Appendix 1: The Trade Expenditure Function

Consider the matrix of second derivatives of the emissions-constrained trade expenditure function with respect to prices, $E_{pp}=e_{pp}-g_{pp}$. The expenditure function e is concave while the GNP function g is convex in prices p. Hence E is concave in p and so the Hessian E_{pp} is negative semi-definite. Recalling that the vector p does not include the price of the untaxed numeraire good, we need only assume that there is some substitutability in either demand or supply between the numeraire and *any* other good to conclude that E_{pp} is negative definite. Identical arguments apply to the second derivatives of the flex-price trade expenditure function:

$$\tilde{E}_{\pi\pi} = \begin{bmatrix} \tilde{E}_{pp} & \tilde{E}_{pt} \\ \tilde{E}_{tp} & \tilde{E}_{tt} \end{bmatrix} = \begin{bmatrix} e_{pp}-\tilde{g}_{pp} & -\tilde{g}_{pt} \\ -\tilde{g}_{tp} & -\tilde{g}_{tt} \end{bmatrix} \qquad \textbf{(42)}$$

The flex-price GNP function is convex in prices *(p,t)*, so $\tilde{E}$ is concave in *(p,t)* and (assuming once again some minimal substitutability) $\tilde{E}_{\pi\pi}$ is negative definite.

To derive equation (11) from (10), write net imports and emissions as a single vector y, and express it as a function of prices and utility only:

$$\tilde{E}_{\pi} = y = \begin{bmatrix} m \\ z \end{bmatrix} = \begin{bmatrix} \tilde{E}_p\left\{\pi, \tilde{E}_t(\pi), u\right\} \\ \tilde{E}_t(\pi) \end{bmatrix} \qquad \textbf{(43)}$$

Totally differentiating gives:

$$dy = \begin{bmatrix} \tilde{E}_{p\pi} + \tilde{E}_{pz}\tilde{E}_{t\pi} \\ \tilde{E}_{t\pi} \end{bmatrix} d\pi + \begin{bmatrix} \tilde{E}_{pu} \\ 0 \end{bmatrix} du = \begin{bmatrix} I & e_{pz} \\ 0 & I \end{bmatrix} \tilde{E}_{\pi\pi}\, d\pi + \begin{bmatrix} x_I \\ 0 \end{bmatrix} e_u\, du \qquad \textbf{(44)}$$

where I have used the fact that $\tilde{g}$ does not depend on z and u to replace $\tilde{E}_{pz}$ and $\tilde{E}_{pu}$ by e_{pz} and e_{pu} respectively; e_{pu} in turn equals the Marshallian income effects x_I deflated by the marginal cost of utility e_u. Substituting into (10) gives equation (11) as required, where the 1-by-$(n+h)$ vector $\hat{\tau}'$ equals the vector $(\tau - \tilde{E}_y)'$ postmultiplied by the square block-triangular matrix on the right-hand side of (44).

Appendix 2: Qualifications to Second-Best Reform

Equations (13) and (14) can be qualified and extended in a number of ways. Note first that they imply results on movements towards the second-best optimum. It can be checked that the change in welfare is proportional to $(r-r^{o})'\tilde{E}_{pp}dr$ if environmental taxes are fixed; and proportional to $(\delta-\delta^{o})'\tilde{E}_{tt}d\delta$ if tariffs are fixed. Hence, as Copeland (1994) notes, a proportionate reduction in either set of instruments towards their second-best values (evaluated at the initial point) must raise welfare.

As always in many-good models, an exact formal statement of the implications of equations (13) and (14) requires a little care. From (13) we can conclude that $(r^{o})'\tilde{g}_{pt}\tilde{\delta}$ must always be negative (since it equals $\tilde{\delta}'\tilde{g}_{tp}(\tilde{E}_{pp})^{-1}\tilde{g}_{pt}\tilde{\delta}$, which is a quadratic form in the negative definite matrix $(\tilde{E}_{pp})^{-1}$). Hence the precise sense in which imports should be subsidised to compensate for lax environmental policy is as follows: a sufficient condition for $r^{o}<0$ is that the vector $\tilde{g}_{pt}\tilde{\delta}$ is positive. This can be interpreted as the combination of (on average) importables which are pollution-intensive ($\tilde{g}_{pt}<0$) and environmental controls which are lax ($\tilde{\delta}<0$). Similar reasoning, using (14) and the fact that $\tilde{g}_{tt}$ is positive definite, implies that the expression $(\tilde{\delta}^{o})'\tilde{g}_{tp}r$ must always be negative. Hence a sufficient condition for $\tilde{\delta}^{o}>0$ is that the combination of (on average) pollution-intensive importables and positive tariffs means that the vector $\tilde{g}_{tp}r$ is negative.

