Hadrons

Patterns are often indications of underlying internal structure – think of the periodic table of the elements, which has groups of similar elements due to their internal electronic structure.

There are also patterns in the masses and spins of the hadrons (the mesons and baryons) which give hints as to their internal structure. The hadrons form multiplets with similar masses. For example there is a doublet of familiar particles – the neutron and proton – which have rather similar masses ≈ 940 MeV. It is tempting to imagine that they can be thought of as being different sides of the same 'object'.

We have previously come across multiplets of states with similar masses. Remember that if the angular momentum operator commutes with the Hamiltonian of some system $[J_i, H] = 0$ then we end up with $2j+1$ states with the same energy and total angular momentum J but different m_j. Let's see if we can understand the states of the baryons and mesons if we assume there are operators – which we will call **isospin** operators, $\mathbf{I}$. These obey the same commutation relations as the angular momentum operators $\mathbf{J}$ (even though isospin has nothing to do with rotations in real space).

Symmetry	Spin	Isospin
Generators	$J_{1,2,3}$	$I_{1,2,3}$
Commutator algebra	$[J_i, J_j] = \epsilon_{ijk} J_k$	$[I_i, I_j] = \epsilon_{ijk} I_k$
Ladder operators	$J_\pm = J_1 \pm J_2$	$I_\pm = I_1 \pm I_2$

The $\{p, n\}$ nucleon system would look like a spin-$\frac{1}{2}$ system

$$
\begin{array}{ll}
I_3|p\rangle = \frac{1}{2}|p\rangle & I_3|n\rangle = -\frac{1}{2}|n\rangle \\
I_-|p\rangle \propto |n\rangle & I_+|n\rangle \propto |p\rangle \\
I^2|p\rangle = \frac{1}{2}(\frac{1}{2}+1)|p\rangle & I^2|n\rangle = \frac{1}{2}(\frac{1}{2}+1)|n\rangle
\end{array}
$$

Table 1: Effect of the isospin operators on protons and neutrons.

We're going to come back to the proton/neutron system shortly, but first let's think about some other baryons: the Δ resonances. There are four of these, with masses ≈ 1230 MeV and different electrical charges: $(\Delta^{++}, \Delta^{+}, \Delta^{0}, \Delta^{-})$. Can we put these in a multiplet? To form a four-fold degenerate multiplet we need the total isospin quantum number to satisfy $(2I+1) = 4$ meaning $I = 3/2$. The ladder

$I_3\|u\rangle = \frac{1}{2}\|u\rangle$	$I_3\|d\rangle = -\frac{1}{2}\|d\rangle$
$I_-\|u\rangle \propto \|d\rangle$	$I_+\|d\rangle \propto \|u\rangle$
$I^2\|u\rangle = \frac{1}{2}(\frac{1}{2}+1)\|u\rangle$	$I^2\|d\rangle = \frac{1}{2}(\frac{1}{2}+1)\|d\rangle$

Table 2: Effect of the isospin operators on up and down quarks.

operators $I_\pm$ can then be used to construct the different states:

$$I_-|\Delta^{++}\rangle \Rightarrow |\Delta^+\rangle \tag{1}$$

$$I_-|\Delta^{+}\rangle \Rightarrow |\Delta^0\rangle \tag{2}$$

$$I_-|\Delta^{0}\rangle \Rightarrow |\Delta^-\rangle \tag{3}$$

So the $\{\Delta^{++}, \Delta^+, \Delta^0, \Delta^-\}$ multiplet is rather like the $m_j = \{\frac{3}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{3}{2}\}$ multiplet we get for a quantum system with $j = \frac{3}{2}$.

We can build a spin-$\frac{3}{2}$ system from three spin-$\frac{1}{2}$ particles. In the same way, we can build the Δ multiplet from $I = \frac{1}{2}$ constituents. We are going to need three of these for each Δ and we will call them **quarks**. The quarks will come in two types, up $|u\rangle$ and down $|d\rangle$, corresponding to $I_3 = \pm\frac{1}{2}$ respectively (Table).

Let's try to build our Δ resonances out of combinations of up $|u\rangle$ and down $|d\rangle$ quarks, which are the basis states of the $I = \frac{1}{2}$ system. We'll do the construction in the same way we did for angular momentum addition. We build the the different states in the Δ multiplet starting with the $|\Delta^{++}\rangle$ built from three up-type quarks, $|u_a u_b u_d\rangle$ and repeatedly hitting it with the lowering operator I_- to generate three-quark states corresponding to the Δ^0 and the Δ^-.

$$I_-|\Delta^{++}\rangle = (I_{-,a} + I_{-,b} + I_{-,c})|u_a u_b u_c\rangle \propto |d_a u_b u_c\rangle + |u_a d_b u_c\rangle + |u_a u_b d_c\rangle$$

To get the electrical charges correct the charge of the u and the d quark must be $+\frac{2}{3}e$ and $-\frac{1}{3}e$ respectively.

Can we build the proton and the neutron from the same quarks? To conserve electrical charges we should construct a proton from a (uud) combination of quarks and a neutron from a (udd) combination.

We are left with the question as to why we cannot apply the raising operator I_+ to the proton to create a doubly-charged (uuu) nucleon, or the I_- operator to the neutron to create a negatively charged (ddd) nucleon.

1 Quark spin and colour

We also need to think about the spin of the quarks. Quarks are spin-half fermions.

In all of the above, we have built multiplets by assuming that the isospin operators commute with the Hamiltonian. In fact this cannot quite be true, since the multiplets contain states of slightly different mass. We would expect similar (but not identical) masses if any isospin-violating parts of the Hamiltonian could be treated as perturbations. Clearly the electromagnetic part of the Hamiltonian is changed by the isospin ladder operators $I_\pm$ which change $d \rightleftharpoons u$ as shown in Table .

Other particles also come in near-degenerate multiplets: for example the pions (π^+, π^0, π^-) have similar masses ≈ 140 MeV.

Quarks are found to come in three different strong interaction charges called colours. We call these three different charges "colours" because combinatons of the three charges "r","g" and "b" are found to have no net colour charge.

Key concepts

Further Reading