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SECOND PUBLIC EXAMINATION

Honour School of Physics – Part B: 3 and 4 Year Courses

Honour School of Physics and Philosophy Part B

B1: ATOMIC STRUCTURE, SPECIAL RELATIVITY AND SUB-ATOMIC PHYSICS

Thursday, 15 June 2006, 9.30 am – 12.30 pm

Answer **five** *questions with at least* **one** *from each section:*

Start the answer to each question on a fresh page.

Start the answer to each section in a fresh book.

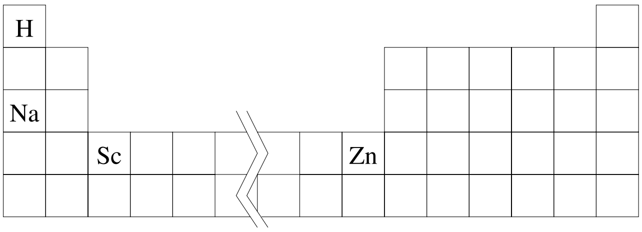
A list of physical constants and conversion factors accompanies this paper.

The numbers in the margin indicate the weight that the Examiners expect to assign to each part of the question.

Do NOT turn over until told that you may do so.

Section A (Atomic Structure)

1. An outline of the periodic table of the elements is shown below. Explain how the structure of the table is connected to the quantum numbers that label the ground configurations of atoms. Give the complete electronic configuration of scandium (Sc). How many elements are there in the series between scandium and zinc (Zn) inclusive? [6]



(a) Sodium (Na) atoms are put into the excited state in which the outer electron is in the 4p configuration. Draw a diagram that shows all the allowed transitions by which these atoms cascade down to the ground state. [7]

(b) The ionisation energy of sodium is 5.14 eV. Following excitation to the 4p configuration, the shortest wavelength radiation observed is $\lambda = 330\,\mathrm{nm}$. The quantum defect of d configurations may be neglected. Find the energies (above the ground state) of the 4p and all lower configurations. [7]

2. For the magnesium atom in a weak magnetic field, state all the interactions and approximations that are important for describing its atomic structure and in what order they should be considered. [6]

When a weak magnetic field is applied to an atom, a spectral line splits into several components (Zeeman effect). Draw a level diagram indicating the allowed transitions for each of the following:

(a) $^1\mathrm{P}_1 \rightarrow {}^1\mathrm{D}_2$ spectral line in magnesium

(b) $^2\mathrm{P}_{3/2} \rightarrow {}^2\mathrm{S}_{1/2}$ spectral line in sodium.

For each of the two spectral lines, find and sketch the relative spacing between the components observed perpendicular to the direction of a weak magnetic field. [10]

Indicate on each of your sketches which components will be observed when viewed parallel to the magnetic field. Explain why. [4]

[The Landé g-factor is

$$g_j = \frac{3}{2} + \frac{s(s+1) - l(l+1)}{2j(j+1)} \, . \qquad]$$

Section B (Special Relativity and Sub-Atomic Physics)

3. Some of the radioactivity in the Earth is accounted for by four distinct decay sequences of heavy elements as shown in the table below.

Name of Series	Mass number	Parent	Half-life (Gyr)	End Product
Thorium	$4n$	$^{232}_{90}\mathrm{Th}$	14	$^{208}_{82}\mathrm{Pb}$
Neptunium	$4n+1$	$^{237}_{93}\mathrm{Np}$	0.0022	$^{209}_{83}\mathrm{Bi}$
Uranium-radium	$4n+2$	$^{238}_{92}\mathrm{U}$	4.5	$^{206}_{82}\mathrm{Pb}$
Uranium-actinium	$4n+3$	$^{235}_{92}\mathrm{U}$	0.72	$^{207}_{82}\mathrm{Pb}$

Explain why the four series are independent of each other. Why is most of the energy that is released converted to heat? Identify any components that do escape. [6]

An unstable element A decays radioactively with mean lifetime τ_{A} into the element B, which subsequently decays with lifetime τ_{B} into the stable element C. Derive an expression for the quantity of element B as a function of time in a sample that initially contains only element A. [5]

