Solving Heterogeneous Agent Models

Solution Methods for Macroeconomic Models

Petr Sedláček

Solution Methods for Macroeconomic Models

- Monday - Tuesday: Solving models with "representative agents"
  - ~~Linearization in theory and practice: Dynare~~
  - ~~Non-linear solutions methods: value function iteration, projection~~
  - Analyzing models: parameterization/estimation, simulation/IRFs
- Wednesday - Thursday: Solving models with "heterogeneous agents"
  - Models without aggregate uncertainty: basic algorithm
  - Models with aggregate uncertainty: key issues and alternatives
- Friday: "Final assignment"
  - Solve/estimate model with heterogeneous firms and aggregate uncertainty

Overview for today

Non-linear solution methods

- Motivation
- Simple heterogeneous agent model without aggregate uncertainty
- Basic solution algorithm

Motivation

Top wealth shares in U.S.

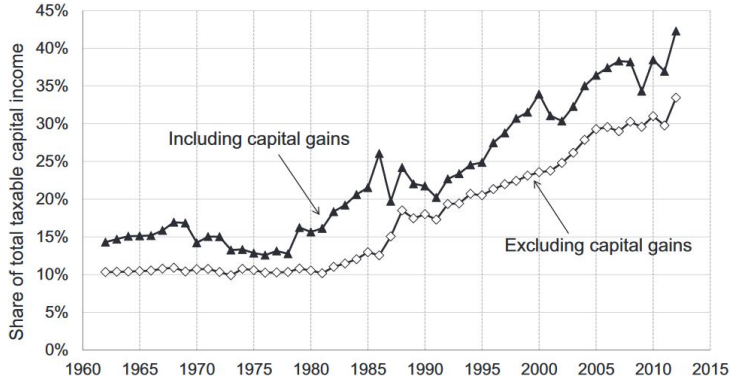
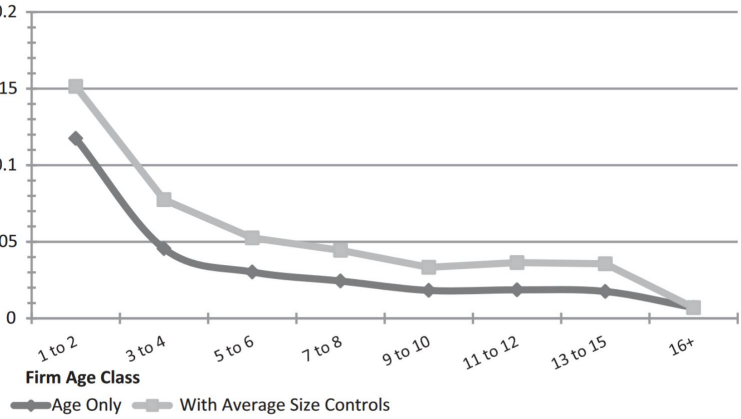


FIGURE III

The Top 0.1% Taxable Capital Income Share in the United States, 1962–2012

Source: Saez, Zucman (2016).

Employment growth rates of firms in U.S.



Source: Haltiwanger et al. (2013).

Motivation

Does heterogeneity actually matter?

Does heterogeneity matter?

Better understanding of existing channels

- effects of monetary policy:
  - following a reduction in (real) rates
  - aggregate consumption increases

In rep-agent models the above is driven by

- direct effects: less saving, more borrowing
- key mechanism: intertemporal substitution
- however, this is at odds with empirics
  - Campbell, Mankiw (1989), Yogo (2004), Canzoneri et al. (2007)

Does heterogeneity matter?

Better understanding of existing channels

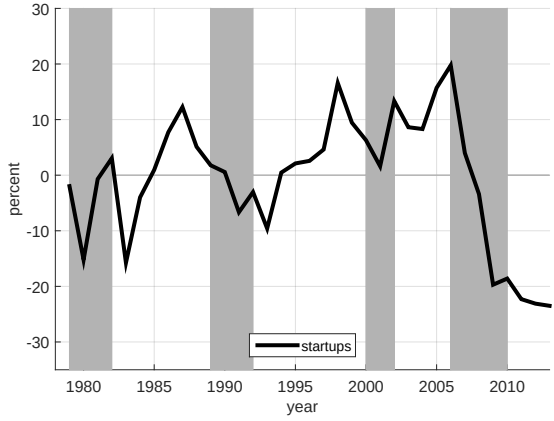
- effects of monetary policy:
  - following a reduction in (real) rates
  - aggregate consumption increases

In hetero-agent models the above is driven by

- indirect effects: in GE labor demand (income) expands
- key mechanism: distribution of earnings and (il)liquid wealth
- consistent with empirical evidence
  - Johnson et al. (2006), Parker (2014), Cloyne and Surico (2016)

