Solution Methods for Macroeconomic Models

Assignment 1

Petr Sedláček

1 Objective

In this assignment you are asked to solve a DSGE model using linearization techniques with Dynare.

2 Model

The model is the standard "neoclassical growth model" with endogenous labor supply. In other words, this is the same model as in the lecture but where, in addition, we allow for a labor supply choice of the household. In particular, we assume

- a representative household maximizing expected life-time utility (derived from consumption and leisure): $\max_{\{c_t,k_{t+1},l_t\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_t \beta^t \left(\frac{c_t^{1-\gamma}}{1-\gamma} - \frac{l^{1+\eta}}{1+\eta} \right)$
  - where γ is the coefficient of relative risk aversion and η is the Frisch elasticity of labor supply
- the household owns the production technology which uses both capital and labor as input factors and is subject to productivity shocks: $y_t = z_t k_t^{\alpha} l_t^{1-\alpha}$
  - where $\alpha \in (0,1)$ and $\ln z_t = \rho \ln z_{t-1} + \epsilon_t$ is aggregate productivity, where $\epsilon_t \sim N(0,\sigma^2)$ and $\rho \in (0,1)$ is a persistence parameter and z_0 is given
- resources are spent on consumption and investment into accumulation of capital and, each period, a fraction (δ) of the capital stock depreciates: $c_t + k_{t+1} = y_t + (1-\delta)k_t$, with k_0 given

3 Assignment

3.1 Derive Optimality Conditions

As a starting point, derive the optimality conditions for capital, labor and consumption. You'll need these to construct the model block in Dynare and to set up the model structure in the DIY linearization.

After you've derived the optimality conditions, have a look through `SolutionMain.m` and `ModelDynare.mod` to make sure you understand the structure of the code. `SolutionMain.m` first sets the model parameters and then proceeds to solve the model using Dynare.

3.2 Dynare Solution

In the first part of `SolutionMain.m` you should do the following:

- set the model parameters to the following values: $\alpha = 1/3$, $\beta = 1.03^{-1/4}$, $\gamma = 2$, $\eta = 2$, $\delta = 0.025$, $\rho = 0.9$ and $\sigma = 0.01$.
- define the system of equilibrium conditions (equations as functions of endogenous variables) you derived above. This includes the Euler equation (we'll call it Ee), the resource constraint (we'll call it Rc) and optimal labor supply (we'll call it Ls).
  - to do so, each of the equilibrium conditions should have the following structure, e.g. "$Ee = @(x)\ \ XYZ$", where XYZ is for instance the Euler condition in steady state such that it is equal to 0 (i.e. put the left hand side of your condition to the right ;))
  - as inputs, we'll use the vector x which consists of consumption (1st element), capital (2nd element) and labor (3rd element). In other words, Ee, Rc and Ls are *functions* of consumption, capital and labor, which we can then manipulate later
- once we solve for the steady state of consumption, capital and labor, let's also define the steady state of output (yss) and investment (Iss)
- now, move on to `ModelDynare.mod` and fill in the model block and the initial values (steady states).
- if all goes well, you should get a figure with impulse response functions. Do they make economic sense?

3.3 Bonus

If you still have time, redo the impulse responses such that they are in percent deviations from steady state. You can either code up your own impulse responses in `SolutionMain.m` using the existing Dynare solutions. Or, you can change the Dynare code slightly to directly generate impulse responses in percent deviations.