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SECOND PUBLIC EXAMINATION

Honour School of Physics – Part B: 3 and 4 Year Courses

Honour School of Physics and Philosophy Part B

B1: ATOMIC STRUCTURE, SPECIAL RELATIVITY AND SUB-ATOMIC PHYSICS

Thursday, 18 June 2009, 9.30 am – 12.30 pm

TRINITY 2009

Answer **five** *questions with at least* **one** *from each section:*

Start the answer to each question on a fresh page.

Start the answer to each section in a fresh book.

A list of physical constants and conversion factors accompanies this paper.

The numbers in the margin indicate the weight that the Examiners expect to assign to each part of the question.

Do NOT turn over until told that you may do so.

Section A (Atomic Structure)

1. Explain what is meant by the *Zeeman effect*. Why is it useful in investigations of atomic structure? Derive an expression for the Landé splitting factor g_J assuming the LS-coupling scheme. [6]

The transition 5s6s $^3S_1$ – 5s5p $^3P_1$ in neutral cadmium has wavelength 480 nm. It is observed from a discharge containing the cadmium isotope of mass number 114 at a temperature of 350 K in a weak magnetic field of 0.3 T. Draw a diagram showing the splitting of the two levels, labelled with appropriate quantum numbers. Show on this diagram the allowed electric dipole transitions. On a separate diagram, show the positions of the Zeeman components of the spectral line on a frequency scale (GHz), relative to the position of the line in zero magnetic field. [10]

Make an estimate of the Doppler broadening of the line under these conditions. Comment on whether the Zeeman structure is resolved in the experiment. Other contributions to the line-width may be neglected. [4]

2. Explain what is meant by the *central field approximation* in the theory of atomic structure and how it leads to the concept of *electron configurations*. Considering the configuration $1s^2 2s^2 2p^6 3s3d$ of magnesium as an example, explain how taking into account the main effects neglected in the central field approximation leads to the splitting of a configuration into *terms* and *levels*. Consider only the situation in which the LS-coupling scheme is appropriate. [9]

The emission spectrum of zinc contains strong lines at 214, 468, 472, 481 and 1106 nm, all arising from transitions between the levels of the $4s^2$, 4s4p and 4s5s configurations. The transition at 214 nm is also observed in absorption. The singlet-triplet splitting in the 4s5s configuration is 211500 m^{-1}. Draw an energy level diagram showing the ordering of these levels. Label each level with appropriate quantum numbers and its energy in units of $10^3\,m^{-1}$, referred to the ground level as zero. Show also on the diagram the transitions giving rise to the spectral lines listed above. [7]

What features of the energy level structure and transitions indicate that the LS-coupling scheme is a good approximation? [4]

Section B (Special Relativity and Sub-Atomic Physics)

3. Consider a general purpose particle detector for e^+e^- collisions at $E_{\text{cm}} \approx 5\,\text{GeV}$. It is made of the following sub-detectors (in no particular order):

- an electromagnetic calorimeter made from CsI scintillator crystals ($Z(\text{Cs}) = 55$),
- a beam pipe made from beryllium ($Z(\text{Be}) = 3$),
- muon drift chambers made from layers of single-wire drift tubes,
- a silicon micro strip vertex detector,
- a hadronic calorimeter made from alternating layers of iron and scintillator,
- a tracking chamber built as a multi-wire drift chamber
- a $2\,\text{T}$ solenoidal magnet.

(a) With arguments and the help of a sketch, order the parts in their optimal sequence, starting from the interaction point, and describe a suitable geometry for their arrangement. [5]

(b) Describe briefly which sub-detectors measure which properties of which types of particles. Include in this the properties of the particle trajectories that may be measured. [7]

(c) In the light of the considerations above, explain the choice of material for the beam pipe. What are the advantages of CsI over NaI as a material for the electromagnetic calorimeter? [3]

(d) Describe briefly how a drift chamber works. [5]

4. The masses and widths of four $b\bar{b}$ resonances produced in e^+e^- collisions at centre-of-mass energies $E_{cm} \approx 10\,\text{GeV}$ are listed in the table below.

