007). In particular, the estimates of the linear terms \(f_1\) and \(f_2\) are closer to zero in our analysis, whereas they are approximately 2 and \(-2\) with very large standard errors in the study mentioned. The estimates of the scale parameters \(e_1\), \(e_2\) and \(e_3\) are also different, as our estimates are close to zero with smaller standard errors compared to Merz and Yashiv (2007) and, importantly for the

### Table 4

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Adjustment cost specification</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1) Capital</td>
</tr>
<tr>
<td>(\alpha)</td>
<td>0.745 (0.0050)</td>
</tr>
<tr>
<td>(\beta)</td>
<td>0.989 (0.0004)</td>
</tr>
<tr>
<td>(\eta)</td>
<td>0.656 (0.0078)</td>
</tr>
<tr>
<td>(\phi)</td>
<td>1.012 (0.0068)</td>
</tr>
<tr>
<td>(f_3)</td>
<td>0.211 (0.0079)</td>
</tr>
<tr>
<td>(e_1)</td>
<td>0.128 (0.0131)</td>
</tr>
<tr>
<td>(e_2)</td>
<td>2.411 (0.0310)</td>
</tr>
<tr>
<td>(e_3)</td>
<td>0.073 (0.0240)</td>
</tr>
<tr>
<td>(\eta_1)</td>
<td>2.597 (0.0106)</td>
</tr>
<tr>
<td>(\eta_2)</td>
<td>3.116 (0.0101)</td>
</tr>
<tr>
<td>(\eta_3)</td>
<td>2.018 (0.0148)</td>
</tr>
</tbody>
</table>

**Notes:** Each entry shows the posterior mean estimate with the standard error in brackets. To approximate the posterior distribution, a random walk Metropolis–Hastings algorithm is used, based on 600,000 replications, whose first 100,000 are discarded as burn-in.

### Table 5

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Adjustment cost specification</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1) Capital</td>
</tr>
<tr>
<td>(a)</td>
<td>1.001 (0.0054)</td>
</tr>
<tr>
<td>(\chi)</td>
<td>1.005 (0.0079)</td>
</tr>
<tr>
<td>(\delta)</td>
<td>0.031 (0.0016)</td>
</tr>
<tr>
<td>(\gamma)</td>
<td>0.057 (0.0020)</td>
</tr>
<tr>
<td>(\tau)</td>
<td>0.388 (0.0080)</td>
</tr>
<tr>
<td>(\rho_a)</td>
<td>0.942 (0.0056)</td>
</tr>
<tr>
<td>(\rho_{\chi})</td>
<td>0.974 (0.0063)</td>
</tr>
<tr>
<td>(\rho_{\delta})</td>
<td>0.959 (0.0064)</td>
</tr>
<tr>
<td>(\rho_{\gamma})</td>
<td>0.934 (0.0070)</td>
</tr>
<tr>
<td>(\rho_{\tau})</td>
<td>0.933 (0.0197)</td>
</tr>
<tr>
<td>(\sigma_{\alpha})</td>
<td>0.074 (0.0083)</td>
</tr>
<tr>
<td>(\sigma_{\beta})</td>
<td>0.106 (0.0049)</td>
</tr>
<tr>
<td>(\sigma_{\eta_1})</td>
<td>0.119 (0.0129)</td>
</tr>
<tr>
<td>(\sigma_{\eta_2})</td>
<td>0.125 (0.0129)</td>
</tr>
<tr>
<td>(\sigma_{\eta_3})</td>
<td>0.078 (0.0118)</td>
</tr>
<tr>
<td>(\sigma_{\phi})</td>
<td>0.066 (0.0137)</td>
</tr>
<tr>
<td>Marginal log-likelihood</td>
<td>507.07</td>
</tr>
</tbody>
</table>

**Notes:** Each entry shows the posterior mean estimate with the standard error in brackets. To approximate the posterior distribution, a random walk Metropolis–Hastings algorithm is used, based on 600,000 replications, whose first 100,000 are discarded as burn-in.
model’s dynamics, as detailed below, the parameter \( e_3 \) is small and positive (equal to 0.073), whereas it is large and negative (equal to \(-103.85\)) in the mentioned study. Finally, we find that the estimates of the elasticity of adjustment costs with respect to capital and labor, \( \eta_1 \) and \( \eta_2 \), show that nonlinearities characterize adjustment costs. In particular, our estimates suggest that \( \eta_1 \) is quadratic, \( \eta_2 \) is cubic and \( \eta_3 \) has a low degree of convexity, whereas Merz and Yashiv (2007) find that \( \eta_1 \) and \( \eta_2 \) are quadratic and \( \eta_3 \) is cubic. Since we condition the estimation of the model on the same dataset as these authors, the differences are due to our system approach, which provides estimation restrictions that link the agent’s decision rules with the coefficients in the asset price equation. Importantly, our estimates are consistent with the rest of the model and generate a steady state consistent with the data. This is immediately apparent if we compare the implied steady-state share of total adjustment costs with respect to output, \( g = y \). From Eq. (23), we can easily derive \( g/y = \delta \psi + e_1/\eta_1 \psi + e_2/\eta_2 \psi \), since \( \delta = \psi \) from Eqs. (5) and (6) respectively. If we calibrate the parameters of this equation with our estimated values, \( g/y \) equals approximately 3.3%, whereas it equals approximately \(-14\%) if calibrated with the estimates in Merz and Yashiv (2007). This shows that accounting for general equilibrium effects in the estimation has two important advantages: first, it improves the accuracy of the estimates and, second, it delivers a steady state consistent with the long-run properties of the data.

