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SECOND PUBLIC EXAMINATION

Honour School of Physics – Part B: 3 and 4 Year Courses

Honour School of Physics and Philosophy Part B

B1: ATOMIC STRUCTURE, SPECIAL RELATIVITY AND SUB-ATOMIC PHYSICS

TRINITY TERM 2010

Thursday, 17 June, 9.30 am – 12.30 pm

Answer **five** *questions with at least* **one** *from each section:*

Start the answer to each question in a fresh book.

A list of physical constants and conversion factors accompanies this paper.

The numbers in the margin indicate the weight that the Examiners expect to assign to each part of the question.

Do NOT turn over until told that you may do so.

Section A (Atomic Structure)

1. Give simple arguments to explain why the hyperfine splitting in an atom can often be successfully described by including a term of the form $A_J \mathbf{I} \cdot \mathbf{J}$ in the atomic Hamiltonian, where J is the total electronic angular momentum quantum number and the nucleus has spin quantum number I. What properties of the system determine the sign of A_J? [4]

A magnetic field of magnitude B is applied to the atom. Explain with reference to the relevant terms in the Hamiltonian what is meant by the terms "strong" and "weak" field in the context of hyperfine structure.

The zero-field hyperfine splitting is 461 MHz in the 4s $^2S_{1/2}$ ground level of the isotope ^{39}K of potassium ($I = 3/2$). An atom in this level is subject to a field of 0.5 T. Show that this falls within the strong-field regime. [3]

Obtain an expression for the energies of the states in this field in terms of the appropriate quantum numbers, relative to the energy the level would have in the absence of field and hyperfine structure. Evaluate your expression in frequency units for all the states of the 4s $^2S_{1/2}$ level, and hence draw a diagram of the structure. Assume that the g-factor of the electron is 2.002. [7]

Derive an expression for the Doppler width (full width at half maximum intensity) of a spectral line, and hence comment on the feasibility of resolving this structure in a transition from 4s $^2S_{1/2}$ to a higher level corresponding to a wavelength of 404.7 nm, using potassium atoms at room temperature. You need not consider the splitting of the upper level by the hyperfine interaction or the external field. [6]

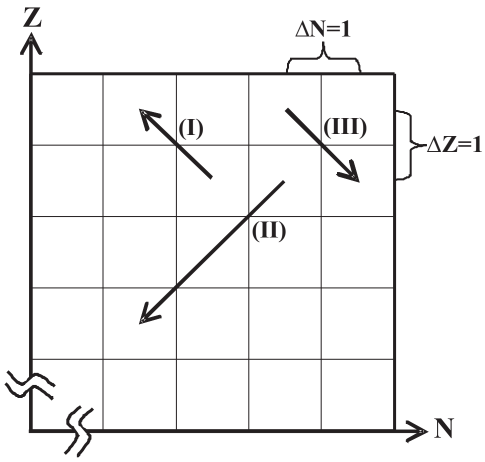
2. Explain the origin of absorption edges in X-ray spectra. Sketch the absorption spectrum you would expect to see with moderate resolution for a heavy element, and explain its principal features. [5]

When $_{10}$Ne is excited with X-rays of energy 1254 eV, corresponding to the K_α line of $_{12}$Mg, primary photoelectrons with energies 384, 1205.5 and 1232.4 eV are observed. Explain the origin of these photoelectrons. What energies would you expect to observe for the characteristic X-rays emitted by neon, for the same excitation conditions? [6]

In this experiment electrons are found also to be emitted from neon as a result of Auger transitions. Their energies fall in three narrow bands. Explain the origin of these electrons, and make a rough estimate of their energies. [5]

The energies are actually 748, 772 and 805 eV. Suggest reasons for any discrepancies between your results and these values. [4]

Section B (Special Relativity and Sub-Atomic Physics)



3. The figure above shows an arbitrary part of the $N - Z$ plane, where N and Z represent numbers of neutrons and protons respectively. The three arrows indicate radioactive decays.