Particle	Mass/ GeV	Width/ keV
$\Upsilon(1s)$	9.460	52
$\Upsilon(2s)$	10.023	43
$\Upsilon(3s)$	10.355	24
$\Upsilon(4s)$	10.580	24000

The following table gives the masses and the products $c\tau$, where τ is the lifetime and c the speed of light, for the lightest B-mesons.

Particle	Mass/ GeV	$c\tau$/ μm
B^{+-}	5.2790	501
B^0	5.2794	460
B^0_s	5.3696	438

(a) Draw example Feynman diagrams of the decays of generic Υ-mesons into final states containing light B-mesons or lepton/anti-lepton pairs. Why is the decay of all Υ-mesons to $q\bar{q}$ final states suppressed? Explain why the $\Upsilon(4s)$ is much broader than the other Υ-mesons. [5]

(b) What type of interaction mediates the decays of the light B-mesons? Why can no other interaction give rise to these decays? Draw lowest-order Feynman diagrams of the decays $B^+ \to \overline{D^0}\rho^+$, $B^- \to D^0\rho^-$ and $B^0 \to D^- e^+ \nu_e$. [4]

(c) Compute the $c\tau$ values for $\Upsilon(1s)$ and $\Upsilon(4s)$. Explain the hierarchy of $c\tau$-values of $\Upsilon(1s)$, $\Upsilon(4s)$ and the light B-mesons. [3]

(d) Some e^+e^- colliders have asymmetric beam energies. Guided by the example below, explain why the beam energy asymmetry is useful. Let E_+ and E_- denote the energies of the e^+ and e^- beams, respectively. An e^+e^- collider operates at the $\Upsilon(4s)$ threshold with $E_+ = \delta\, E_-$ $(0 < \delta < 1)$. Consider the case in which a B^+ from the decay of an $\Upsilon(4s)$ travels in the direction of the e^- beam. Find an expression for the velocity ratio $\beta = v/c$ and the average distance L that a B^+-meson would travel in the laboratory frame before it decays. Calculate the values of E_+, E_-, v/c and L for $\delta = 0.1$.

[Note: You may assume that the location of the primary interaction point can be reconstructed.] [8]

5. The force between nucleons can be described by the exchange of π-mesons between the nucleons.

(a) Obtain an estimate for the range of this force based on the observation that the number density ρ_{N} of nucleons is approximately 1.6×10^{44} nucleons m^{-3} for nuclei of all sizes. From this range, estimate the mass of the pions in MeV. [4]

(b) Draw quark level diagrams representing elastic neutron-neutron (n-n) and neutron-proton (n-p) scattering in lowest-order pion exchange. Show in your diagrams how colour can be conserved in the interactions. How do your diagrams explain that the nuclear force is referred to as being partially an exchange force? How can your diagrams explain the fact that the elastic cross-section σ_{np} for n-p scattering is significantly larger than σ_{pp} for p-p scattering? [8]

(c) Consider the reaction $\mathrm{A} + \mathrm{a} \rightleftharpoons \mathrm{B} + \mathrm{b}$ involving four spinless particles. We refer to the left- and right-hand channels of this reaction as α and β, respectively. The principle of detailed balance relates the forward cross-section $\sigma(\alpha \to \beta)$ to the reverse cross-section $\sigma(\beta \to \alpha)$ at the same total energy:

$$p_\alpha^2 \sigma(\alpha \to \beta) = p_\beta^2 \sigma(\beta \to \alpha),$$

where p_α and p_β are the centre-of-mass momenta of A and B, respectively. Explain any relationship between the p^2 factors and the densities of states in the channels.

How does the density of states change when you consider particles with non-zero spin? Suggest and make plausible an extension of this equation to the case where the particles have spins J_{A}, J_{a}, J_{B} and J_{b}.