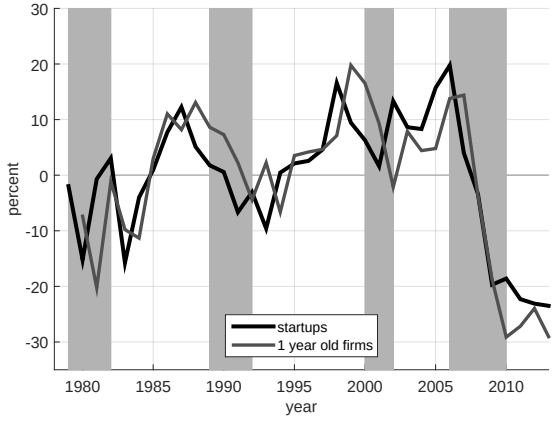
Discovering new channels

Employment in deviations from mean (by firm age)



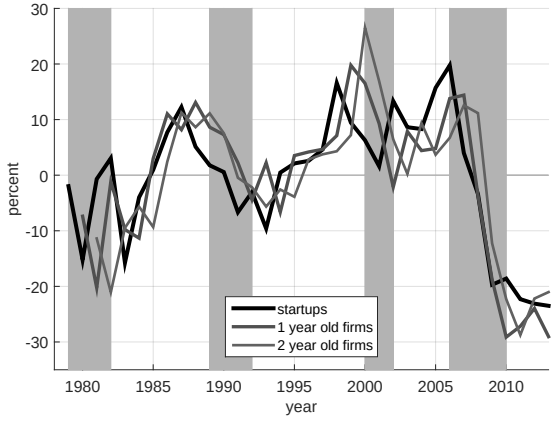
Discovering new channels

Employment in deviations from mean (by firm age)



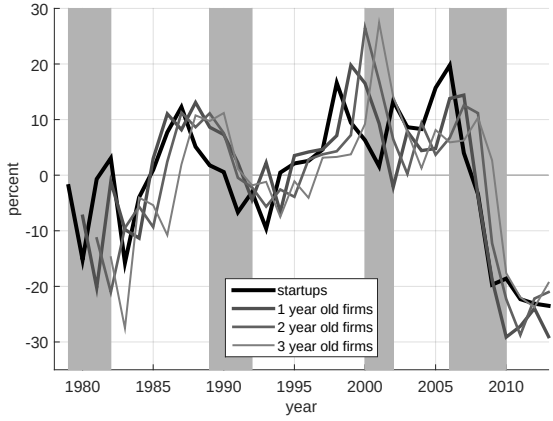
Discovering new channels

Employment in deviations from mean (by firm age)



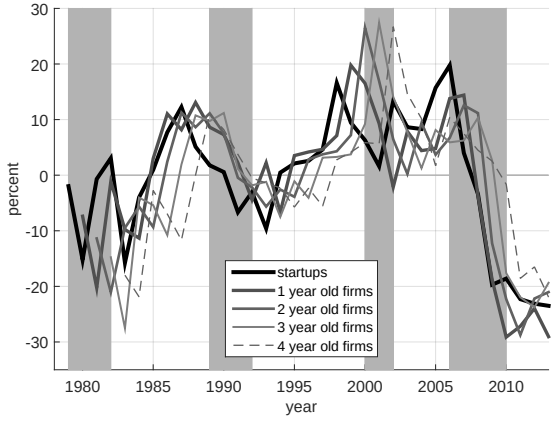
Discovering new channels

Employment in deviations from mean (by firm age)



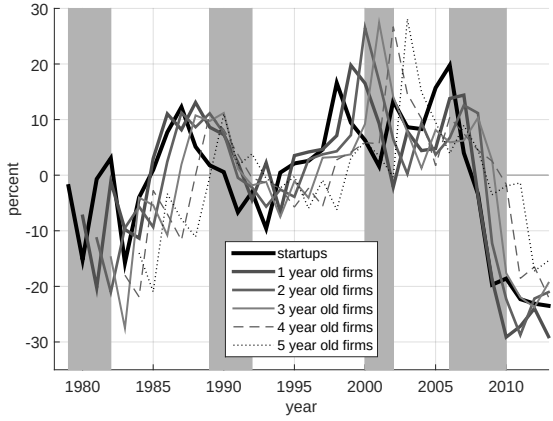
Discovering new channels

Employment in deviations from mean (by firm age)



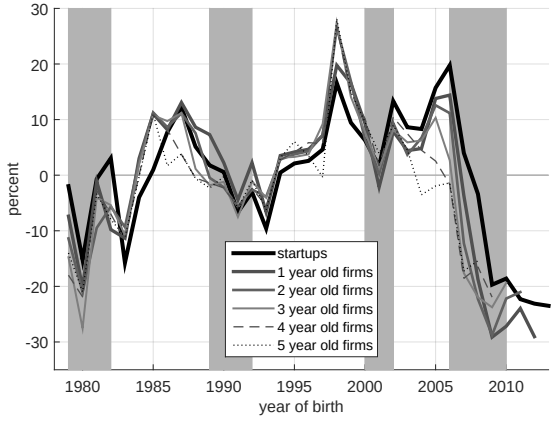
Discovering new channels

Employment in deviations from mean (by firm age)



Discovering new channels

Employment in deviations from mean (by firm age)



Does heterogeneity matter?