We now evaluate the plausibility of these adjustment costs estimates exploiting the long-run properties of the general equilibrium model. The steady state of the model’s variables depends on the preferences and technologies as well as the parameters’ estimates of the adjustment costs function. As documented, these estimates generate total adjustment costs of approximately 3.3% of total output per quarter \( g/y \) in the model, which is within the range of estimates between 0.5% and 6% based on disaggregate data reported in Shapiro (1986) and Gilchrist and Himmelberg (1995), and in line with the estimates in Bloom (2009). It is also interesting to gauge the plausibility of the marginal cost of hiring in terms of average output per worker \( (gh)/(y/n) \). This value is equal to 0.33% in the model, which is equivalent to approximately 15% of the quarterly wage, implying that the firm pays about 2 weeks of wages to hire a marginal worker. This is in line with Shimer (2005), who finds that it is reasonable to assume that the firm needs to employ a worker for about 1.4 weeks of wages to recoup hiring costs. Similarly, Hagedorn and Manovskii (2008) decompose hiring costs into, first, the capital flow cost of posting a vacancy, estimated equal to 3.7% of quarterly wages and, second, the labor cost of hiring one worker, estimated between 3% and 4.5% of quarterly wages. Together these components imply around 1.1–1.3 weeks of wages. Hence, overall our estimate is close to those of these existing studies. The marginal cost of investing in terms of average output per unit of capital \( (gi)/(y/k) \) is equal to 0.21. Such a value is within the range of estimates in the literature that vary between 0.04 and

![Fig. 1. Prior and posterior distribution of structural parameters. Each panel shows the prior (grey line) and posterior (black line) distributions of one of the model’s parameter.](image-url)
0.98, as reported in Gilchrist and Himmelberg (1995) and Cooper and Haltiwanger (2006). Hence, overall our parameters' estimates of the adjustment costs function generate plausible adjustment costs, whose magnitude is in line with estimates based on disaggregate data.

The advantage of conducting the analysis with a structural model is that we can use the model to recover estimates of the individual shocks using a Kalman smoothing algorithm, which relies on information contained in the full sample of data. By feeding the recovered structural shocks into the theoretical model we are able to generate estimated time series for the model's variables, which we use to provide some additional insights on the model's dynamics and evaluate the model's performance to replicate the firm's market value in the data. One key finding in Merz and Yashiv (2007) is that factor adjustment costs enable a partial equilibrium model estimated on aggregated data to closely replicate movements in the firms' market value. Would this result hold in a general equilibrium framework? Fig. 3 shows the firms' market value from the data (dark line) against the equivalent series from the theoretical model (dashed-gray line) and the 68% confidence band (shaded area). The series generated by the model are the one-period-ahead forecast obtained from the Kalman filter, computed using the posterior mean reported in column 1 of Tables 4 and 5. The confidence bands are derived from the posterior distribution of parameters. The figure provides a visual diagnostic on the in-sample fit of each model. It clearly emerges that the model with the adjustment costs function that

Fig. 2. Prior and posterior distribution of shock parameters. Each panel shows the prior (grey line) and posterior (black line) distributions of one of the model's parameter.

Table 6
Posterior odds ratios.

<table>
<thead>
<tr>
<th>Adjustment cost specification</th>
<th>Posterior odds ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capital</td>
<td>e^{86.96}</td>
</tr>
<tr>
<td>Labor</td>
<td>e^{36.10}</td>
</tr>
<tr>
<td>Quadratic, no Interaction</td>
<td>e^{45.35}</td>
</tr>
<tr>
<td>Quadratic, Interaction</td>
<td>e^{24.74}</td>
</tr>
</tbody>
</table>

Notes: Each entry reports the posterior odds ratios, computed as the difference between the marginal log-likelihood of the model that allows for the broader set of parameters and each of the marginal log-likelihoods of the alternative specifications.
allows for both linear and convex capital and labor adjustment costs, and that also allows for their joint interaction, is able to replicate the firms’ market value in the data more accurately than models with alternative specifications of adjustment costs function. This finding corroborate the results from the marginal-likelihood analysis that also establish that the model with convex adjustment costs which allows for the interaction between capital and labor costs outperforms alternative specifications. More generally, the analysis shows that factor adjustment costs improve the performance of a prototype general equilibrium model to replicate the firm’s market value in the data.