Finally, an interesting special case is where pollution levels are directly associated with output levels. By suitable choice of units for x and z, we can then set $z=x$ and so write the

GNP function as $g(p-t)$. Outputs and emissions are now *perfect* complements in production and hence tariffs and pollution taxes are perfect alternatives as far as their effects on production are concerned. This simplifies many of the results. For example, optimal second-best pollution taxes are exactly equal to the given tariffs: $\delta^0=r$; and, irrespective of the level of tariffs, welfare will improve if pollution taxes are lowered in proportion to the *difference* between tariffs and pollution distortions: $(1-r'x_I)e_u du = -(r-\delta)'g_{tt}dt$.

Appendix 3: Taxes versus Emission Standards in Oligopoly

I wish to prove that the slopes of both firms' reaction functions are algebraically smaller when pollution is regulated by taxes rather than by standards. To save on notation I consider the home firm's reaction functions; obviously identical conclusions apply to the foreign firm's. (To avoid some unnecessary complications in the Bertrand case, I assume that output is not subsidised: $s=0$.) The proof, adapted from Neary (1985), proceeds by defining a new operating profit function, which takes account of the optimal adjustment of emissions in the face of a given tax rate:

$$\tilde{\pi}(a,a^*,t) \quad \equiv \quad \underset{z}{Max}\ [\pi(a,a^*,z) - t.z] \qquad \textbf{(45)}$$

The first-order condition is simply:

$$\pi_z(a,a^*,z) \ = \ t \qquad \textbf{(46)}$$

which can be solved for z as a function of (a,a^*,t), provided π is strictly concave in z. (This is necessary for an interior optimum, and is ensured by the assumption made in the text that pollution abatement costs are convex in z.) Now, take an initial point with given initial values of a and a^*, and consider two otherwise identical firms, one facing an emissions tax

t and the other facing an emissions standard z, where the values of the two instruments are consistent with (46). We wish to compare the reaction function slope of the standards-constrained firm, da/da^*, with that of the tax-constrained firm, $d\tilde{a}/da^*$.

To derive the reaction function slopes we need to consider the second derivatives of $\tilde{\pi}$ and π. First, differentiate (45) with respect to a (using (46) to simplify):

$$\tilde{\pi}_a = \pi_a \tag{47}$$

which reflects the envelope theorem. Now differentiate this with respect to a:

$$\tilde{\pi}_{aa} = \pi_{aa} + \pi_{az}\frac{dz}{da} = \pi_{aa} - \pi_{az}\pi_{zz}^{-1}\pi_{za} \tag{48}$$

The second-order conditions require that, at an optimum, $\tilde{\pi}$ must be concave in a and π must be concave in both a and z. Hence (48) implies that:

$$\pi_{aa} < \tilde{\pi}_{aa} < 0 \tag{49}$$

Next, differentiate (47) with respect to a^*:

$$\tilde{\pi}_{aa^*} = \pi_{aa^*} + \pi_{az}\frac{dz}{da^*} = \pi_{aa^*} - \pi_{az}\pi_{zz}^{-1}\pi_{za^*} \tag{50}$$

In the Cournot case, the second term on the right-hand side is zero, since pollution abatement costs are independent of foreign output and so $\pi_{za*}=0$. Hence $\tilde{\pi}_{aa*}=\pi_{aa*}$, and both are negative, given the assumption that foreign output is a strategic substitute for home output. Combining this with (49), it follows that the home firm's reaction function has a greater slope (in absolute value) when emissions are controlled by taxes than when they are controlled by standards:

$$\left.\frac{d\tilde{x}}{dx^*}\right|_t = -\frac{\tilde{\pi}_{aa^*}}{\tilde{\pi}_{aa}} < -\frac{\pi_{aa^*}}{\pi_{aa}} = \left.\frac{dx}{dx^*}\right|_z < 0 \quad \textbf{(51)}$$

As for the Bertrand case, the assumption that the foreign price is a strategic complement for the home price means that both $\tilde{\pi}_{aa*}$ and π_{aa*} are positive. Direct calculation now yields:

$$\left.\frac{d\tilde{p}}{dp^*}\right|_t - \left.\frac{dp}{dp^*}\right|_z = \frac{-\tilde{\pi}_{aa^*}\pi_{aa} + \pi_{aa^*}\tilde{\pi}_{aa}}{\pi_{aa}\tilde{\pi}_{aa}} \quad \textbf{(52)}$$

$$= \frac{\pi_{az}\pi_{zz}^{-1}}{\pi_{aa}\tilde{\pi}_{aa}}\left[\pi_{za^*}\pi_{aa} - \pi_{aa^*}\pi_{za}\right] \quad \textbf{(53)}$$

$$= \frac{\pi_{az}\pi_{zz}^{-1}\pi_{za}}{q_p\pi_{aa}\tilde{\pi}_{aa}}\left[q_{p^*}\pi_{aa} - q_p\pi_{aa^*}\right] \quad \textbf{(54)}$$

$$= \frac{\pi_{az}\pi_{zz}^{-1}\pi_{za}}{q_p\pi_{aa}\tilde{\pi}_{aa}}\left[q_p q_{p^*} + (q_{p^*}q_{pp} - q_p q_{pp^*})(p - C_x - \bar{C}_x)\right] \quad \textbf{(55)}$$