Discovering new channels

- firm cohort effects incredibly persistent
- recession-born cohorts remain weak
- even after economy recovers and vice versa
- key mechanism: changes in distribution of growth potential

Macro model with firm dynamics

- consistent with above empirical pattern
- $\rightarrow$ startup conditions explain aggregate trends
  - Sedláček and Sterk (2017)

Heterogeneity matters!

Important distinction between

- theoretical/qualitative results
- their quantitative implications

Krusell-Smith (1998) is the classical example

- incomplete markets
- $\rightarrow$ not possible to aggregate to rep-agent economy
- still, aggregate dynamics very similar to RA economy

Heterogeneity matters! But...

Effects on

- aggregates and asset prices are often small
  - infinitely-lived agents
  - general equilibrium feedback effects
- individual outcomes typically very important
  - e.g. costs of business cycle fluctuations very large individually
  - but still difficult to make them matter at the aggregate level

Motivation

Brief history of heterogeneous agent models

A brief history of hetero-agent models

- "first-generation" hetero-models 1990's and early 2000's
- "second-generation" hetero-models after Great Recession

First generation hetero-models

Move away from representative agent framework

- incorporate heterogeneity from micro data
- mainly income and wealth heterogeneity

Macro models with a distribution

- that distribution potentially moves over time
- and responds to macro policies and shocks
  - e.g. Aiyagari, Bewley, Huggett, Krusell-Smith, den Haan, Hopenhayn-Rogerson ...

Can speak to issues such as

- who gains (looses) most from growth (recessions)?
- welfare analysis

First generation hetero-models cont.

Typically, however, these models find that

- heterogeneity doesn't matter for aggregates
  - mainly because rich are just scaled up poor!
  - large are just scaled up small!
  - $\rightarrow$ inequality does not alter aggregate dynamics
  - $\rightarrow$ firm distributions don't matter for aggregate dynamics
- hard to believe that heterogeneity does not matter in the data
  - rich are not just scaled up poor!
  - and large are not just scaled up small!

Second generation hetero-models

These models take micro data more seriously

- household balance sheets, credit constraints, non-convexities
- life-cycle dynamics, fluctuations in composition of types etc

Typically find that heterogeneity does matter for macro!

- monetary policy example: Kaplan, Moll, Violante (2018)
- firm dynamics example: Sedláček, Sterk (2017)

Motivation

Terminology and avoiding complexity

Some terminology

Types of heterogeneneity

- ex-post heterogeneity
  - ex-ante they are identical
  - they are, however, facing idiosyncratic shocks
  - $\rightarrow$ making them ex-post heterogeneous
- ex-ante heterogeneity
  - from the onset, agents are different
  - and possibly also face idiosyncratic shocks

Which is easier to handle conceptually?

Before you start writing your model

Many models with heterogeneity are “simple” to handle

- search models
- several “two-agent” models
- model with heterogeneity only within the period
- partial equilibrium models

Overview of this lecture

Intro into heterogeneous agent models

1. ~~Motivation~~
2. Simple heterogeneous agent model without aggregate uncertainty
3. Basic solution algorithm

Simple heterogeneous agent ("Aiyagari") model

Agents

- representative firm
  - uses aggregate labor and capital in production

$$Y_t = K_t^{\alpha} L_t^{1-\alpha}$$

  - pays competitive wages and interest rate for labor and capital
- ex-ante identical workers
  - each supply unit of labor to firm
  - hit by idiosyncratic productivity shocks $\epsilon_{i,t}$
  - $\rightarrow$ ex-post heterogeneous

Environment

Markets are incomplete

- cannot insure away individual risk
- can save in capital
- with borrowing constraint $k_{i,t+1} \geq 0$

For now, no aggregate uncertainty

Firm problem

Maximizes profits

- choose aggregate capital and labor inputs
- results in standard competitive prices

$$r_t = \alpha K_t^{\alpha-1} L_t^{1-\alpha}$$

$$w_t = (1-\alpha) K_t^{\alpha} L_t^{-\alpha}$$

Maximize utility s.t. budget and borrowing constraint

$$\max_{\{c_{i,t},k_{i,t+1}\}_{t=0}^{\infty}} \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \ln(c_{i,t})$$

s.t.

$$c_{i,t} + k_{i,t+1} = r_t k_{i,t} + w_t \epsilon_{i,t} + (1-\delta) k_{i,t}$$

$$\epsilon_{i,t+1} = 1 - \rho + \rho \epsilon_{i,t} + \eta_{i,t+1}, \quad \eta_{i,t} \sim N(0, \sigma_\epsilon^2)$$

$$k_{i,t+1} \geq 0$$

Equilibrium

No aggregate risk

- $K_t = K$ and $L_t = L$
- also means that $w_t = w$ and $r_t = r$

What are the equilibrium masses of labor and capital?

