Lecture 3: Banks

Heterogeneous Agents in Macroeconomics

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Coimbra and Rey (2023)

Ex-ante heterogeneous financial intermediaries (banks, for short), indexed by i, that live for 2 periods.

Limited liability, risk neutrality, deposit insurance, complete markets.

The role of banks: source deposits d_{it} from households, combine with own endowment ω, and invest into risky capital k_{it} or risk-free storage s_{it}.

The banking block is embedded into a standard general equilibrium macroeconomic framework.

TECHNOLOGY

Representative firm produces the final good:

$$Y_t = Z_t K_t^{\theta} \tag{1}$$

Z_t follows AR(1) process in logs and CDF of shocks ϵ is $F(\exp(\epsilon))$:

$$\log Z_{t+1} = (1 - \rho^z)\mu^z + \rho^z \log Z_t + \epsilon^z_{t+1}, \quad \epsilon^z_{t+1} \sim N(0, \sigma_z) \tag{2}$$

Return on capital under depreciation rate δ:

$$R^K_{t+1} = \theta Z_{t+1} K_t^{\theta - 1} + (1 - \delta) \tag{3}$$

Competitive wage:

$$W_t = (1 - \theta) Z_t K_{t-1}^{\theta - 1} \tag{4}$$

Preferences

Representative household solves the following problem:

$$\max_{\{C_t, S_t^H, D_t^H\}_{t=0}^{\infty}} \mathbb{E}_t \sum_{t=0}^{\infty} \mathcal{U}(C_t) \tag{5}$$

s.t.

$$C_t + D_t + S_t = R_t D_{t-1} + S_{t-1} + W_t - T_t \tag{6}$$

Exogenous labor supply, normalized to unity.

Value-at-Risk Heterogeneity

Occasionally binding Value-at-Risk constraint (VaR):

$$\mathcal{P}(\pi_{i,t+1} < \omega) \leq \alpha_i \tag{7}$$

where π is profit and ω the endowment.

Heterogeneity in the maximal probability of incurring losses, the VaR parameter: α_i.

$\alpha_i \in [\underline{\alpha}, \bar{\alpha}]$ is distributed according to continuous measure G.

Bank Balance Sheet

Balance sheet:

$$k_{it} + s_{it} = \omega + d_{it} \tag{8}$$

Cash flow statement:

$$\pi_{i,t+1} = R^K_{t+1} k_{it} + s_{it} - R_t d_{it} \tag{9}$$

R^K is the common Neoclassical return on capital. Why? Technological homogeneity + complete markets.

R is the common interest rate on household deposits. Why? Deposit insurance.

Bank Problem

Partial-equilibrium problem of the bank under limited liability:

$$\max_{\{k_{it}, d_{it}\}} \mathbb{E}_t \left[\max \left(0, \pi_{i,t+1} \right) \right] \tag{10}$$

subject to:

$$\mathcal{P}(\pi_{i,t+1} < \omega) \leq \alpha_i \tag{11}$$

$$k_{it} + s_{it} = \omega + d_{it} \tag{12}$$

$$\pi_{i,t+1} = R^K_{t+1} k_{it} + s_{it} - R_t d_{it} \tag{13}$$

Financial Stability

Limited liability truncates the profit function at zero.

Conditional on a given expected return, higher variance increase the option value of default:

$$\mathbb{E}_t\left[\max(0, \pi_{i,t+1})\right] \geq \mathbb{E}_t\left[\pi_{i,t+1}\right] \tag{14}$$

with strict inequality whenever probability of default is positive.

Extensive Margin

Potential entrants know their intrinsic α_i and take aggregate prices $\{R^K, R\}$ and quantities as given. For any $\mathbb{E}_t(R^K_{t+1}) \geq 1$:

Non-participating banks choose not to enter the capital market, only hold storage, and source zero deposits.

Safe banks choose to enter but invest only using the endowment, setting the (book) leverage ratio to unity.

Risky banks choose to enter, source deposits, lever up, and hold risky capital until the VaR constraint binds because they are risk neutral.

Risky Banks

Banks with the ex-ante α_i that is greater than some threshold α_t^L are risk-loving enough to become risky bankers.

Franchise value of the risky bank:

$$V_{it}^L = \max_{\{k_{it}, d_{it}\}} \mathbb{E}_t \left[\max \left(0, R_{t+1}^K k_{it} + s_{it} - R_t d_{it} \right) \right] \tag{15}$$

Safe Banks

Banks with ex-ante α_i that is lower than α_t^L but still greater than the threshold of non-participation α_t^N choose to invest into capital but do not source deposits.

