Lecture 0: Foundations

Heterogeneous Agents in Macroeconomics

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Gorman (1961)

Finite set $\mathcal{N}$ with cardinality N of agents, indexed by i. Denote $y_i(p, \omega_i)$ as preference for a homogenous good of agent i, given price $p \in \mathbb{R}^1$, and wealth ω_i. Then, aggregate demand:

$$Y(p, \omega_1, \ldots, \omega_{N)} = \sum_i^N y_i(p, \omega_i) \underbrace{\equiv Y(p, \sum_i^N \omega_i)}_{?}$$

Gorman (1961)

Aggregate-wealth-preserving re-distribution. Total differentiation of Y:

$$\frac{d\,Y(p,\sum_i^N \omega_i)}{d\,\omega_i} = 0 \to \sum_i^N \frac{\partial\, y_i(p,\omega_i)}{\partial\, \omega_i} d\,\omega_i = 0$$

True iff:

$$\frac{\partial\, y_i(p,\omega_i)}{\omega_i} = \frac{\partial\, y_j(p,\omega_j)}{\omega_j} \quad \forall i,j \in \mathcal{N}$$

Strong static aggregation if linear Engel curves = MPC homogeneity.

Argument in Gorman (1961) is through indirect utility functions.

Rubinstein (1974)

Agent maxes consumption c_t given HARA utility, discount factor β, wealth ω_t, risky asset allocation α_t, risky return R_t^a, risk-free return R_t^f, and linear absolute risk aversion $RA(c) \equiv -U(c)''/U(c)' = \frac{1}{A+Bc}$:

$$\max_{\{c_t, \alpha_t\}} \mathbb{E} \left(\sum_t^T \beta^t \mathcal{U}(c_t) \right)$$

$$\text{s.t.} \quad \omega_{t+1} = (\omega_t - c_t) \left(\alpha_t R_t^f + (1 - \alpha_t) R_t^a \right)$$

Rubinstein (1974)

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Strong dynamic aggregation if:

1. Homogeneous curvature B (necessity).
2. Homogeneous ω_0, β, and $\mathcal{U}$ (sufficiency).
3. Homogeneous β and $B \neq 0$ (sufficiency).
4. Complete markets & $B = 0$ (sufficiency).
5. Complete markets & $\omega_{0j} = \omega_0$ & $B = 1$ & $A = 0$ (sufficiency).

Need condition (1) plus any of (2)-(5).

Constantinides (1982)

Debreu (1959) model: *m* consumers indexed by i, *n* firms indexed by j, l goods indexed by h, wealth endowments $\omega_i \equiv (\omega_{i,1}, \ldots, \omega_{il})$, shares in firms s_{in} with $\sum_m s_{ij} = 1$, consumption $x_i \equiv (x_{i1}, \ldots, x_{il}) \in \mathcal{X}_i$, production $y_j \equiv (y_{j1}, \ldots, y_{jl}) \in \mathcal{Y}_j$, concave utility $\mathcal{U}$ over consumption of goods, price vector $p \equiv (p_1, \ldots, p_l)$.

Equilibrium is the price vector p^* such that $(m+n)$-tuple $[(x_i^*)_{i=1}^m, (y_j^*)_{j=1}^n]$ achieves: (1) consumers max utility subject to budget constraint and $\mathcal{X}$, (2) firms max profit subject to $\mathcal{Y}$, (3) markets clear. Under standard Arrow-Debreu assumptions, the equilibrium exists and is Pareto optimal.

Constantinides (1982)

There exists a vector of positive numbers $\lambda \equiv (\lambda_1, \ldots, \lambda_m)$ such that the solution to (1) below is $(x_i) = (x_i^*)$ and $(y_j) = (y_j^*)$:

$$\max_{x,y} \sum_{i=1}^{m} \lambda_i \mathcal{U}_i(x_i)$$

subject to

$$\begin{aligned} & y_j \in \mathcal{Y}_j, \quad \forall j \\ & x_i \in \mathcal{X}_i, \quad \forall i \\ & \sum_i x_{ih} = \sum_j y_{jh} + \sum_i \omega_{ih}, \quad \forall h \end{aligned} \tag{1}$$

Constantinides (1982)

Problem (1) is equivalent to (2) below:

$$\max_y \left[\max_x \sum_{i=1}^{m} \lambda_i \mathcal{U}_i(x_i) \right]$$

subject to

$$\begin{aligned} y_j &\in \mathcal{Y}_j, \quad \forall j \\ x_i &\in \mathcal{X}_i, \quad \forall i \\ \sum_i x_{ih} &= \sum_j y_{jh} + \sum_i \omega_{ih}, \quad \forall h \end{aligned} \tag{2}$$

Constantinides (1982)