(a) State all decay processes represented by the arrows and give their associated nuclear reaction equations in the form ${}^{A}_{Z}\mathrm{E} \to {}^{A'}_{Z'}\mathrm{E}' + X$ where E and E′ denote the parent and daughter nuclei and X denotes any further decay products. [3]

(b) Describe all processes associated with decays of type (I) at the nucleon and quark level, using Feynman diagrams for the latter. State the conditions on the nuclear masses under which these decays are energetically possible. [4]

(c) Use the semi-empirical mass formula (SEMF, given below) to derive the Z of the most stable nucleus for a given odd A. The lightest odd A nuclei with two naturally occurring isobars are ${}^{87}_{37}\mathrm{Rb}$ and ${}^{87}_{38}\mathrm{Sr}$. What does the SEMF predict for their stability against decays of type (I). Explain for this case the very rare existence of two naturally occurring odd A isobars. [7]

(d) How can the SEMF explain the facts that some nuclei can undergo decays of types (I) and (III) and that many even-A nuclei have two naturally abundant isobars. [6]

[The semi-empirical mass formula (SEMF) states:

$$M(A,Z)c^2 = M_\mathrm{p}c^2 Z + M_\mathrm{n}c^2(A-Z) - B(A,Z),$$

where

$$B(A,Z) = \alpha A - \beta A^{2/3} - \epsilon Z^2 A^{-1/3} - \gamma \frac{(A-2Z)^2}{A} + \delta,$$

and the parameters are $\alpha = 15.56\,\mathrm{MeV}$, $\beta = 17.23\,\mathrm{MeV}$, $\epsilon = 0.697\,\mathrm{MeV}$, $\gamma = 23.285\,\mathrm{MeV}$ and $\delta = \pm 12A^{-1/2}\,\mathrm{MeV}$ or 0.]

4. The first direct observations of neutrinos from the Sun were made using the Kamiokande-II neutrino detector. It used a large volume of water as the target mass and was located in a Zinc mine approximately 1 km below ground. With the aid of a diagram describe a detector for solar neutrinos, such as Kamiokande-II, which uses H_2O as a target material. Why was Kamiokande-II located in a deep mine? Draw the Feynman diagrams of those solar neutrino interaction that are detectable in an H_2O-based detector and describe how they lead to a signal in the detector. How can such a detector distinguish solar neutrinos from atmospheric neutrinos? [7]

The three nuclear reactions of the pp-cycle in the Sun are:

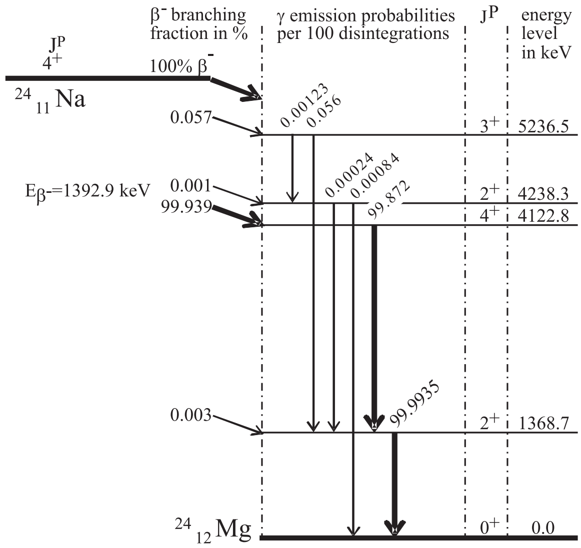
$$\begin{aligned} {}^1_1\mathrm{p} + {}^1_1\mathrm{p} &\rightarrow {}^2_1\mathrm{d} + \mathrm{e}^+ + \nu_\mathrm{e}, \\ {}^2_1\mathrm{d} + {}^1_1\mathrm{p} &\rightarrow {}^3_2\mathrm{He} + \gamma, \\ {}^3_2\mathrm{He} + {}^3_2\mathrm{He} &\rightarrow {}^4_2\mathrm{He} + 2\,{}^1_1\mathrm{p}. \end{aligned}$$