Franchise value of the safe bank:

$$V_{it}^N = \max_{k_{it} \leq \omega} \mathbb{E}_t \left[R_{t+1}^K k_{it} + s_{it} \right] \tag{16}$$

Non-Participating Banks

Banks with ex-ante α_i that is lower than α_t^N choose to invest into the storage technology.

Franchise value of the non-participating bank:

$$V_{it}^O = \omega \tag{17}$$

Intensive Margin

Conditional on being a risky bank, risk neutrality guarantees the constraint always binds:

$$\mathcal{P}\left[\pi_{i,t+1} < \omega\right] = \alpha_i \tag{18}$$

One can solve for the book leverage ratio $\lambda_{it} \equiv \frac{k_{it}}{\omega}$:

$$\lambda_{it} = \frac{R_t}{R_t - \theta Z_{t+1} K_t^{\theta-1} F^{-1}\left(\alpha_i\right) + \delta} = \frac{R_t}{R_t - R_t^{\alpha_i}} \tag{19}$$

with R^{α_i} the realized return on investment for bank type i. One can show:

$$\frac{\partial \lambda_i}{\partial \alpha_i} > 0, \quad \frac{\partial \lambda_i}{\partial R} < 0, \quad \frac{\partial \lambda_i}{\partial R^K} > 0, \quad \frac{\partial \lambda_i^2}{\partial R \partial \alpha_i} < 0 \tag{20}$$

Extensive Margin in Equilibrium

Threshold of non-participation:

$$\alpha_t^N = F\left(\frac{\delta K_t^{1-\theta}}{\theta Z_{t+1}}\right) \tag{21}$$

Aggregate capital stock:

$$K_t = \int_{\underline{\alpha}}^{\bar{\alpha}} k_{it} dG(\alpha_i) = \int_{\alpha_t^L}^{\bar{\alpha}} k_{it} dG(\alpha_i) + \left[G(\alpha_t^L) - G(\alpha_t^N)\right]\omega \tag{22}$$

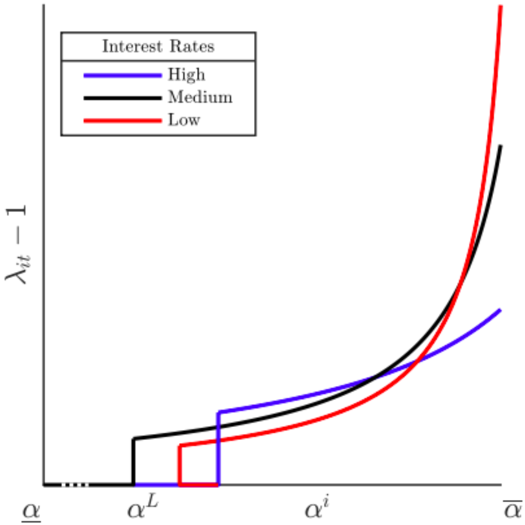
Conditional on (R_t, Z_t), implicit function for α^L:

$$\alpha_t^L = A(R_t, Z_{t+1}, K_t) \tag{23}$$

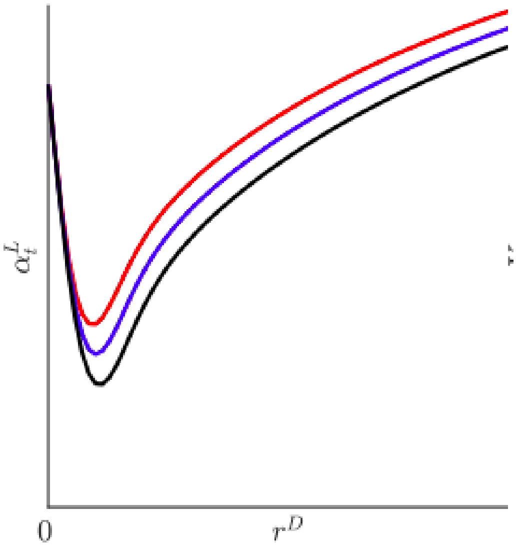
Assume $\alpha^N = 0$ and $\omega = 1$, then:

$$\frac{\partial \alpha^L}{\partial R}\left(\lambda^{\alpha^L} - 1\right) = \int_{\alpha^L}^{\bar{\alpha}} \frac{\partial \lambda^{\alpha}}{\partial R} dG(\alpha) - \frac{\partial K}{\partial R} \tag{24}$$

