Lecture 2: Firms

Heterogeneous Agents in Macroeconomics

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HOPENHAYN (1992)

A continuum of firms that produce a homogenous good.

Exogenous demand system: inverse demand function $D(Q)$ and price of labor input $W(N)$. $D(.)$ is continuous, strictly decreasing, and $\lim_{x\to\infty} D(x) = 0$. $W(.)$ is continuous, non-decreasing, and strictly bounded above zero.

Firms take output and input prices p and w as given. Firm-level output: $q = f(a, n)$.

$a \in S \equiv [0, 1]$ is firm-level productivity, first-order Markov, independent across firms, with conditional distribution $F(a'|a)$. If $a_1 > a_2$ then $F(.|a_1)$ first-order stochastic dominates $(F.|a_2)$.

Hopenhayn (1992)

Define $\pi(a,p,w,)$, $q(a,p,w)$, and $n(a,p,w)$ as profit, output supply, and input demand functions, respectively.

$q(.)$ and $n(.)$ are single valued, strictly increasing in a, and continuous. $\pi(.)$ is continuous and strictly increasing in a.

Incumbent firms must pay a period-by-period fixed cost of operation c_f.

Static, "intratemporal" profits:

$$\pi(a,p,w) := \max_n \left[pf(a,n) - wn - c_f\right] \tag{1}$$

Define $n^*(a,p,w)$ as optimal employment and $q^*(a,p,w)$ as optimal output policy functions.

Hopenhayn (1992)

Incumbent firms choose to exit if their state a falls below some reservation level x.

Exiting firms leave with a present value of net worth equal to 0.

Entrants must pay an entry fee c_e, which is a sunk cost. Afterwards, entrants draw an initial productivity shock.

Initial productivity shocks a_0 are drawn from a distribution $G(a)$.

Incumbent

Incumbent firms take price sequences $z \equiv \{p_s, w_s\}_{s \geq 0}$ as given and solve the recursive problem below:

$$v_t(a, z) = \max\left[\pi(a, p_t, w_t)\right] + \beta \max\left[0, \int v_{t+1}(a', z) dF(a'|a)\right] \tag{2}$$

where $0 < \beta < 1$ is a discount factor.

Exits

Firms decide to exit *before* observing next period's productivity draw.

The exit threshold is:

$$x_t = \begin{cases} \text{inf}\left[a \in S : \int v_{t+1}(a', z)dF(a'|a) \geq 0\right] \\ 1 \quad \text{if the set is empty} \end{cases} \tag{3}$$

The firm exits immediately after $a_t < x_t$ the first time this occurs.

Entrants

Entrants come from a pool of ex-ante identical candidates.

Define M_t the mass of entrants.

Entry occurs until expected discounted profits net of entry cost are zero:

$$v_t^e(z) := \beta \int v_t(a,z)dG(a) \leq c_e \tag{4}$$

with strict equality when $M_t > 0$.

Distribution

Denote $\mu_t(S)$ the measure of incumbents. Total size of the industry.

Denote $\mu_t(\mathcal{A})$ the mass of firms with shocks in $\mathcal{A}$ for any borel set $\mathcal{A} \subset S$.

μ_t is an endogenous, time-varying state variable.

Law of motion of the distribution of incumbents with $a \in [0, a')$:

$$\mu_{t+1}([0, a')) = \int_{a \geq x_t} dF(a'|a) d\mu_t(a) + M_{t+1} G(a') \tag{5}$$

where the dependency on z is dropped for presentability.

Distribution in Matrix Notation

$$\boldsymbol{\mu}_{t+1} = \boldsymbol{\Phi}_t \boldsymbol{\mu}_t + M_{t+1} \boldsymbol{g} \tag{6}$$

$\boldsymbol{\Phi}_t$ defines a bounded linear operator on the space of bounded measures defined by $\boldsymbol{\Phi}_t \mu_t(\mathcal{A}) = \int \boldsymbol{\Phi}_t(a, \mathcal{A}) d\mu_t(a)$ for all $\mathcal{A} \subset S$.

With N elements, $\boldsymbol{\Phi}_t$ is an $N \times N$ transition matrix, determined by a_t process and exit threshold x_t.

$\boldsymbol{\mu}_t$ and $\boldsymbol{g}$ are $N \times 1$ vectors.

M_{t+1} is a scalar.

Aggregation

Aggregate labor input demand function:

$$N^d(\mu, p, w) = \int n(a, p, w) d\mu(a) \tag{7}$$

Recall that aggregate demand $p = D(Q^d)$ is exogenous.

Industry supply:

$$Q^s(\mu, p, w) = \int q(a, p, w) d\mu(a) \tag{8}$$

Market clearing ensures that $Q^d = Q^s$.