The electromagnetic power radiated by the Sun is 3.845×10^{26} W, 95% of which is produced in the above reactions. The neutrinos produced in those reactions have a continuous energy spectrum with an average energy of 0.26 MeV and a maximum energy of 0.42 MeV. The total kinetic energy of all particles in these reactions is 24.056 MeV for each ${}^4_2\mathrm{He}$ produced. How many neutrinos are produced by these reactions in the Sun per day? Assuming that the average cross-section for elastic scattering between these neutrinos and electrons is $1.16 \times 10^{-45}\ \mathrm{cm}^2$, how many scattering events will occur in a detector with an H_2O target mass of 10^6 kg. [5]

Can the highest energy neutrinos from the above reactions be detected in principle by a detector using H_2O as target material? What minimum energy would a neutrino need to have to be detectable in principle by such a detector. The refractive index of water is $n(H_2O) = 1.345$. Why could those working on Kamiokande-II not claim in practice that they had observed solar neutrinos from the pp-cycle. [8]

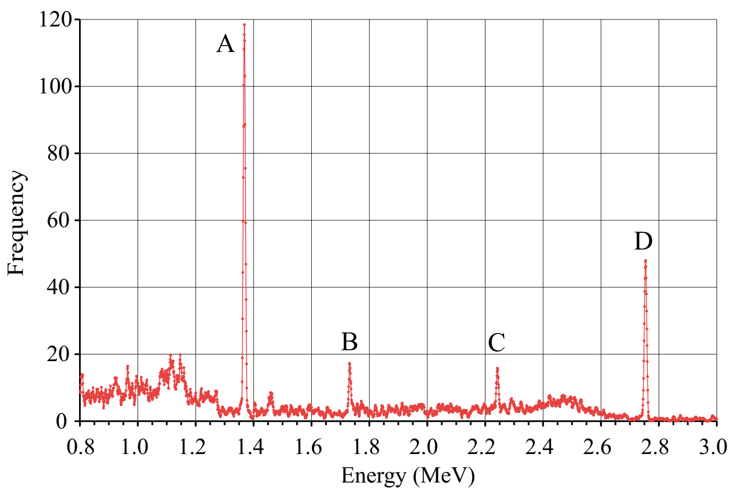
5. In the context of detecting photons describe briefly the three main processes by which photons interact with matter, using Feynman diagrams where appropriate. [5]

Below is the decay scheme of $^{24}_{11}$Na. The total energy released in the transition to the ground state of $^{24}_{12}$Mg is 5515.8 keV. Explain why $^{24}_{11}$Na predominantly decays to the 4122.8 keV excited state of $^{24}_{12}$Mg. Suggest how $^{24}_{11}$Na can be made from naturally abundant $^{23}_{11}$Na. [3]



The figure below shows the energy spectrum of $^{24}_{11}$Na measured using a cryogenically cooled germanium detector. Explain what type of particles the detector will detect given that it is housed in a cryostat and the source is outside the cryostat. Describe how the energy of the primary particle is deposited in a germanium detector and hence explain why it has excellent energy resolution. [8]

Explain how the features labelled A–D in the energy spectrum arise. What can you learn about the energy dependence of the pair-production cross-section in germanium from the absence of a significant peak around 858 keV. [4]



6. In 1851 Armand Hippolyte Louis Fizeau tried to measure the speed of light in moving water. He found that it depended on the speed of the water in the following way:

$$u \cong \frac{c}{n} + v(1 - \frac{1}{n^2})$$

where v is the velocity of the water with respect to Fizeau's detector, u is the speed of light as measured by Fizeau's detector and n is the refractive index of the water. Derive this result from a relativistic treatment of this so called 'drag effect', explaining any approximations you make. [4]