Define aggregate consumption $z \equiv (z_1, \ldots, z_l)$ and good-specific total consumption $z_h \equiv \sum_i x_{ih}$. Now solve (3) below:

$$\mathcal{U}(z) \equiv \max_x \sum_{i=1}^{m} \lambda_i \mathcal{U}_i(x_i)$$

subject to

$$\begin{aligned} & x_i \in \mathcal{X}_i, \quad \forall i \\ & \sum_i x_{ih} = z_h, \quad \forall h \end{aligned} \tag{3}$$

Constantinides (1982)

Finally, solve (4) below:

$$\max_{z,y} \mathcal{U}(z)$$

subject to

$$\begin{aligned} y_j &\in \mathcal{Y}_j, \quad \forall j \\ \sum_j y_{jh} + \omega_h &= z_h, \quad \forall h \end{aligned} \tag{4}$$

Given $\lambda \equiv (\lambda_1, \ldots, \lambda_m)$, if m consumers are replaced by one representative agent with utility over aggregate consumption $\mathcal{U}(z)$, endowment equal to the sum of m consumers' endowments, and shares the sum of the m consumers' shares, then price vector p^* and the associated $(1+n)$-tuple $(\sum_i x_i^*, y_j^*)$ is an equilibrium of Problem (4).

Weak static aggregation under complete markets.

Deaton and Paxson (1994)

Discount factor is $\delta = \frac{1}{(1+r)}$, C^* is the bliss level, $\omega_0 \geq 0$ is initial endowment, quadratic utility. Permanent and transitory shocks. Agent solves:

$$\max_{C} \mathbb{E}_0 \left[-\frac{1}{2} \sum_{t=1}^{T} \delta^t \left(C^* - C_t\right)^2 \right]$$

subject to:

$$\begin{aligned} \sum_{t=1}^{T} \delta \left(y_t - C_t\right) + \omega_o &= 0 \\ y_t &= y_t^p + \epsilon_t \\ y_t^p &= y_{t-1}^p + \eta_t \end{aligned} \tag{5}$$

Deaton and Paxson (1994)

First-differencing the solution:

$$\Delta C_t = \eta_t + \frac{1}{\sum_{\tau=0}^{T-t} \delta^\tau} \epsilon_t \approx \eta_t \tag{6}$$

Assume $\text{cov}_i \left(C_{t-1}^i, \eta_t^i\right) = 0$. Then:

$$\text{var}_i \left(C_t^i\right) - \text{var}_i \left(C_{t-1}^i\right) \approx \text{var}\left(\eta_t\right) \tag{7}$$

Within-cohort rise in consumption inequality is the variance of permanent shocks. Complete markets benchmark: consumption inequality should not rise within cohorts.

The permanent income model:

1. Consumption inequality rises linearly within cohorts.
2. Substantial, uninsured idiosyncratic risk.

Deaton and Paxson (1994)

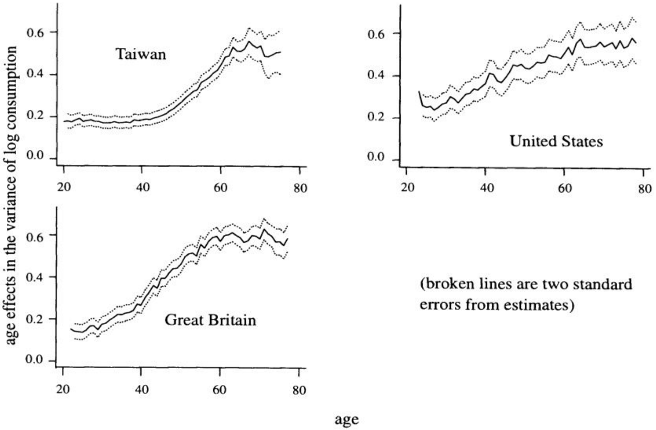

Fig. 4.—Age effects (and confidence bands) for the variance of log consumption

Blundell, Pistaferri, and Preston (2008)

General empirical model:

$$\Delta c_t = \alpha \eta_t + \beta \frac{1}{\sum_{\tau=0}^{T-t} \delta^\tau} \epsilon_t$$

1. $\alpha = \beta = 1$: permanent income model.
2. $\alpha = \beta = 0$: complete markets.

BPP (2008) estimate: $\hat{\alpha} = \frac{2}{3}$.

Three general identification problems:

Issue 1: accumulation of wealth and the precautionary savings motive lower α.

Issue 2: permanent shocks are anticipated, i.e. news about future shocks.

Issue 3: shocks are less persistent and AR(1) process decays exponentially.

Takeaway

A representative agent can be constructed with complete markets.

Moreover, with linear Engel curves, equilibrium does not depend on (re-distribution).

Existence of a representative agent by itself does not imply irrelevance of (re-)distribution.

The choice of λ and the distribution of ω can influence the aggregate outcome.

TAKEAWAY

Complete markets are unrealistic.

The permanent income model may be a better “standard model”.

Substantial idiosyncratic risk and a single risk-free asset to insure.

May be too extreme. Reality is more flexible.

Either agents have access to devices to partially insure against shocks or ...

... idiosyncratic risk is persistent but perhaps not permament.