- labor is fixed, normalize it to $L = 1$
- equilibrium capital given by demand and supply
  - firm demands in accordance with $r = \alpha K^{\alpha-1}$
  - individuals take prices as given
  - and decide on consumption and savings

Overview of this lecture

Intro into heterogeneous agent models

1. ~~Motivation~~
2. ~~Simple heterogeneous agent model without aggregate uncertainty~~
3. Basic solution algorithm

Basic solution algorithm

What do we need to know?

What do we need to know when solving for K?

- the entire joint distribution of
  - capital holdings and
  - idiosyncratic productivity shocks
- $\rightarrow$ gives us individual choices
- $\rightarrow$ next period's aggregate capital

Main idea of solution algorithm

1. guess value for r
   - implies values for K^D and w
2. solve individual problem with given r and w
3. simulate economy and calculate aggregate capital K^S
4. compare K^D and K^S
   - if $K^D = K^S \rightarrow$ stop
   - if $K^D \neq K^S \rightarrow$ update guess for r and go to 2

How to update r?

- if $K^S < K^D \rightarrow \uparrow$ r
- let r^q be the guess of r in the qth iteration
- $r^{q+1} = r^q + \lambda(K^D - K^S)$
  - the above may not be very efficient
  - λ may need to be very small to ensure convergence
- alternative (for) updating?
  - $r^{q+1} = r^q(1 + \lambda(K^D - K^S)/K^D)$
  - use equation solver to get r from $K^D(r) = K^S(r)$
  - bisection method

Basic solution algorithm

Using perturbation to solve model?

How to solve the individual problem

- borrowing constraint makes it a bit problematic
- $\rightarrow$ projection methods or VFI (or continuous time)
- what about perturbation and Dynare?

Need to smooth the borrowing constraint for perturbation

- replace it with a "penalty function"
- introduce an additional term to utility

$$-\frac{\zeta_1}{\zeta_0}\exp(-\zeta_0 k_{i,t}) - \zeta_2 k_{i,t}$$

- Euler equation becomes

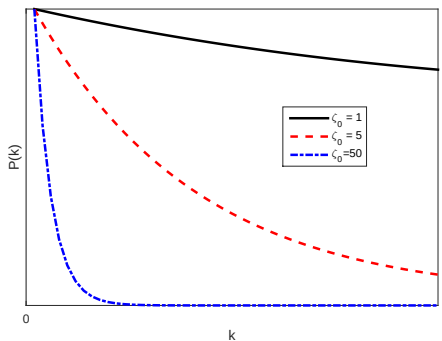
$$c_{i,t}^{-1} = \zeta_1 \exp(-\zeta_0 k_{i,t}) - \zeta_2 + \mathbb{E}_t \beta c_{i,t+1}^{-1}(r_{t+1} + 1 - \delta)$$

Interpretation of penalty function $P(k)$

- true constraint
  - $P(k) = 0$ if $k \geq 0$
  - $P(k) = \infty$ if $k < 0$
- penalty function more flexible
  - $P(k) = \frac{\zeta_1}{\zeta_0} \exp(-\zeta_0) + \zeta_2 k$
    - inequality constraint implemented for high ζ_0
    - what is the role of ζ_2?
    - from Euler equation, if $\zeta_2 = \zeta_1 \exp(-\zeta_0 \overline{k})$
    - steady states of true and penalty-function model conincide

Penalty function parametrizations

- different values of ζ_0 with
  - ζ_1 to normalize penalty at minimal capital
  - ζ_2 to equalize steady states across parametrizations



Aiyagari wrap-up

- we know how to solve the Aiyagari model
- we can implement the solution even with perturbation
- what makes it different (tougher) from a rep-agent model?
- nevertheless, it wasn't so difficult
  - and it is relatively fast
- so what's the big deal about heterogeneous agent models?

Taking Stock

Heterogeneous agent model without aggregate uncertainty

- solution of “individual problem” same as before
- key question is model equilibrium
- simple algorithm (guess-verify-guess again)

Overview for today

Intro into heterogeneous agent models

1. ~~Motivation~~
2. ~~Simple heterogeneous agent model without aggregate uncertainty~~
3. ~~Basic solution algorithm~~


That's all Folks!