Derive the formula for the relativistic Doppler effect that relates the emitted frequency (ν_0) to the observed frequency (ν) of light from a source moving at instantaneous velocity $\mathbf{v}$ in vacuum with respect to the observer. Your result should be expressed in terms of the radial velocity component (v_r) and the speed v. [5]

What are the three major differences between this relativistic Doppler effect and its classical counterpart for waves propagating in a medium? [2]

An observer located at the origin O of an inertial frame S observes a sample of radioactive nuclei moving with constant velocity $\mathbf{v}$ in the direction of positive x on a trajectory with $y = y_0$ and $z = 0$. The nuclei emit gamma photons of energy E_0 as measured in their own rest frame. At time $t = 0$ the observer sees a gamma ray moving in the direction of the negative y-axis. Calculate the observed energy E of the gamma rays as detected by the observer as a function of the observer's time t, E_0 and $|\mathbf{v}|$. How fast does a sample with $E_0 = 1\,\text{MeV}$ have to move for the minimum observed energy E_{min} to be $0.9\,\text{MeV}$. [6]

Describe the frequency spectrum of a cloud of nuclei if its centre of mass is stationary with respect to the observer but the nuclei move isotropically at high speed. [3]

7. Draw three Feynman diagrams for the main modes of production of W-bosons in e^+e^--collision and list their decays. Explain how fully leptonic decays of these W-bosons can be distinguished from background events. [7]

The Tevatron collides protons with anti-protons at a centre of mass energy of 1.96 TeV. Explain how W-bosons can be produced at the Tevatron and draw appropriate Feynman diagrams. Considering other reactions likely to occur in $p\bar{p}$-collisions explain which of the decays of the W-bosons are most readily detectable at the Tevatron. [3]

Neglecting the coupling of the W-boson to pairs of quarks from different generations calculate the ratio R of the partial width of real W-bosons into hadrons (Γ_{had}) and into leptons (Γ_{lep}) where $R = \Gamma_{\text{had}}/\Gamma_{\text{lep}}$. [3]

Write down the propagator of the W-boson in terms of the W-mass m_{W} and its four momentum $\mathbf{q}$. Given that the Fermi coupling constant can be written as

$$G_{\text{F}} = \frac{\sqrt{2}}{8}(\hbar c)^3 F(g_{\text{W}}, m_{\text{W}}),$$

where g_{W} is the weak coupling constant and the value of the Fermi coupling constant is $G_{\text{F}} = 1.166 \times 10^{-5}\,\text{GeV}^{-2}(\hbar\text{c})^3$, estimate the order of magnitude of the mass of the W-boson and find an expression for the function $F(g_{\text{W}}, m_{\text{W}})$ in terms of g_{W} and m_{W}. [4]

[You should assume that the weak and electromagnetic coupling constants (g_{W} and g_{EM}) are equal.]

How could W-bosons be produced at the LHC which collides protons with protons? [3]

8. State how hyper-charge Y is related to baryon-number B and strangeness S in the $SU(3)$-flavour model of the light quarks. Copy and fill the table of quantum numbers of the quarks given below. [4]

		d	u	s	$\overline{\text{d}}$	$\overline{\text{u}}$	$\overline{\text{s}}$
baryon number	B						
electric charge	Q						
isospin	I						
third component of I	I_3						
strangeness	S						
hyper-charge	Y						
parity	P						
spin	s						

Explain how in the $SU(3)$-flavour model, the two multiplets of the 18 lightest mesons arise and explain how the total angular momentum J and parity P of the mesons in both multiplets are derived from the properties of their constituents. Construct the weight diagrams of the lightest mesons in the (Y, I_3)-plane, stating the quark content and names of all particles. [10]

Explain how one may measure the lifetime τ_ρ of ρ-mesons produced in e^+e^- collisions, given that these decay nearly 100% to $\pi^+\pi^-$ pairs. Consider which quantities of which interactions you wish to measure, how to produce and identify these interactions and how to obtain a good estimate of the lifetime from the data. [6]