4.3. Impulse response functions and variance decomposition

To investigate how adjustment costs and other key variables of the model react to each exogenous disturbance, Fig. 4 plots the impulse responses of selected variables to one standard deviation of each of the shocks. The solid line reports the mean responses and the dashed lines report the 2.5 and 97.5 percentiles of the responses. A few interesting results stand out. First, for shocks to technology, disutility of labor and tax rate, the reaction of the total adjustment costs, $g$, is driven by movements in output, which affects the overall costs of adjusting capital and labor by changing the size of production. For instance, in reaction to the technology shock, output rises, expanding production, thereby increasing the total costs of investing and hiring, whereas the effect is the opposite for shocks to the preference in the disutility of labor and the tax rate. Second, for shocks to capital depreciation and job separation rates, the reaction of the total adjustment costs is driven by movements in gross investment and hiring rates, $i/k$ and $h/n$ respectively. For instance, in reaction to an increase in the capital destruction rate the gross investment rate rises, pushing total adjustment costs upwards, despite the fall in the size of production. Third, across all shocks the reaction of the marginal costs of investing and hiring, $g_i$ and $g_h$, determine the response of the asset prices for capital and labor, $q_k$ and $q_n$, as from Eqs. (9) and (10). Moreover, movements in the firm’s market value, $s$, mirror the dynamics of the asset prices for capital and labor. For instance, in reaction to the technology shock both $g_i$ and $g_h$ increase, thereby triggering similar movements in the asset values of capital and labor, which in turn increase the firm’s market value.

To understand the extent to which each shock explains movements in the variables, Table 7 reports the asymptotic forecast error variance decompositions. The entries show that technology shocks explain the bulk of short-run movements
in output and in the firm’s market value, while they compete with preference and job separation rate shocks to explain fluctuation in the marginal cost of investing. Shocks to the preference and the job separation rate explain a sizeable fraction of short-run fluctuations in the total adjustment costs and the marginal cost of hiring. In the long run, technology shocks continue to play a prime role on output and the firm’s market value, and they explain a sizeable portion of the marginal cost of hiring and investing. Shocks to the job separation rate and preferences compete with technology shocks to explain movements in total adjustment costs and contribute to fluctuations in the marginal cost of investing, whereas shocks to technology explain together with taxation shocks the bulk of the fluctuations in the marginal cost of hiring.

5. Conclusion

This paper has studied factor adjustment costs functions estimating a general equilibrium model using Bayesian methods on US aggregate data. The theoretical framework is a standard production-based model enriched with labor market frictions and factor adjustment costs. The estimation finds that adjustment costs are convex in both capital and labor costs, and it is important to allow for the joint interaction of capital and labor in the adjustment costs function. We also found that adjustment costs are small, as they represent 3.3% of total output, in line with estimates based on disaggregated data.

Using the fully-defined general equilibrium model we uncovered some interesting results. We identify the effect of exogenous disturbances to technology, labor supply, job separation and capital destruction rates and tax changes. In this respect, we found that factor adjustment costs are pro-cyclical for all shocks, except for shocks to job separation and capital destruction rates. Forecast error variance decompositions show that technology shocks drive output and factor adjustment costs in the short run, whereas shocks to the job separation rate compete with technology shocks to explain factor adjustment costs in the long run. Finally, by simulating the system over the sample period we find that the adjustment costs function that allows for both linear and convex capital and labor adjustment costs, and that also allows for their joint interaction, is able to replicate more closely the fluctuations in the firm’s market value in the data.

As outlined in the article, our system approach presents several advantages over single equation approaches. However, the results have to be qualified with respect to the specific structural model employed. Despite the fact that the underlying theoretical
model is a prototype production-based model, its setting may be potentially misspecified. In particular, to keep the analysis simple we made the standard assumption of period-by-period Nash bargaining over wages, whereas a staggered multiperiod wage contracting may provide a more detailed description of the labor market, as suggested in Gertler and Trigari (2009) and Faccini et al. (2013). Also, Beaudry and Portier (2006) show that future expectations of technological changes are key drivers of the firm’s market value, whereas Hall (2001) and Hall (2004) suggest that a large stock of intangibles may explain fluctuations in the firm’s market value. To enrich the model with these additional features and evaluate their interaction with factor adjustment costs and the firm’s market value is certainly an interesting task for future research.

Finally, it would also be interesting to enrich the model with nominal price rigidities, such as to include nominal variables in the analysis, and investigate the interaction between nominal and real adjustment costs on a broader set of macroeconomic aggregates. This extension also remains an outstanding task for future research.

Acknowledgments

The authors thank James Bullard, an Associate Editor and two anonymous referees for extremely helpful suggestions and Emelie Jensert and Lydia Silver for very useful comments.

Appendix A. Data sources

The time series used to construct the five observable variables in the estimation are

1. Real gross domestic product, y: NIPA accounts, Table 1.1.6, line 1
2. Labor share of income, wn/y: NIPA Table 1.16, lines 19 and 24
3. Employment, n: CPS data, computed as employment level in non-agricultural industries (mnemonics LNS12032187) less government workers (LNS12032188), less self-employed workers (LNS12032192), less unpaid family workers (LNS12032193).


5. Investment, i: BEA and Fed Flow of Funds data.


7. Hiring, h: based on BLS data, adjusted as explained in Bleakley et al. (1999).

8. Separation rate, ψ: based on BLS data, adjusted as explained in Bleakley et al. (1999).


References


