

# AAHK Mini-Course Maths, Comp Sci

## Sheet 1 — Summer 2022

### Statements and Proofs

Please submit your answers to the Google Classroom by **19 Aug 17:00 HKT**. Please attempt every question. Even if you do not know the answer, please write down your ideas (what you tried to do, why you think your method didn't work etc). You should also indicate your preferred time slots. Feel free to include questions about the problems or the course if you have any.

**Please fill in the google excel sheet for a tutorial timeslot.** You would not be invited to a tutorial otherwise.

1. (Oxford Problem Sheet) Each card in a pack has a number on one side and a letter on the other. Four cards are placed on the table.



You are permitted to turn just two cards to test the following hypothesis: **a card that has an even number on one side has a vowel on the other**. Which two cards should you turn? Or is it impossible? Justify your answer.

2. Using truth tables
  - (a) Prove that  $\neg(A \implies B)$  is logically equivalent to  $A \wedge \neg B$
  - (b) Prove that  $(C \wedge D) \vee (C \wedge \neg D)$  is logically equivalent to  $C$ <sup>1</sup>
  - (c) Prove that  $((\neg A \implies B) \wedge (\neg A \implies \neg B)) \implies A$  is a tautology
3. Using any method, show that  $(P \wedge Q) \implies R$  and  $P \implies (Q \implies R)$  are equivalent.

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<sup>1</sup>This is a way of explaining why we can prove something by splitting it into cases

4. (What is the name of this book? - The riddle of Darcula and other logical puzzles)

**There are only two people, A and B, each of whom is either a knight or a knave.<sup>2</sup> Knights always tell the truth. Knaves always lie.**

**A makes the following statement: “At least one of us is a knave.” What are A and B?**

(Feel free to reason this in your head / do it your own way! But please try to do the parts below as well so that you’re using what we’ve learnt in the lecture!)

Let P be the statement letter indicating that A is a knight. Let Q be the statement letter indicating that B is a knight. Draw a truth table.

For each of the following statement forms, express it using P, Q and connectives. Then draw a column representing its truth values in the truth table.

- (a) “At least one of A and B is a knave”
- (b) “A is a knight and A says “At least one of us is a knave””
- (c) “A is a knave and A says “At least one of us is a knave””
- (d) “A says “At least one of us is a knave””

Can you solve the problem now?

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<sup>2</sup>We mean the exclusive or here, not the inclusive or that we’ve covered in the lectures. I.e. you cannot be a knight and a knave at the same time.