

## Lecture 4: General Covariance and gauge theories

### 4.1 What's special about GTR?

Since the theory's inception, the general covariance of GTR has often been taken to be one of its substantial, defining characteristics. But equally, since Kretschmann's (1917) response to Einstein's original claims for general covariance, many see the requirement that a theory should be formulated generally covariantly as physically empty: (virtually?) any theory can be so formulated. Must one of these two points of view be rejected, or can they be reconciled?

General relativity is distinguished from other dynamical field theories by its invariance under *active* diffeomorphisms. Any theory can be made invariant under *passive* diffeomorphisms. Passive diffeomorphism invariance is a property of the *formulation* of a dynamical theory, while active diffeomorphism invariance is a property of the dynamical theory *itself*. (Gaul & Rovelli 2000)

One task for this lecture is to understand and assess this claim.

### 4.2 Pre-GR theories and the Hole Argument

#### 4.2.1 Local spacetime theories

Earman & Norton (1987) state a preference for *local spacetime theories*; i.e. formulations of theories

1. in terms of models  $\langle M, O_1, \dots, O_n \rangle$
2. satisfying field equations  $O_k = 0, \dots, O_n = 0$
3. such that every such  $(n + 1)$ -tuple satisfying (2) is a model.

Examples include GTR and "special relativistic electromagnetics".

Note that, although they advocate identifying  $M$  with spacetime, rather than  $M$  plus additional structure, they argue that "manifold-plus-absolute-structure" substantialists must deny the physical equivalence of models related by hole diffeomorphisms.

#### 4.2.2 Stachel's criticisms

Stachel intends to offer an interpretation of the hole argument that leads to a concept of general covariance which, like the hole argument itself, is supposed not to be applicable to pregeneral-relativistic theories.

**chronogeometric structures** "characterize the behavior of (ideal) clocks and measuring rods" (Stachel 1993, 134)

**affine structures** "characterize the behavior of freely falling ... test particles" (Stachel 1993, 134)

**dynamical structures** characterize “the behavior of physical fields and/or particles in spacetime. . . usually specified by requiring that the dynamical variables be subject to a set of differential equations” (Stachel 1993, 135)

In any physical theory that is either globally special-relativistic, like special relativity, or locally special-relativistic, like general relativity, there is a certain ten-component tensor field encoding all information about the chronogeometrical and affine structures of spacetime. It is called the metric tensor field, or “the metric” for short. In general relativity, the affine structure of spacetime is identical with the inertio-gravitational field. Consequently the metric tensor plays a dual role in general relativity: It engenders not only all the spacetime structures, but also all the gravitational structures of the theory. It is worthwhile to emphasize this point since it is crucial to the hole story: Spacetime and gravitational field in general relativity are like love and marriage in the old song. In its dual role as source of all gravitational structures, the metric is a dynamical structure, obeying certain gravitational field equations. In its chronogeometric role, this implies that all the spacetime structures of the general theory are also dynamical structures. (Stachel 1993, 136)

**individuating field** *any* structure on a differentiable manifold that individuates the points of the manifold by some property or properties that characterize(s) each of the points (*cf.* Stachel 1993, 139; N.B. the issue of symmetries is dodged on p. 143)

. . . we can now identify the missing element in the resolution of the hole argument . . . To justify the identification of a whole equivalence class of drag-along metrics with *one* gravitational field, we must stipulate that, in general relativity, there is no structure on the differentiable manifold that is *both* independent of the metric tensor *and* able to serve as an individuating field (Stachel 1993, 140)

[In 1905, Einstein gave a] careful definition of a physical frame of reference. He defined it in terms of a network of measuring rods and a set of suitable-synchronized clocks, all at rest in an inertial system. He used such a system of rods and clocks to give physical meaning to a preferred coordinate system associated with the inertial frame of reference. In my language (admittedly anachronistic), the rods and clocks serve to establish a nondynamical individuating field for the points of Minkowski spacetime. (Stachel 1993, 141)

**The Principle of General Covariance** (1) There are no individuating fields in spacetime that are independent of the metric tensor field; (2) the metric field determines both chronogeometrical and inertio-gravitational structures of spacetime; and (3) The metric field obeys a set of generally covariant field equations. (Stachel 1993, 142)

A major weakness of [Earman and Norton's] discussion is that Earman and Norton do not consider the role of individuating field. . . When individuating fields are introduced, previously physically indistinguishable models often become distinct. For example, on Earman and Norton's definition of a "special relativistic electromagnetic" model, there is only one generic model of the Coulomb field generated by a given charge. The field of such a charge at *one* place and the field of a similar charge at *another* place are indistinguishable, as are the fields of the charge at rest and a similar one in uniform motion. The distinctions between such fields cannot be formulated in their definition of a model, which certainly makes life in the laboratory rather difficult! (1993, 146–7)