Show that when $\tau_{\mathrm{A}} \gg \tau_{\mathrm{B}}$ the ratio $n_{\mathrm{B}}/n_{\mathrm{A}}$ of the numbers of atoms of A and B tends to a steady state at times $t \gg \tau_{\mathrm{B}}$, and give the limiting value of $n_{\mathrm{B}}/n_{\mathrm{A}}$. [4]

The $^{238}\mathrm{U}$ decay chain proceeds through radon gas ($^{222}_{86}\mathrm{Rn}$) which has a half life of 3.8 days, via several isotopes with very short lifetimes, to lead $^{210}_{82}\mathrm{Pb}$ with a half life of 22.2 years then several other isotopes before reaching the end of the chain. 77.3 disintegrations of $^{210}_{82}\mathrm{Pb}$ per second are found to occur per cubic metre of rainwater from Milford Haven in Wales. The annual rainfall at Milford Haven is 1.10 m. Calculate the quantity of radon gas which escapes into the atmosphere in atoms $\mathrm{m}^{-2}\,\mathrm{s}^{-1}$. [5]

4. Nuclei can undergo a change $M(A,Z) \to M(A,Z\pm 1)$. Which interaction is responsible for this change? What are the processes involved for (i) β^- decay, (ii) β^+ decay and (iii) electron capture. Why it is possible for certain even-A nuclei to exhibit double beta decay even though single beta decay is not energetically possible? [6]

The rate for β^+ decay in which the emitted positron has momentum p and energy E is given by integrating the expression

$$P(p)\,\mathrm{d}p = \frac{2\pi}{\hbar} g^2 |M|^2 \frac{p^2}{2\pi^2\hbar^3} \frac{(E_0 - E)^2}{2\pi^2\hbar^3 c^3}\,\mathrm{d}p\ ,$$

where E_0 is the maximum positron energy. Outline the basis for this expression. [5]

Show that when the positron is highly relativistic, the decay rate is proportional to E_0^n, where n is a constant, and find the value of n. [4]

The following table gives various β decays. Show that processes (1) and (2) are consistent with each other and comment on any deviation in their consistency with each of processes (3) and (4). [5]

	Decay	Mean lifetime	E_0	Nuclear spins
(1)	μ^+ lepton decay	2.2×10^{-6} s	105 MeV	
(2)	${}^{20}_{9}\mathrm{F} \to {}^{20}_{10}\mathrm{Ne}^*$	11.00 s	4368 keV	$2^+ \to 2^+$
(3)	${}^{26}_{13}\mathrm{Al} \to {}^{26}_{12}\mathrm{Mg}^*$	7.17×10^5 years	1173 keV	$5^+ \to 2^+$
(4)	τ^+ lepton decay	2.9×10^{-13} s	1777 MeV	

5. The concept of an exchange particle may be used to describe forces between particles. Describe how the properties of the force change depending on whether the exchange particle has mass. [3]

Use a Feynman diagram to show how $e^+e^- \to \mu^+\mu^-$ may proceed through γ exchange. What factors appear in the expression for the cross section of the above reaction from each vertex and from the γ propagator? [3]

(a) At around 10 GeV, the cross section of $\mathrm{e^+e^-}$ collisions goes through three narrow resonances. Account for these peaks and their unusual widths. Calculate the ratio of cross sections R

$$R = \frac{\sigma(\mathrm{e^+e^-} \to \text{hadrons})}{\sigma(\mathrm{e^+e^-} \to \mu^+\mu^-)}$$

at an energy just below these peaks. [6]

(b) As the beam energy is raised to suitably high energies, the reaction $\mathrm{e^+e^-} \to \mu^+\mu^-$ may also proceed through Z-boson exchange. Neglecting any differences in vertex factors, at what beam energy (below M_Z) does the contribution to the cross section from the γ exchange diagram become equal to the contribution from the Z exchange diagram? What contribution comes from interference between the two diagrams at this energy? [6]

(c) For each of the three resonances near 10 GeV, the total width is found to be equal to the sum of the partial widths of the individual measured final states (hadrons, $\mathrm{e^+e^-}$, $\mu^+\mu^-$ and $\tau^+\tau^-$) whereas for the Z-boson resonance, this is not true. Account for this difference. [2]

6. The following list gives some combinations of quarks. State which combinations occur in nature, and in each such case give an example.