The reactions $\mathrm{d} + \pi^+ \rightleftharpoons \mathrm{p} + \mathrm{p}$ (d=deuteron) have been studied in fixed target collisions. Beams of π^+ and p with kinetic energies of $T_{\mathrm{lab}}^{\pi} = 29\,\mathrm{MeV}$ and $T_{\mathrm{lab}}^{\mathrm{p}} = 340\,\mathrm{MeV}$ were used. The cross-sections were measured as $\sigma(\mathrm{d} + \pi^+ \to \mathrm{p} + \mathrm{p}) = (3.1 \pm 0.3)\,\mathrm{mbarn}$ and $\sigma(\mathrm{p} + \mathrm{p} \to \mathrm{d} + \pi^+) = (0.18 \pm 0.06)\,\mathrm{mbarn}$. Compute the centre-of-mass energies $E_{\mathrm{cm}}^{\pi\mathrm{d}}$ and $E_{\mathrm{cm}}^{\mathrm{pp}}$ and momenta p_{cm}^{π} and $p_{\mathrm{cm}}^{\mathrm{p}}$. Assuming knowledge of the spins of p and d, use the principle of detailed balance to estimate the spin of the π^+. You may assume that any energy dependence the two cross-sections may have are small enough to ignore the differences in the centre-of-mass energies of both experiments. [8]

6. The semi-empirical mass formula (SEMF) states:

$$M(A,Z)c^2 = M_{\mathrm{p}}c^2 Z + M_{\mathrm{n}}c^2(A-Z) - B(A,Z),$$

where

$$B(A,Z) = \alpha A - \beta A^{2/3} - \epsilon Z^2 A^{-1/3} - \gamma\frac{(A-2Z)^2}{A} + \delta,$$

and the parameters are $\alpha = 15.56\,\mathrm{MeV}$, $\beta = 17.23\,\mathrm{MeV}$, $\epsilon = 0.697\,\mathrm{MeV}$, $\gamma = 23.285\,\mathrm{MeV}$ and $\delta = \pm 12A^{-1/2}\,\mathrm{MeV}$ or 0.

(a) Describe briefly the two major models of the nucleus underlying the SEMF. Explain briefly which terms in the SEMF stem from these models and how they arise. Hence explain the A-dependence of the terms involving α and β. [7]

(b) Derive the term involving ϵ and compute a value for ϵ in MeV. You may assume that nucleons are associated with a spherical volume of radius $R_0 \approx 1.2\,\mathrm{fm}$. [5]

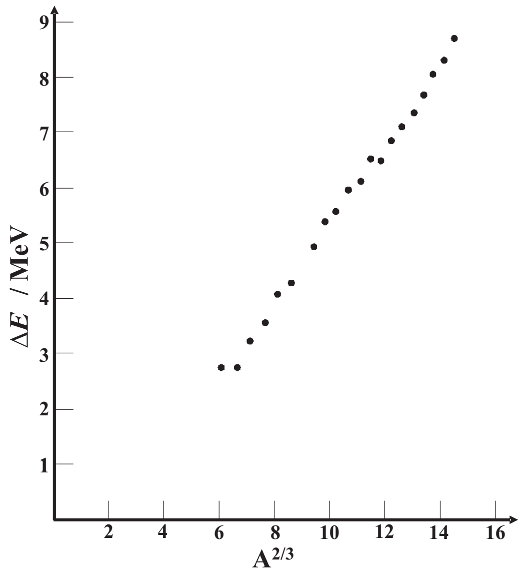
(c) A $^{209}_{82}\mathrm{Pb}$ nucleus has a radius of $R_{\mathrm{Pb}} \approx 6.75\,\mathrm{fm}$. Estimate a value for ϵ_{Pb} for this nucleus. From fits of the SEMF to the masses of many nuclei one finds $\epsilon_{\mathrm{fit}} = 0.697\,\mathrm{MeV}$. Compare both your values for ϵ and ϵ_{Pb} with ϵ_{fit} and suggest arguments explaining the relative sizes. [8]

7. The difference in the Coulomb energy $\Delta E_{\rm C}$ of two nuclei with equal radius R and $\Delta Z = 1$ is approximately

$$\Delta E_{\rm C} = \frac{3}{5}\frac{\rm e^2}{4\pi\epsilon_0 R} \times 2(Z-1).$$

