

Functions of a complex variable (S1)

Answers for Problem Sheet 4

1. (a) $2\pi/(3\sqrt{3})$ (b) $(\pi/n)/\sin(\pi/n)$
2. (a) π (b) π
3. (a) $-i\pi/3$ (b) $7\pi i/180$ (c) 0 (d) $2\pi i/3$
4. (a) $J = 0$; $P = 1$, $N = 1$ (b) $J = -2$; $P = 2$, $N = 0$
5. (a) 3 (b) 6
6. 1
7. (a) $\pi^2/12$ (b) $\pi^3/32$
8. (a) $\pi/\sqrt{2}$ (b) $2\pi/\sqrt{3}$
9. (a) $-\pi/4$ (b) $\pi \ln 2$
10. 1 for $t > 0$; 0 for $t < 0$
11. (a) $\tilde{f}(\omega) = 2 \sin \omega / \omega$ (b) $\tilde{f}(\omega) = \sin^2(\omega/2) / (\omega/2)^2$
12. (a) $F(z) = 1/(1+z^2)$ (b) $F(z) = 2z/(1+z^2)^2$ (c) $F(z) = z/(z^2-1)$
13. $1/\sqrt{\pi t}$
14. (a) $\partial \tilde{u} / \partial t = -\lambda p^2 \tilde{u}$, $\tilde{u}(p, 0) = \tilde{h}(p)$
 (b) $\tilde{u}(p, t) = \tilde{h}(p) e^{-\lambda p^2 t}$
 (c) $u(x, t) = (1/\sqrt{4\pi\lambda t}) \int_{-\infty}^{\infty} dy e^{-(x-y)^2/(4\lambda t)} h(y)$
15. (a) $\tilde{y} + \tilde{y}/z^2 = 2/(z^2+4)$; $\tilde{y}(z) = 2z^2/[(z^2+1)(z^2+4)]$
 (b) $y(t) = (4/3) \sin 2t - (2/3) \sin t$