Equation (53) follows by substituting from (48) and (50) into (52); equation (54) follows by differentiating (17) to get $\pi_{za*}=-\bar{C}_{zx}q_{p*}=\pi_{za}q_{p*}/q_p$; and equation (55) follows by differentiating (17) to evaluate π_{aa} and π_{aa*}. The term outside the square brackets in (55) is positive and the first term inside is negative. The second term inside the square brackets is also negative provided demand is concave in both prices ($q_{pp}<0$ and $q_{pp*}<0$), since $p-C_q-\bar{C}_x$ is the price-cost margin and must be positive. Hence we may conclude that the home firm's reaction function has a smaller positive slope when emissions are controlled by taxes than when they are controlled by standards:

$$\left.\frac{dp}{dp^*}\right|_z = -\frac{\pi_{aa^*}}{\pi_{aa}} > -\frac{\tilde{\pi}_{aa^*}}{\tilde{\pi}_{aa}} = \left.\frac{d\tilde{p}}{dp^*}\right|_t > 0 \qquad \textbf{(56)}$$

provided demands are concave or linear in both prices.

Appendix 4: Strategic Investment in Pollution Abatement: The General Case

When the link between investment in abatement technology (denoted by *k*) and emissions *z* depends on the level of output, the home firm's profits are given not by (24) but by:

$$\Pi = \pi(a,a^*,k) + s.x(a,a^*) - tz, \qquad \text{where:} \qquad z = z(k,x) \qquad \textbf{(57)}$$

Investment is costly, so $\pi_k<0$; and it reduces emissions, so $z_k<0$. I also assume that higher output raises emissions, so $z_x>0$, and reduces the effectiveness of investment, so $z_{kx}<0$. (These restrictions are satisfied by the special case considered by Ulph and Ulph (1996), where $z=x\bar{z}(k)$. Section 2 above considered the very special case where $z=-k$.) Compared to (18), the first-order condition for the firm's action *a* contains an extra term which takes account of the extra emissions tax liability incurred by an increase in output:

$$\Pi_a = \pi_a + s.x_a - tz_x x_a = 0 \qquad \textbf{(58)}$$

By contrast, the first-order condition for investment is a simple transformation of (26):

$$\Pi_k = \pi_k + \Pi_{a^*}\frac{da^*}{dk} - tz_k = 0 \qquad \textbf{(59)}$$

The welfare function becomes, instead of (19), the following:

$$W = \pi(a,a^*,k) - D(z) \qquad \textbf{(60)}$$

To derive the optimal policies, totally differentiate the welfare function, use the differential of $z(k,x)$ to eliminate dk, and use the first-order conditions to eliminate π_a and π_k as before. After some derivations this yields:

$$\text{(i)} \quad s.x_a \;=\; \{\pi_{a^*} - D_z z_x x_{a^*}\} A_a^* + (t - D_z) z_x x_a \quad \text{and} \quad \text{(ii)} \quad t \;=\; D_z + \Pi_{a^*} \frac{da^*}{dk} z_k^{-1} \tag{61}$$

The key point to note is that, except for the switch in strategic variable from z to k, the first-order condition for the emissions tax is identical to (27) (ii), implying an over-strict policy in Cournot competition and an over-lax one in Bertrand competition. The formula for the optimal export subsidy is now more complicated, however. It still contains the rent-shifting term $\pi_{a*}A_a^*$. In addition, it has some extra terms reflecting the extra complexity of the general case. In effect, the optimal emissions tax is targeted towards ensuring that investment is efficient; this leaves the export subsidy with the dual role of shifting rents and enuring that (given the level of k) the level of emissions is at its optimal level. Fortunately, these two effects tend to reinforce each other, so the optimal subsidy must be positive in Cournot competition and is likely to be negative in Bertrand competition.

Turning to the second-best case, the optimal emissions tax becomes:

$$t \;=\; D_z + \Pi_{a^*} \frac{da^*}{dk} z_k^{-1} + (s - \bar{s}) x_a \frac{da}{dz} \tag{62}$$

(where the term da/dz should now be interpreted as $\{da/dt\}/\{dz/dt\}$). Substituting back into (59) yields the first-order condition for investment at the optimum, which, except for the switch in strategic variable from z to k, is identical to (23):

$$\pi_k z_k^{-1} \;=\; D_z + (s - \bar{s}) x_a \frac{da}{dz} \tag{63}$$

It follows therefore that the conclusions in the text continue to hold.