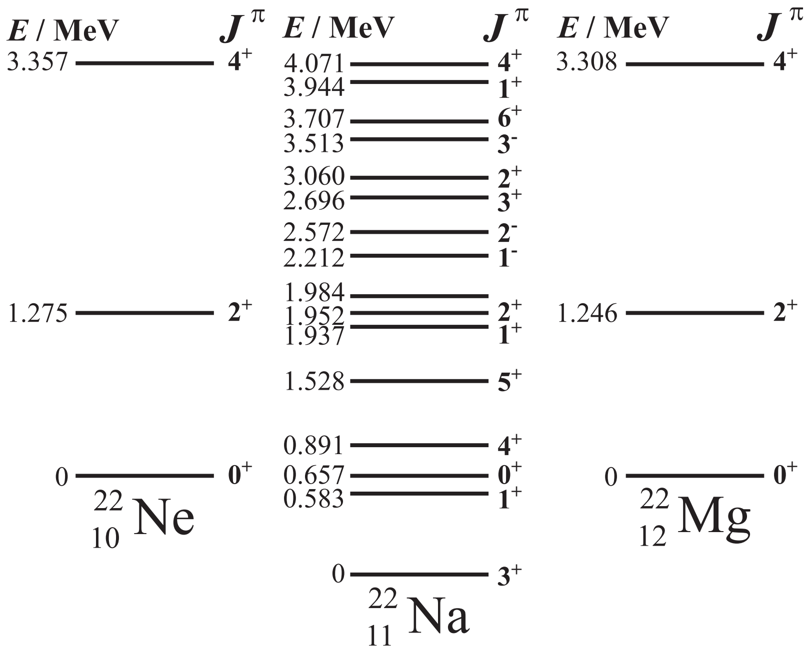
Two nuclei for which the number of neutrons in either one is equal to the number of protons in the other one are called mirror nuclei.

(a) The graph below shows the mass differences of mirror nuclei with $A > 15$, corrected for the neutron and proton mass differences. Which conclusions can you draw from the data about the dependence of the nuclear force on the charge of the particles it acts upon. From the data and the expression for $\Delta E_{\rm C}$ estimate the radius of a nucleon R_0. [Hint: Use suitable approximations.]



[10]

(b) The figure below shows the lower energy levels of a triplet of nuclei with identical A and Z ranging from $A/2-1$ to $A/2+1$ (J is the total angular momentum and π the parity). The levels have been corrected for the neutron/proton mass difference and ΔE_{C}.



(i) Explain qualitatively why $^{22}_{11}$Na has more energy levels than the other two nuclei.

(ii) In addition to your arguments above, what can you learn from the fact that, for identical J^{π} configuration, all three nuclei have very similar energy levels?

(iii) Why is the ground state of $^{22}_{11}$Na lower than those of the other two nuclei? Use the same arguments to explain why the deuteron is bound but the di-neutron and di-proton are not and what you expect for the spin of the deuteron. [10]

8.

(a) Write down the Lorentz transformations relating energy E, momentum $\mathbf{p}$, time t, and position $\mathbf{r}$ in a frame S to those in a frame S', moving uniformly in the positive x-direction with velocity βc. Write down the co-ordinate representation of two four-vectors that contain the above quantities and show that the squares of both four vectors are Lorentz invariant. Explain in detail how the square of the four-momentum is formed and explicitly state the form of the metric you use. [7]

(b) The expression $E_{\mathrm{t}} = mc^2/\sqrt{1-\frac{v^2}{c^2}}$ defines the relativistic total energy of a particle with mass m traveling at speed v. Relate E_{t} in the low-speed limit to the non-relativistic kinetic energy E_{kin}. Explain how the low-energy limit of E_{t} is consistent with the classical expression for energy. Derive an expression for E_{t} in the case of a massless particle. [5]

(c) Particle A (at rest in frame S') decays into two photons G_1 and G_2. Using the Lorentz transformations of E and $\mathbf{p}$, or otherwise, find the photon energies E_1, E_2 in frame S (both S and S' are the same as in part a) as a function of the angle θ^* which G_1 makes with respect to the positive x-axis of frame S'. Find the average of E_1 and E_2 over all angles θ^* and compare your result to what is expected from energy conservation. [8]