$$q,\quad qq,\quad qqq,\quad qqqq,\quad q\overline{q},\quad qq\overline{q},\quad qqq\overline{q},\quad qqqq\overline{q},\quad q\overline{q}\overline{q},\quad \overline{q}\overline{q}\overline{q}$$ [3]

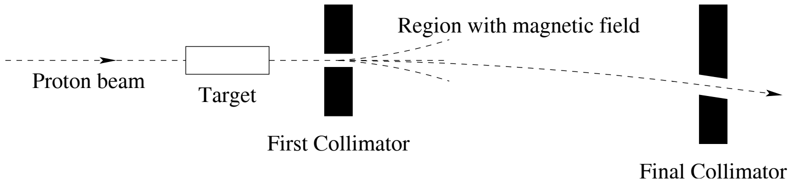
The D^+ meson is the lowest-mass charmed meson with a mass of $1869\,\mathrm{MeV}/c^2$. The $(\mathrm{D}^+)^*$ meson has a mass of $2010\,\mathrm{MeV}/c^2$ and has spin-parity $J^P = 1^-$. Give the spin and parity of the D^+ and explain how the D^+ and $(\mathrm{D}^+)^*$ mesons fit into the quark model. [3]

The D^+ may decay into $\overline{\mathrm{K}}^{*0}\mu^+\nu_\mu$. What is the maximum possible momentum the muon could have in a decay in which the D^+ has total energy of 1885 MeV? [9]

Enumerate the likely decay modes of the $(D^+)^*$ meson and draw a Feynman diagram or quark flow diagram for each. [5]

[The mass of $\overline{\mathrm{K}}^{*0}$ is $892\,\mathrm{MeV}/c^2$.]

7. A beam of protons with momentum $400\,\mathrm{GeV}/c$ hits a carbon target. Two collimators (shields with holes in them) are placed downstream of the target as shown in the diagram, and a uniform magnetic field is present between them. The position of the final collimator is adjusted so that positively charged particles emerge with momentum around $75\,\mathrm{GeV}/c$. By discussing the relevant interactions and conservation laws, explain what types of particles are produced at the target, and why the only particles still in the beam 1 m downstream of the final collimator are Σ^+, p, K^+ and π^+. Why in principle should there be very few muons? [6]



Calculate the length of target required so that 5% of the incoming beam interacts. [5]

The composition of the beam at this point is 30% protons, 5% K^+ and 65% π^+ and a negligible number of Σ^+. What is the composition of the beam 500 m further downstream? [4]

Discuss how Cerenkov radiation could be used to tag the kaons in the beam and have a negligible contamination of mistagged pions or protons. [5]

[The interaction cross section for protons on carbon at $400\,\mathrm{GeV}/c$ is $\sigma = 225\,\mathrm{mb}$. The density of carbon is $\rho = 2265\,\mathrm{kg/m^3}$. The mean lifetimes of K^+ and π^+ are 1.2×10^{-8} s and 2.6×10^{-8} s, respectively.]

8. It is proposed to generate a pure beam of either electron neutrinos or electron antineutrinos by accelerating ions of unstable nuclei to relativistic speeds and then allowing them to decay in a long straight section of the accelerator.

An unstable ion of rest mass M decays after it has been accelerated to total energy E and Lorentz factor $\gamma = E/Mc^2$ and emits a neutrino of energy E_ν at an angle of θ to the beam direction. (i) Derive an expression for the neutrino's energy E_ν^* in the rest frame of the ion in terms of E_ν, θ and the velocity of the ion βc. (ii) Show that in the rest frame of the ion, the neutrino's path is inclined to the beam direction by the angle θ^* that satisfies

$$\cos\theta^* = \frac{\cos\theta - \beta}{1 - \beta\cos\theta} \; .$$

[6]

Ions are accelerated to $\gamma = 100$ and decay in the straight section of the accelerator. A cylindrical detector that is coaxial with the beam and has radius $r = 30\,\mathrm{m}$, is placed $D = 300\,\mathrm{km}$ downstream. Show that the angle between the beam direction and the edge of the detector θ^* as viewed in the rest frame of the ion is approximately given by

$$\cos\theta^* = \frac{1 - \gamma^2\theta^2}{1 + \gamma^2\theta^2 - \theta^2/2}$$

where $\theta \simeq r/D$. Given that the emission of neutrinos is isotropic in the ion rest frame, find the fraction of the neutrinos that pass through the detector. [14]