The symmetry group of a differentiable manifold is the group of diffeomorphisms of that manifold; so only a theory that has the diffeomorphism group as the symmetry group of its spacetime structures requires a differentiable manifold for its most appropriate. . . models. As we have seen, Newtonian and special-relativistic theories have much smaller space-time symmetry groups, so manifold-plus models of these theories introduce "empty or superfluous" elements into these models. (Stachel 1993, 150)

The reason why Earman and Norton's concept of general covariance is trivial should be clear. . . By considering the drag along of *all* physical fields over a bare manifold, we have passed beyond the real physics. (Stachel 1993, 153)

- Stachel's own examples of individuating fields in pre-GTR theories (e.g. Einstein's inertially moving network of rods and clocks in STR) are *neither* nondynamical *nor* independent of the spacetime structure fields.
- In the GTR hole construction one considers the drag along of "*all* physical fields over a bare manifold": why isn't the general covariance of GTR just as trivial?
- Earman and Norton are not committed to there being only one generic model of the Coulomb field generated by a given charge in any worrying sense if the models are understood as containing solutions of the dynamical fields representing, e.g., laboratories.
- Even pre-GTR theories satisfy Stachel's Principle of General Covariance (although the generally covariant equations that the spacetime structure fields satisfy (that do not couple these fields to the dynamical fields) are rather uninteresting, e.g., 'curvature = 0').

### 4.3 Background independence

Active diff invariance should not be confused with passive diff invariance, or invariance under change of coordinates. . . . A field theory is formulated in manner invariant under passive diffs (or change of coordinates), if we

can change the coordinates of the manifold, re-express all the geometric quantities (dynamical *and non-dynamical*) in the new coordinates, and the form of the equations of motion does not change. A theory is invariant under active diffeomorphisms, when a smooth displacement of the dynamical fields (*the dynamical fields alone*) over the manifold, sends solutions of the equations of motion into solutions of the equations of motion. Distinguishing a truly dynamical field, namely a field with independent degrees of freedom, from a nondynamical field disguised as dynamical (such as a metric field  $g$  with the equations of motion  $\text{Riemann}[g] = 0$ ) might require a detailed analysis (for instance, Hamiltonian) of the theory. (Rovelli 2001, original emphasis)

- Ban absolute objects
- When made generally covariant, the action should contain no fields that are functions of the independent variables which are not themselves varied. (But do these two notions always coincide? See Sorkin 2002.)

#### 4.4 Getting a fix on gauge

In a series of recent papers (Earman 2003, 2002a,d,b), John Earman has suggested that GTR satisfies a strong version of general covariance—that the space-time diffeomorphism group be a gauge group of the theory—that familiar pre-GTR theories fail to satisfy.

##### 4.4.1 The Lagrangian formalism and Noether's 2nd theorem

If the action principle is (quasi-)invariant under a group of transformations  $\mathcal{G}_{\infty r}$  that depend on  $r$  arbitrary functions of all the dependent variables, then Noether's 2nd theorem tells us that there are  $r$  linearly independent mathematical identities constructed from the Euler-Lagrange expressions and their derivatives. The Euler-Lagrange equations are not all independent. There are more unknowns than independent equations.

One standardly treats the elements of  $\mathcal{G}_{\infty r}$  as *gauge transformations*, i.e. transformations that map solutions of the theory onto mathematically distinct solutions that represent the exactly same physical states/histories.

##### 4.4.2 The constrained Hamiltonian formalism

Gauge transformations are those generated by the first class constraints (those constraints whose Poisson bracket with any constraint vanishes on the constraint surface). (See Belot & Earman 2000, 2001)

Note that when time evolution is pure gauge, gauge invariant quantities do not change in time. The dynamics is “frozen” (see Earman 2002c).

##### 4.4.3 Absolute objects

A group of spacetime transformations are gauge if they contain arbitrary functions of the spacetime variables and leave invariant all of the absolute elements of the spacetime structure.

[A theory's satisfying weak general covariance] does not guarantee that the theory satisfies the strong requirement; indeed, as judged by the light of the constrained Hamiltonian formalism the gauge freedom of a weakly generally covariant theory may fall far short of diffeomorphism invariance, and as a result the observables of the theory may be much richer than the class of diffeomorphism invariants. ([Earman 2002c](#), 15–6)

Suppose the genuine observables of GTR are diffeomorphic-invariant quantities. Is the set of observables of any familiar pre-GTR theories really much richer than the class of diffeomorphism invariants?

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