Partial Equilibrium



Partial Equilibrium



Monetary Policy

Monetary policy manages a deposit spread γ_t over wholesale funding $l_{i,t}$

$$R_t^L = R(1 - \gamma_t) \tag{25}$$

Wholesale funding a fixed proportion of deposits:

$$l_{it} = \chi d_{it} \tag{26}$$

γ_t follows an AR(1) process. Total liabilities:

$$f_{it} = (1 + \chi) d_{it} \tag{27}$$

Total cost of funding:

$$R_t^F = \frac{1 + \chi(1 - \gamma_t)}{1 + \chi} R_t \tag{28}$$

General Equilibrium

$$K_t = K^*(R_t^F, Z) \quad (29)$$

$$\alpha^L = \alpha^{L,*}(R_t^F, Z) \quad (30)$$

$$F_t = \int_{\alpha_t^L}^{\bar{\alpha}} (k_{it} - \omega) dG(\alpha_i) \quad (31)$$

$$D_t = \int_{\alpha_i^L}^{\bar{\alpha}} d_{it} dG(\alpha_i) = \frac{\int f_{it} dG(\alpha_i)}{1+\chi} \quad (32)$$

$$D_t = D_t^H \quad (33)$$

$$S_{t-1}^H + \int s_{it} dG(\alpha_i) + Y_t = C_t^H + \int c_{it} dG(\alpha_i) + I_t + S_t^H + \int s_{it} dG(\alpha_i) + T_t^F \quad (34)$$

$$T_t^F \equiv \int l_{it} dG(\alpha_i) - R_{t-1}^L \int l_{i,t-1} dG(\alpha_i) \quad (35)$$

Impact of Productivity Shocks

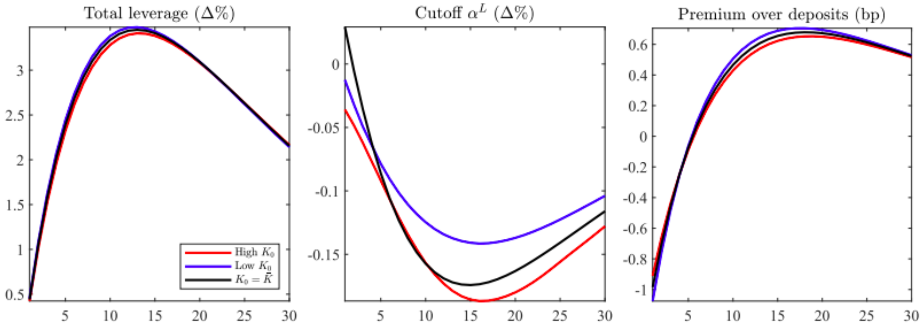


Figure 10

Shock to exogenous productivity

Impact of Monetary Shocks

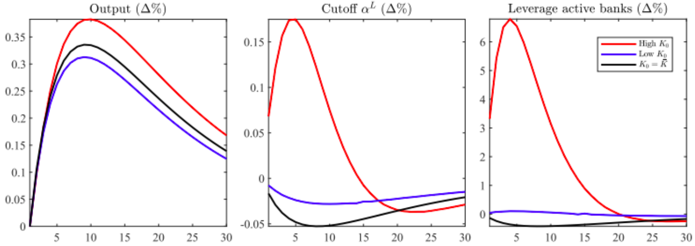


FIGURE 8

Monetary policy shock of 100 basis points to γ_t

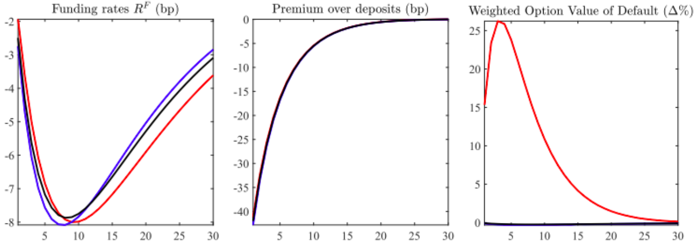


FIGURE 9

Monetary policy shock of 100 basis points to γ_t: financial variables

Takeaway

Risk aversion heterogeneity for financial intermediaries as induced via VaR differences.

Direct violation of the Gorman-Rubinstein necessity conditions for strong demand aggregation.

The representative agent assumption fails: behavior of the average bank $\neq$ average behavior in the distribution.

Aggregate state-dependency: the state of the financial cross section determines the total impact of aggregate shocks.