Newton summarized, when apples have caused damage to heads

All prospective physicists have learned the contents of this short note in school. As time has gone by and some of us have had apples fall on their heads, we may have forgotten crucial details about Newton's laws of motion. This short note may be found useful to remind us of the basics when dealing with simple mechanical systems. We have certainly found it useful when dealing with predictions of microscope stage travel times or robot arm action times!

Newton's First law:

The velocity of a body remains constant unless the body is acted upon by an external force.

Newton's Second law:

The acceleration a of a body is parallel and directly proportional to the net force F and inversely proportional to the mass m, i.e., $F = ma$.

Newton's Third law:

The mutual forces of action and reaction between two bodies are equal, opposite and colinear.

When dealing with steady state or accelerated motions of bodies, a very few simple equations will suffice to derive positions, velocities and accelerations, as shown below:

Steady state motion

To find:	As a function of:	Use:
r (position)	*v* (velocity) and t (time)	$r = v.t$
v (velocity)	*r* (position) and t (time)	$v = r/t$
t (time)	*r* (position) and v (velocity)	$t = r/v$

Accelerated motion

To find:	As a function of:	Use:
r (position)	*a* (acceleration) and *t* (time)	$r = a.t^2 / 2$
r (position)	*v* (velocity) and *a* (acceleration)	$r = v^2 / 2a$
v (velocity)	*a* (acceleration) and *t* (time)	$v = a.t$
v (velocity)	*r* (position) and *a* (acceleration)	$v = (2.a.r)^{1/2}$
a (acceleration)	*v* (velocity) and *t* (time)	$a = v / t$
a (acceleration)	*r* (position) and *t* (time)	$a = 2.r / t^2$
a (acceleration)	*v* (velocity) and *r* (position)	$a = v^2 / 2.r$
t (time)	*v* (velocity) and *a* (acceleration)	$t = v / a$
t (time)	*r* (position) and *a* (acceleration)	$t = (2.r / a)^{1/2}$

When the acceleration is time dependent, then the situation becomes slightly more involved. The relationships between position, velocity and acceleration can be expressed in terms of derivatives or integrals.

	Derivative form	Integral form
Position	$r(t)$	$r(t) = r_o + \int_0^t vdt'$
Velocity	$v(t) = dr/dt$	$v(t) = v_o + \int_0^t adt'$
Acceleration	$a(t) = dv/dt = d^2r/dt^2$	$a(t)$

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