REVENUE-CONSTRAINED STRATEGIC TRADE AND INDUSTRIAL POLICY

J. Peter Neary

and

Dermot Leahy

University College Dublin and CEPR

29 May 2003

Abstract

We characterise optimal revenue-constrained trade and industrial policy towards dynamic oligopolies, prove that total net subsidy payments at the optimum are decreasing in the social cost of funds, and illustrate the implications in Cournot and Bertrand special cases.

Corresponding Author: J. Peter Neary, Department of Economics, University College Dublin, Belfield, Dublin 4, Ireland. Tel.: (+353) 1-716 8272; fax: (+353) 1-283 0068; email: peter.neary@ucd.ie.

1. Introduction

The theory of strategic trade policy highlights differences in commitment ability between firms and governments as a key motivation for intervention in oligopolistic markets. The theory has also been extended to many periods, allowing consideration of industrial as well as trade policy, but only on the assumption that subsidies can be financed costlessly by non-distortionary taxation, so the marginal social cost of funds is unity.[1] In this paper we extend the theory of strategic trade and industrial policy to allow for divergences between the social valuations of corporate profits and subsidy revenue foregone. We characterise optimal revenue-constrained trade and industrial policy towards dynamic oligopolies, prove that total net subsidy payments at the optimum are decreasing in the social cost of funds, and illustrate the implications in special cases where investment precedes either Cournot or Bertrand competition. Finally, we note the wide range of issues to which our approach can be applied.

2. Strategic Trade and Industrial Policy with and without Revenue Constraints

In the absence of a government revenue constraint, the setting is the same as in Neary and Leahy (2000). We consider a market with two firms, denoted "home" and "foreign", which compete over a finite number T of time periods. In each period t, each firm takes an action, choosing the value of some variable, a_t for the home firm and b_t for the foreign firm. This specification allows for many alternative types of oligopolistic interaction: in each period the

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[1] See Brander (1995) and Neary and Leahy (2000) for syntheses and further references. Neary (1994) considers asymmetries between the private and social cost of funds in one-period models of strategic trade policy.

decision variables might be output, price, R&D, etc. The home firm seeks to maximise the present value of its profits, π, equal to the sum of the present values of its gross profits (i.e., sales revenue less total costs) R and its subsidy income S:

$$\underset{a}{Max} \; \pi(a,b,s) \quad = \quad R(a,b) \; + \; S(a,b,s)\,, \tag{1}$$

where a and b are the vectors of the home and foreign firm's actions respectively. Both R and S are twice differentiable functions of own and rival actions; in addition, S depends on a vector of subsidy rates, one for each period, chosen by the home government, s. In many applications, the subsidy is directly related to the firm's decision variable, so the vector of partial derivatives S_a equals s. An exception is the case of an output subsidy in Bertrand competition, where some elements of S_a have the opposite sign to those of s.[2] As for the foreign firm, we assume that it is not assisted by its government, so its decision problem is simply to maximise $\pi^*(b,a)=R^*(b,a)$ by choice of b.

The home government seeks to maximise welfare:

$$W(a,b,s,\delta) \quad = \quad \pi(a,b,s) \; - \; \delta S(a,b,s)\,, \tag{2}$$

where δ is the marginal social cost of funds, so the excess of δ over one measures the distributional preference for subsidy revenue over corporate profits. To focus on strategic issues, we assume that all home output is exported, so welfare does not depend on consumer surplus.

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[2] In Bertrand competition, the firm's decision variable a_t is its price p_t, but its subsidy income in period t, S^t, equals $s_t x_t$, where x_t, the period-t demand function, depends in general on all current and past actions (prices). Hence, $\partial S^t/\partial a_t = s_t \partial x_t/\partial p_t$, which has the opposite sign to x_t.

We also assume that the government commits to all future subsidies before firms take any actions. The government's problem is therefore to choose the vector of subsidies s to maximise (2), anticipating the game played between the firms. Formally, we can view the government as solving the first-order conditions of the two firms for their actions as functions of the subsidies, $\tilde{a}(s)$ and $\tilde{b}(s)$, and substituting into (2). Since the number of instruments or subsidies equals the number of targets or actions chosen by the home firm, and provided (as we assume) the function $\tilde{a}(s)$ is invertible, this is equivalent to the government's choosing a. It is as if the government solves $\tilde{a}(s)$ and $\tilde{b}(s)$ for $\hat{b}(a)$ and $\hat{s}(a)$, and then substitutes these into (2) to give a reduced-form welfare function which depends only on a and δ:

$$\hat{W}(a,\delta) \equiv W\left[a,\hat{b}(a),\hat{s}(a),\delta\right] = \hat{R}(a) - (\delta-1)\hat{S}(a), \tag{3}$$

where the reduced-form gross profits and subsidy revenue functions $\hat{R}$ and $\hat{S}$ are defined analogously to $\hat{W}$: e.g., $\hat{R}(a)\equiv R[a,\hat{b}(a)]$. Maximising welfare therefore leads to the government's first-order condition:

$$\hat{W}_a = \hat{R}_a - (\delta-1)\hat{S}_a = 0, \tag{4}$$

while its second-order condition is that the matrix $\hat{W}_{aa}$ is negative definite.