Competitive Equilibrium

Given an initial measure μ_0, the competitive equilibrium for the industry consists of bounded sequences $\{p_t^*, w_t^*, Q_t^*, N_t^*, M_t^*, x_t^*, \mu_t^*\}_{t=0}^{\infty}$ such that:

1. Markets clear
   1.1 $p_t^* = D(Q_t^*)$ and $w_t^* = W(N_t^*)$.
   1.2 $Q^s(\mu_t^*, p_t^*, w_t^*) = Q_t^*$ and $N^d(\mu_t^*, p_t^*, w_t^*) = N_t^*$.
2. Exit rule x_t^* satisfies (3).
3. New entry condition holds: $v_t^e(z^*) \leq c_e$, with equality if $M_t^* > 0$.
4. Distributions are optimal given the entry and exit rule: μ_t^* is defined by (6) conditional on μ_0, M_t^*, and x_t^*

In the rest of the lecture we study *stationary* equilibria, defined by:

$$\{p^*, w^*, Q^*, N^*, M^*, x^*, \mu^*\}$$

Stationary Equilibrium

Given any μ with $\mu(S) > 0$ there exists a unique aggregate input-output vector (N, Q) and prices $(p(\mu), w(\mu))$ that satisfy condition 1 of equilibrium's definition.

There exists a unique solution v to the below program:

$$v(a,\mu) = \pi(a,\mu) + \beta \max \left[0, \int v(a',\mu)dF(a'|a)\right] \tag{9}$$

The exit point $0 < x < 1$ satisfying (3) and $(p(\mu), w(\mu))$:

$$\int v(a',\mu)dF(a'|x) = 0 \tag{10}$$

Necessary condition for equilibrium with positive entry:

$$\int v(a,\mu)dG(a) = c_e \tag{11}$$

Stationary Equilibrium

For some $x > 0$ and $M > 0$ there exists a unique time-invariant measure $\mu^s(x, M)$ that satisfies:

$$\boldsymbol{\mu} = \boldsymbol{\Phi}_x \boldsymbol{\mu} + M\boldsymbol{g} \tag{12}$$

Solving the above yields:

$$\boldsymbol{\mu} = M\left(\boldsymbol{I} - \boldsymbol{\Phi}_x\right)^{-1} \boldsymbol{g}$$

with $\boldsymbol{I}$ the identity matrix.

A stationary equilibrium with positive entry is given by $(x, M, \boldsymbol{\mu})$ that satisfy (9)-(12).

Some Equilibrium Properties

Define $M_1(x)$ as the entry rule such that the exit rule x is optimal.

Define $M_2(x)$ as the entry rule that satisfies the free entry condition.

Equilibrium with positive entry exists iff $M_1(x^*) = M_2(x^*)$.

Corner 1: $\mu^* = 0$ (empty industry) is an equilibrium. Occurs when c_e too high. Implies *no* exit.

Corner 2: $x = 0$ (firms live forever) is an equilibrium.

Equilibrium Computation

Set the price of labour as the numeraire. Denote iteration count j:

1. Guess output price $p_{j=0}$. Conditional on the price, solve the partial-equilibrium problem of the incumbent:

$$v_t(a, p_{j=0}) = \max\left[\pi(a, p_{j=0})\right] + \beta \max\left[0, \int v_{t+1}(a', p_{j=0}) dF(a'|a)\right]$$

2. Verify that the free-entry condition holds:

$$\beta \int v(a', p_{j=0}) dG(a') = c_e$$

   If expected discounted profits are too low, then raise the price to $p_{j=1} > p_{j=0}$. Iterate until p^* that satisfies both steps 1. and 2. is found. Store the optimal exit rule $x^*(p^*)$.

Equilibrium Computation

3. Conjecture entrants $M_{j=0}$. Solve for the stationary distribution $\mu_{j=0}$ for all a':

$$\mu_{j=0}([0,a')) = \int_{a \geq x^*(p^*)} dF(a'|a) d\mu_{j=0}(a) + M_{j=0} G(a')$$

4. Given the obtained $\mu_{j=0}$, compute the aggregates and check market clearing:

$$Q^s(\mu_{j=0}, p^*) = \int q(a, p^*) d\mu_{j=0}(a) = Q^d(p^*)$$

5. If supply is too low, increase new entrants $M_{j=1} > M_{j=0}$, and re-do Step 3. Iterate until optimal market-clearing M^* is found.

Comparative Statics

Unanticipated, exogenous increase in the cost of entry c_e:

1. From the free-entry condition, discounted profits must $\uparrow$.
2. Number of new entrants $M \downarrow$.

Comparative Statics

The price effect:

1. Equilibrium price rises, as seen from the equilibrium computation Step 2.
2. Given higher prices, output and demand for labour increase.

The selection effect:

1. Incumbent makes more profit, the reservation value falls, likelihood of exit falls, $x \downarrow$.
2. Less selection. Barriers to entry protect the incumbent.
3. Average firm age $\uparrow$. More "deadwood" firms.
4. Share of firms with low productivity increases. Less "cleansing of the unproductive".

The net effect of the two forces depends on calibration.

Some empirical evidence that the selection effect channel is strong.

Takeaway

Hopenhayn (1992) is a workhorse model of firm dynamics.

Idiosyncratic firm productivity risk belongs to the “ex-post” heterogeneity class of models.

Endogenous distribution of firms analogous to what we learnt with households.

Endogenous intensive and extensive (entry and exit) margins. The latter is new.

Numerous implications for cross-sectional and time-series empirical tests.