When the marginal social cost of funds is unity, only the first term $\hat{R}_a$ on the right-hand side of (4) matters. This term can be expressed as the deviation of S_a, the vector of subsidy terms evaluated at an arbitrary point, from $\bar{S}_a$, the vector of optimal subsidy formulae in the standard case when δ equals one:[3]

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[3] See Neary and Leahy (2000), equations (6), (8) and (24).

$$\hat{R}_a' \;=\; R_a' + R_b'\hat{b}_a \;=\; -(S_a - \overline{S}_a)' \tag{5}$$

The second term in (4) can be written:

$$\hat{S}_a' \;=\; S_a' + S_b'\hat{b}_a + S_s'\hat{s}_a. \tag{6}$$

Substituting into the first-order condition (4) gives the optimal subsidies in the general case:

$$S_a' \;=\; \tfrac{1}{\delta}\overline{S}_a' \;-\; \tfrac{\delta-1}{\delta}(S_b'\hat{b}_a + S_s'\hat{s}_a). \tag{7}$$

This shows that $\overline{S}_a$, the standard formula for the optimal subsidies, must be modified in two respects. First, the usual strategic motive for intervention (summarised by $\overline{S}_a$) is diluted, counting for less the higher is δ. Second, there is an additional group of terms which reflect the deadweight loss of raising revenue. These terms are likely to contribute to a reduction in the optimal subsidies. (Consider the one-period case for example. If competition is Cournot, S_a is the optimal subsidy, while S_s and $\hat{s}_a$ are both positive; whereas if competition is Bertrand, then S_a is negatively related to the optimal subsidy, S_s is positive and $\hat{s}_a$ is negative.) There is no general presumption that this must always be so. (Figure 1 below provides a counter-example in a simple case.) However, we can show that *total* subsidy payments are unambiguously related to δ:

Proposition 1:   Total subsidy payments fall as δ rises.

Proof: Totally differentiate the subsidy revenue function $\hat{S}(a)$ with respect to δ and evaluate at the optimal a, using the total differential of the government's first-order condition (4):

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$$\frac{dS}{d\delta} \;=\; \hat{S}_a' \frac{da}{d\delta} \;=\; -\hat{S}_a' \hat{W}_{aa}^{-1} \hat{W}_{a\delta} \;=\; \hat{S}_a' \hat{W}_{aa} \hat{S}_a \tag{8}$$

This is a quadratic form in a matrix which must be negative definite from the government's second-order condition. Hence, S and δ must be inversely related along the revenue-constrained iso-welfare locus.

Q.E.D.

Figures 1 and 2, drawn in subsidy space, illustrate a two-period linear-quadratic example. Firms choose the level of cost-reducing R&D in the first period and then compete on either output (in Figure 1) or price (in Figure 2).[4] In both figures iso-R contours are ellipses centred on point A, the optimum in the absence of revenue constraints, while iso-S contours are downward-sloping concave loci. Revenue-constrained optima must lie on the locus, passing through A, of tangency points between these two sets of contours. A higher marginal social cost of funds (i.e., a rise in δ) implies a movement downwards along this locus from point A. However, as Figure 1 illustrates, this implies in the Cournot case that the investment subsidy becomes *less* negative (though it never attains a positive value). The export subsidy falls in absolute value, as Proposition 1 requires. The Bertrand case in Figure 2 is more intuitive. Here,

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[4] The cost of R&D k equals γk^2 and the cost of production c equals $c_0 - \theta k$. In Figure 1, the demand function is $p = a_0 - X$, and the marginal efficiency of R&D $\eta \equiv \theta^2/\gamma$ is set at *0.2*. In Figure 2, goods are symmetrically differentiated in demand, the demand function for home output is $x = \alpha - (p - p^*)$, where p and p^* denote home and foreign prices respectively, and η equals *0.4*. For very high or very low subsidies, one or other firm is unprofitable.

both subsidies fall in algebraic value as δ rises, though the investment subsidy never actually becomes negative.

3. Concluding Remarks

In conclusion, we may note the wide applicability of our approach. When δ is interpreted as the marginal social cost of funds, it may exceed unity for a variety of reasons, including the deadweight loss from financing subsidies by distortionary taxation, pure distributional considerations, or the fact that the home firm is partly foreign-owned.[5] All these imply a value of δ greater than one. The model can also be interpreted in ways which imply a value of δ less then one. The first case is where the home firm's profits at the unconstrained optimum would be insufficient to induce it to enter the market, and a non-distorting subsidy to fixed costs is unavailable. Hence the government's problem is to choose the optimal policy package to maximise welfare subject to a profit constraint. Forming the Lagrangian, the government's problem is to maximise $\mathcal{L}=W+\lambda(\pi-\pi_m)$, where π_m is the minimum required profit level. Replacing W by R and dividing by $1+\lambda$, this is equivalent to (3), except that the coefficient of the deadweight-loss term is now positive rather than negative: $\lambda=(1-\delta)/\delta$, or $\delta=1/(1+\lambda)<1$. A second case, following Grossman and Helpman (1994) is where the incumbent government seeks to maximise a political support function, which is a weighted average of welfare and political contributions. Assuming that the latter equal profits in equilibrium, the political support function is $\Phi=\alpha W+(1-\alpha)\pi$, which (replacing W by R), can again be written in the form of (3), with $\delta=\alpha<1$. In both these cases, Proposition 1 implies that total subsidy payments must be greater

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[5] See Neary (1994).

than in the unconstrained optimum, and, in Figures 1 and 2, the optimum lies along the same revenue-constrained loci as before, but *above* rather than *below* point *A*.

Acknowledgements

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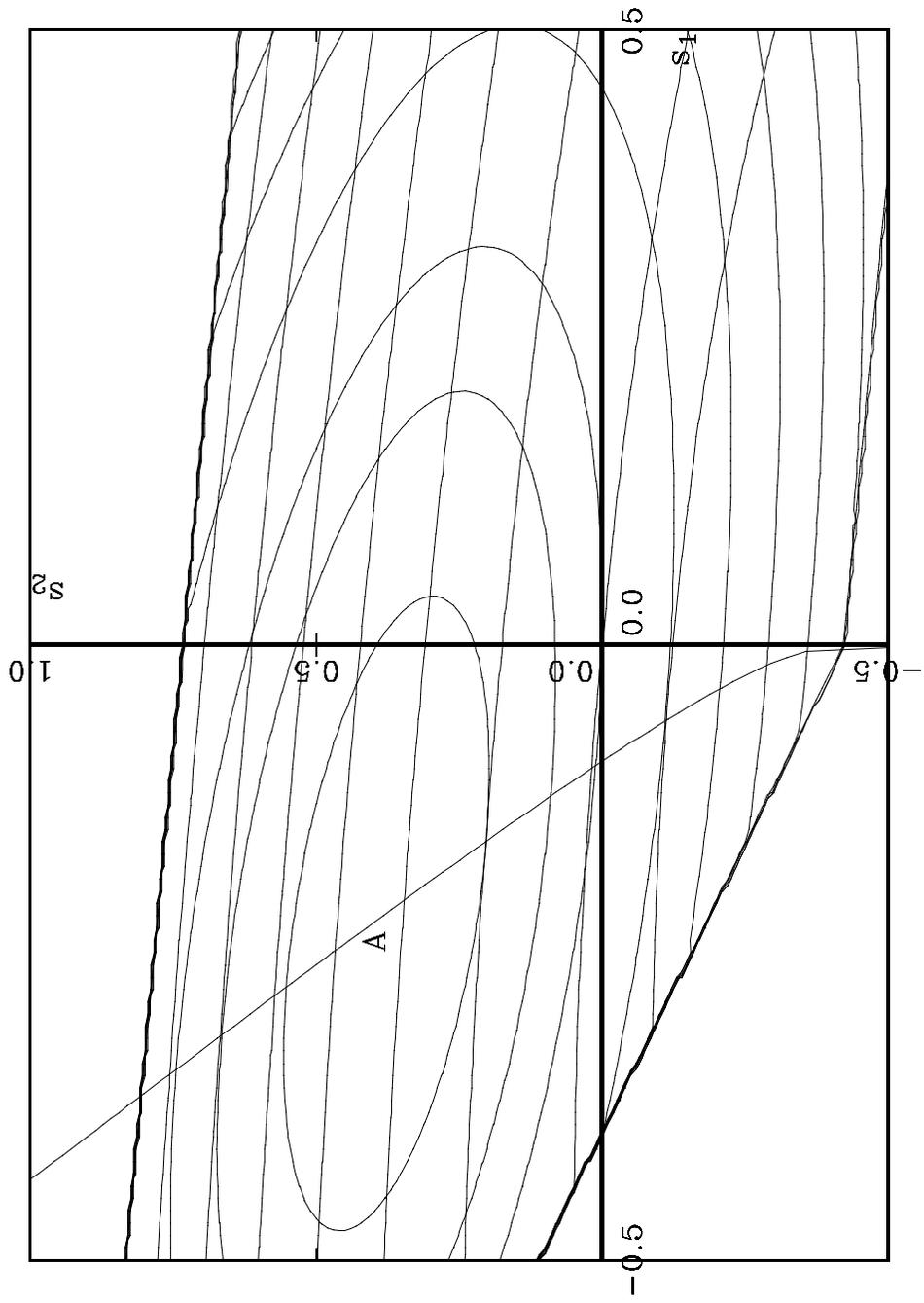Fig. 1: Revenue–Constrained Optimal Subsidy Locus in R&D–Cournot Game

Fig. 2: Revenue–Constrained Optimal Subsidy Locus in R&D–Bertrand Game